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A History of Ancient Mathematical Astronomy

Part Two

V . 2

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Part Two

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Book III

Egypt

§ 1. Introduction and Summary

Egypt has no place in a work on the history of mathematical astronomy. Nevertheless I devote a separate "Book" on this subject in order to draw the reader's attention to its insignificance which cannot be too strongly emphasized in comparison with the Babylonian and the Greek contribution to the development of scientific astronomy.

Egypt provides us with the exceptional case of a highly sophisticated civilization which flourished for many centuries without making a single contribution to the development of the exact sciences. In fact, however, this is not the exception but the rule. Nowhere within ancient civilizations known to us did the sciences originate independently, neither in pre-Hellenic nor in early Greek civilization, in the ancient Near East, on the Iranian plateau, nor in pre-Arian or Arian India – with the sole exception of Mesopotamia, probably in the early second millenium.[1] It is at this single center that abstract mathematical thought first appeared, affecting, centuries later, neighbouring civilizations, and finally spreading like a contagious disease.

The non-existence of any Egyptian science does not mean that Egypt has not left its traces on later developments. The mathematically very simple Roman techniques of land measurement (reflected in the writings of the so-called *agrimensores*) and the corresponding Greek literature (much of which is associated with the name of Heron) probably took its authority mainly from Egyptian tradition. The primitive, strictly additive, Egyptian way of computing with unit fractions had a detrimental effect throughout, even on Greek astronomy. For example in the Almagest one can find many cases where final results are given in unit fractions, abolishing for no good reason a more accurate value determined by consistent sexagesimal computation.

The most important contribution of Egypt to astronomy is undoubtedly the "Egyptian year" of a fixed length of 365 days. In antiquity this is the only larger time scale which satisfies the basic requirement of any reasonable unit of measurement, namely constancy of length. All other calendaric systems entangled time-reckoning with religious and political considerations, or with requirements of unforeseeable astronomical complexity (as the luni-solar calendars), or with both.

In Egypt the absence of scientific speculation kept an early primitive calendar alive which had the added advantage of practical simplicity. The Hellenistic astronomers fully realized the usefulness of the Egyptian calendar for computational purposes – just in time before the Augustan "reform" abolished for the civil calendar the intercalation-free arrangement. Thus Egyptian years remained the

[1] As far as I can see only China has developed a theoretical astronomy totally independent of Mesopotamian influences.

backbone for the computation of astronomical tables, not only in antiquity but deep into the Renaissance.

During the main part of Egyptian history, i.e. down to the Persian period, the civil calendar in Egypt was arranged in three seasons of four months each. The names of these seasons, "inundation" – "emergence" (of the fields from the water) – "dryness" demonstrate their agricultural origin, totally unrelated to astronomy. The "Sothic cycle" of 1460 years can be explained[2] as the result of connecting the agricultural year with a yearly recurring astronomical phenomenon, the heliacal rising of Sirius (= Sothis), which roughly coincides with the beginning of the inundation, whereas the beginning of the schematic Egyptian year of 365 days was "wandering" in the course of time through all seasons.

Only in the latest period appear the month-names which became familiar through Greek and Coptic documents and remain associated with the Egyptian years in Arabic and Latin astronomical treatises.[2a] The following table shows the Greek forms of the month-names in their relation to the original seasonal calendar[2b]

I	Thoth	'ḫt	I
II	Phaophi		II
III	Athyr		III
IV	Khoiak		IV
V	Tybi	prt	I
VI	Mekheir		II
VII	Phamenoth		III
VIII	Pharmouthi		IV
IX	Pachon	šmw	I
X	Payni		II
XI	Epiphi		III
XII	Mesore		IV
	5 epagomenal days		

Among the enormous mass of Egyptian inscriptions and papyri from all periods of Egyptian independent history there has not been found a single record of astronomical observations. It is significant that no reference is ever made to an Egyptian eclipse or to any other observation anywhere in the astronomical literature of the Greeks, including the Almagest. What is today commonly referred to as "Egyptian astronomy" consists mainly of schematic arrangements for the division of the night into "hours" (accidentally of rather uneven lengths) by means of extremely crude observations of certain stars. A cursory summary of such methods more than suffices for our present purposes.

The earliest of these schemes, known to us only from coffin lids (of the time from about 2100 to 1800 B.C.) relates stars, or groups of stars, by their risings to

[2] Cf. Neugebauer [1939].
[2a] These names are derived from divinities or their festivals associated with the month in question; cf. Černý [1943].
[2b] The customary pronounciation of the Egyptian names of the seasons is akhet, peret, shemu, respectively.

36 decades (of 10 days each) of the civil calendar and then uses these "*decans*" to indicate by their rising the consecutive hours of night. One can show that these constellations belong to a belt roughly parallel and somewhat south of the ecliptic.[3] When the Babylonian zodiac was introduced during early Hellenistic times into Egyptian celestial iconography these 36 decanal constellations were simply located as 36 ten-degree sections in the zodiac. These, then, are the "decans" or "faces" of Hellenistic, Hindu, and mediaeval astrology.

Although probably none of the Egyptian devices for measuring time ever attempted more than to secure some regularity in the temple services even these modest requirements could not be satisfied for any length of time by the simple scheme of consecutively rising decans. Thus, after some intermediate attempts still involving the decanal constellations, one changed to the observation of transits based on another selection of stars. Obviously this could have led to an accurate definition of sidereal time; but actually the procedure was so incredibly crude—observation of "transits" with respect to the head, the ears, and the shoulders of a sitting man as the reference system—that also this method had no scientific consequences whatever. From the position of Sirius within the calendar we can conclude that this method was introduced in the 15th century B.C.; in the actually preserved monuments,[4] the ceilings of royal tombs of the Ramesside period three centuries later, these lists of stars only serve as funeral decorations, exactly as the decanal stars were preserved only on coffin lids, in all probability in imitation of now lost earlier ceiling decorations of royal tombs, long after their actual use in the temple services.

Primitive sun dials and equally inaccurate water clocks[5] supplement the "astronomical" methods of Egyptian time reckoning. It is possible for us to reconstruct in outlines the unintentional steps which led from the decanal hours to a 12-division of night and daytime and thus produced eventually the "seasonal hours" of Hellenistic astronomy[6] and the 24-hour division of the day.

Of the Egyptian constellations only the distorted names of the decans were preserved in astrological context whereas the transit stars left no trace in later astronomical or astrological literature. The extreme inaccuracy of all aspects of Egyptian astronomy makes it impossible to identify any of its constellations, except for Sirius by means of its calendaric significance, for Orion from its characteristic arrangement of stars in the propinquity of Sirius, and for the seven stars of the Great Dipper depicted by the Egyptians as outlining an ox-leg.[7]

The well established fact that the Greeks had nothing to learn from native astronomy makes it desirable to investigate the basis for numerous ancient references to astronomical activities of "Egyptians". I do not mean, of course, the ever repeated stories about the hoary wisdom of the ancient orient, be it Egypt or Babylonia or any other region of fame. But it seems worth while to make an attempt to clarify what Ptolemy had in mind when he in his "Phaseis" refers to "the Egyptians" as observers of meteorological data in relation to fixed star phases.

[3] Cf. Neugebauer-Parker, EAT I, p. 97ff.
[4] Cf. Neugebauer-Parker, EAT II.
[5] Cf. Borchardt, Zeitm.
[6] Neugebauer-Parker, EAT I, p. 116ff. Cf. also below p. 706.
[7] Cf. Neugebauer-Parker, EAT I, p. 28 (Pl. 4, 6, 8, etc.); EAT III, p. 183ff.

One group of references to "Egyptians" is easy to explain: to the Romans all inhabitants of Egypt were simply "Egyptians".[8] This is nicely illustrated by a remark of Valerius Probus (end of first cent. A.D.) who calls the well-known courtier and astronomer Conon, Samius mathematicus, "Aegyptius natione".[9] Even a Byzantine encyclopaedist of the tenth century could have the same attitude and call Theon of Alexandria "*Αἰγύπτιος*".[10] In other cases as well it is obvious that the "Egyptians" are actually Greeks. When, e.g., Firmicus Maternus (in the 4th cent.) says[11] that the Egyptians call Saturn "Faenon", Jupiter "Faethontem", etc., then he only contrasts the original Greek names with the later astrological terminology. Similarly the term "comets" is plainly Greek although Theophrastus considers it a name given by the Egyptians.[12] The widest influence of this ambiguity of the term "Egyptians" was exercised to the present day by a remark of Macrobius about a supposedly Egyptian planetary model that assumed a heliocentric motion for the inner planets.[13]

The only case that does not readily fall into the preceding pattern is the reference by Ptolemy to "Egyptians" as observers of meteorological data in relation to fixed star phases.[14] It can hardly be doubted that for Ptolemy this meant native Egyptians in contrast to the Greek astronomers whom he mentions by name.[15] On the other hand it is clear that these observations concern only Greek constellations and have nothing to do with the ancient Egyptian configurations. Rehm suggested for these observations a period between Eudoxus and Hipparchus in view of the terminology used for wind directions and similar criteria.[16] At any rate it is clear that Ptolemy must have relied for the "Egyptian" observations on much older sources. This is confirmed by the fact that Pliny as well knows about "Egyptian" parapegmata.[17] But in contrast to Ptolemy, who says that the Egyptians observed in Lower Egypt, Pliny expressly includes Phoenicia, Cyprus, and Cilicia. Rehm considers this as reflecting the political situation at the height of the power of the Ptolemies,[18] but one could also think of an astronomical range of validity similar to the use of "Phoenicia" for planetary phases.[19] Finally a parapegma on an inscription in Miletus (early first cent. B.C.) mentions among its authorities Eudoxus and the Egyptians.[20] It is tempting to assume as the common source of these references a treatise that originated, perhaps in the second century B.C., in hellenistic Egypt and was therefore called "Egyptian" in contrast

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[8] Cf. H. Idris Bell, Egypt from Alexander the Great to the Arab Conquest (Oxford 1948), p. 70.
[9] Cf. Servius, in Vergilii carmina commentarii, ed. Thilo-Hagen, Vol. 3, 2 (1902), p. 330, 10. For Conon cf. also below p. 572.
[10] Suidas, ed. Adler I, 2, p. 702, 10.
[11] Mathes. II, 2 ed. Kroll-Skutsch, p. 42, 8–14.
[12] Weather Signs 57 (Loeb, Plants II, p. 432/433).
[13] Cf. below p. 695.
[14] Ptolemy refers nowhere to Egyptians for astronomical observations or theories (cf. the index in Ptol. Opera I, 2, p. 271 f.), only to calendaric concepts and to some astrological doctrines which he traditionally accepts as of Egyptian origin (cf. the index in Ptol., Opera III, 2, p. 71, edition of 1952).
[15] Cf. below p. 929.
[16] Rehm, Parap., p. 35, p. 101–104; RE Par., col. 1351 f.
[17] Pliny NH XVIII, 57 (Loeb V, p. 325/327); cf. below p. 612.
[18] Rehm, RE Par., col. 1351, 35.
[19] Almagest XIII, 10 (Manitius II, p. 383); cf. above p. 234.
[20] Cf. below p. 588.

to the older "Greek" astronomers, Eudoxus, Meton and Euctemon, etc. Ptolemy may have simply accepted an established older terminology.

§ 2. The 25-year Lunar Cycle

From a demotic papyrus of the Roman period, P. Carlsberg 9, we know of the existence of a cycle which equates 25 Egyptian years, i.e. 9125 days, with 309 synodic months. This is indeed a very close approximation since 309 mean synodic months are not quite 5/100 of one day shorter than 9125 days. Hence it takes more than five centuries before the accumulated error amounts to one day. As we have seen before[1] Ptolemy made use of this close agreement for the computation of his table of mean syzygies.

The papyrus text, as we have it, concerns six cycles from A.D. 19 (Tiberius 6) to 144 (Antoninus 7). Parker has given good reasons[2] for dating the origin of the scheme to the fourth century B.C. because at that time the dates provided by the text would coincide with the date of the last visibility of the moon which defines the beginning of the Egyptian lunar months.[3] For the Roman period the small error of the cycle has moved the dates close to the dates of first visibility which, however, are of no significance for the Egyptian way of counting days or lunar months.

That the 25-year cycle originated from truly Egyptian calendaric concepts is not only evident from the significant role of the Egyptian year but also by the fact that the papyrus calls years which contain 13 beginnings of lunar months "*great years*," a term which is already attested in the twelfth dynasty (around 1900 B.C.).

The papyrus lists for one 25-year cycle a date in the Egyptian civil calendar for every second, or occasionally third, lunar month. The interval for two months is ordinarily 59 days but 60 days, five times. In every "great year," i.e. nine times in one cycle, an interval contains not only two but three months. For example in the year 1 of the cycle we have between the date II, 1 and IV, 30 a distance of $2 \cdot 29\,1/2 + 30 = 89$ days. The complete scheme for one whole cycle is shown in the Table on p. 564. The next date after XII, 7 in the year 25 would again be II, 1 as in the year 1.

The reason for giving calendar dates only for every second month seems to indicate that one had a free choice in the arrangement of consecutive full and hollow months. I cannot give a plausible explanation for such partial freedom within an otherwise rigid scheme.[4]

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[1] Above p. 119.

[2] Parker, Cal., p. 15–23.

[3] The reckoning of the lunar months from the day of last visibility (or of invisibility) is in all probability caused by the Egyptian reckoning of the day from sunrise, a procedure which in itself is most natural and does not require any astronomical motivation. Cf. Sethe, Zeitr., p. 130 to 138 for the evidence for the morning epoch of the day from the different periods of Egyptian history; that the same norm is still valid during the Roman period follows from the dates given in the Almagest (cf. below p. 1068 and Fig. 2, p. 1433).

[4] Parker, Cal., p. 24 ff. reconstructed, on the basis of actually attested lunar dates, rules for the length of intermediate months. I do not see, however, why P. Carlsberg 9 omitted the intermediate dates if they had to follow definite rules.

Year of Cycle	Month											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1*		1		30		29		28		27		26
2		20		19		18		17		16		15
3*		9		8		7		6		5		4
4		28		27		26		25		24		23
5		18		17		16		15		14		13
6*		7		6		5		4		3		2
7		26		25		24		23		22		21
8		15		14		13		12		11		10
9*		4		3		2		1		30		29
10		24		23		22		21		20		19
11		13		12		11		10		9		8
12*		2		1		30		29		28		27
13		21		20		19		18		17		16
14*		10		9		8		7		6		5
15		30		29		28		27		26		25
16		19		18		17		16		15		14
17*		8		7		6		5		4		3
18		27		26		25		24		23		22
19		16		15		14		13		12		11
20*		6		5		4		3		2		1
21		25		24		23		22		21		20
22		14		13		12		11		10		9
23*		3		2		1		30		29		28
24		22		21		20		19		18		17
25		12		11		10		9		8		7
1*		1										

As we have remarked before, the dates furnished by the cycle agree with the beginnings of the Egyptian lunar months in the fourth century B.C. Since dates provided by the cycle and actual lunar dates will only agree in the mean and since the difference between cycle dates and facts will vary only very slowly, one cannot exclude a date of origin of the cycle, say, in the fifth century B.C. If this date were correct it would constitute a curious parallel to the contemporary development of mathematical astronomy in Mesopotamia. In neither case is there the slightest indication of Greek or any other foreign influence. Nevertheless it looks as if the creation of the Persian empire stimulated intellectual life everywhere in the ancient world of which the Hellenistic world was to become the heir.

In Hellenistic astrology, in particular in predictions concerning the length of life, it is quite common to associate certain integer numbers with the celestial bodies.[5] It is from the Egyptian 25-year cycle that the moon becomes related to this number; the sun commands the number 19 in view of the approximate coincidence of 19 solar (not Egyptian) years with 235 lunar months, the so-called

[5] Neugebauer-Van Hoesen, Gr. Hor., p. 10.

Metonic cycle; for the planets the Babylonian "goal-year" periods had to serve.[6] As usual even the most absurd astrological doctrines contain sound astronomical elements which are indicative for the historical conditions under which these doctrines were invented.

§ 3. Concluding Remarks

We know very little about Egyptian astronomy after the Ramesside period. Zodiacs in temples and tombs appear only from the Hellenistic period on – imported, together with the basic astrological doctrines, from Mesopotamia. Demotic papyri from the Roman period continue in part a much older Egyptian tradition, in part they are exactly of the same type as the Greek astronomical and astrological papyri.

The most notable example of the first class of texts is the great papyrus Carlsberg 1 which contains a commentary of astronomical-mythological texts which are still extant in the cenotaph of Seti I (about 1300 B.C.) and in the tomb of Ramses IV (about 150 years later).[1]

A definite date for the Egyptian adaptation of Babylonian astronomical omina is provided by another demotic papyrus.[2] The relation between Egyptian civil months and Babylonian lunar months in this text corresponds to the 6th century B.C., i.e. to the Persian period.

Beginning with the Hellenistic period the rapid development of Greek astrology is also reflected in Egyptian documents. From the time around the beginning of our era we have demotic horoscopes and demotic planetary tables, all close parallels to Greek texts. We also have demotic tables which use the sexagesimal number system and apply Babylonian methods. It is clear that these texts can no longer be considered as belonging to "Egyptian" astronomy although they were undoubtedly written by Egyptians. The methods, however, are purely Hellenistic and have no connection with the Egyptian past.[3]

During the period of disintegration of the Roman empire the Egyptian population became increasingly christianized and adopted a slightly modified Greek alphabet for the writing of Late-Egyptian dialects. This "Coptic" literature, almost exclusively oriented toward religious subjects, shows no trace of interest in scientific matters. In Islamic astronomical treatises the "Coptic" names of the months figure in the calendaric sections. In fact these so-called Coptic names of the months are the above listed names,[4] used since the Persian period, in particular by the Greek-writing population.

Coptic-Hellenistic influence extended also to the christianized population of Ethiopia. As far as can be seen at present Ethiopia preserved an astronomical tradition of a slightly larger extent than the Coptic literature and received another impetus in the Islamic period, again from Egypt.

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6 Cf. above p. 554.
1 Cf. Neugebauer-Parker, EAT I, p. 36.
2 Published by Parker, Vienna Pap. (1959).
3 Cf. for these texts below p. 787 f.
4 Thoth, Phaophi, ... etc. (cf. above p. 560).

It is a commonly adopted procedure to postulate "secret" knowledge when the available material is too elementary or too trivial to suit the taste of the modern historian. But we know well how "secrets" were kept from the time of the Pyramid Texts in the Old Kingdom to the Jewish Kabbala in the High Middle Ages: all these "secrets" were eagerly written down and have survived in countless copies and in an enormous variety of forms: the "spells" of the Egyptian literature for the use in the Nether World, the Greek magical papyri and the gnostic texts, the secrets of Nechepso-Petosiris and of Hermes Trismegistos, the Jewish and Christian "secret" names of God, of saints and demons in all Semitic tongues, the amulets and curses – nothing is less secret than "secret" literature. It would be absurd to think that secrecy was effective alone for the exact sciences: not a line has ever been committed to writing and the non-secret sources never contain a hint which implies underlying secret knowledge. Obviously the assumption of secret scientific knowledge has no basis whatever.

Epilogue. For the amusement of the reader I shall mention a case of unexpected influence exercised by non-existent Egyptian astronomy on modern astronomy. In A.D. 1962 the section for celestial dynamics of the Jet-Propulsion Laboratory of the California Institute of Technology busied itself with the computation of highly accurate planetary tables. As farthest distant checking point these computations utilize positions of the planets supposedly recorded in an "Egyptian horoscope" dated to −2034 July 14. This extraordinary piece of evidence comes from a book, called "A Scheme of Egyptian Chronology" by Duncan Macnaughton (London 1932) which contains absurd interpretations of perfectly plain Egyptian texts which say nothing of the things which Mr. Macnaughton reveals on the basis of his private interpretation of hieroglyphic signs.[5] Thus one may say that modern tables, computed with the finest methods known, are required to produce over some 4000 years exactly the same error as Macnaughton's computations, carried out with some primitive tables constructed by modern astrologers.

§ 4. Bibliography

In view of the general situation in Egyptian "astronomy" we included some topics here which we would ignore elsewhere in these volumes (e.g. time measurement or astrology) and we also count as "Egyptian" material which is only linguistically Egyptian (the majority of the demotic and coptic texts) though actually following hellenistic patterns.

A. General

Neugebauer-Parker, EAT I to III (1960 to 1969). It was our goal to compile a corpus of Egyptian astronomical texts, excluding cosmogonic mythology, calen-

[5] For the characterization of Macnaughton's astronomy it may suffice to mention that he finds (p. 250) for a demotic (!) horoscope on a coffin lid the date −879 Sept. 25/27. Actually it belongs to a person who died in A.D. 125 and who was born A.D. 93, about October 16 (Neugebauer [1943], p. 115).

daric problems and time reckoning as well as astrology. Vol. I deals mainly with the diagonal star clocks from coffin lids of the Middle Kingdom, based on the rising of the decans. Vol. II is concerned with the Ramesside star tables for "transits" (cf. above p. 561). Vol. III discusses the later monuments, in particular the decans and finally the hellenistic zodiacs. This volume presents also the demotic astronomical papyri (cf. below).

Van der Waerden, Anf. d. Astr.; English version, BA, 1974. This work contains a summary of the earlier phases of Egyptian astronomy, mainly based on vols. I and II of the preceding work. Discussion of the planetary texts of the Roman period under the viewpoint of their dependence on Babylonian methods: cf. for this problem below V A 1, 2.

Neugebauer [1939]. On the origin of the so-called "Sothic period" of the Egyptian wandering year and the modern misinterpretation as an early astronomical achievement.

Parker, Calendars. Discussion of the historical development of the Egyptian calendaric concepts, in particular concerning the role of a lunar calendar beside the schematic civil calendar.

Borchardt, Zeitm. Construction and theory of sundials and waterclocks preserved from ancient Egypt.

B. Demotic and Coptic Texts

Texts of this group were obviously written in the latest period of ancient Egyptian history but may be based on much older sources. This is in particular true of P. Carlsberg 1, a text which is a commentary to the representation of the sky goddess Nut and of the decans in the cenotaph of Seti I in Abydos (about 1300 B.C.). Last publication: EAT I, p. 36ff.

P. Carlsberg 9; cf. above p. 563f. Latest publication EAT III, p. 220ff.

Demotic astronomical texts from the Roman period: P. Berlin 8279 and the Stobart Tables are planetary tables; with planetary motion are also concerned P. Carlsberg 32 and P. Florence 44. P. Vienna D 4876 deals with the moon, P. Florence 8 with zodiacal signs. These texts are published, or republished, in EAT III, p. 225ff. Cf. also the discussion below V A 1, 2 in relation to similar Greek texts. An ostrakon,[1] probably from the third century B.C., gives the names of the planets and of the zodiacal signs.

Astrological demotic texts. Modern scholars have long been influenced by the ancient propaganda which ascribed the origin of astrological doctrines not only to the "Chaldeans" but also to the Egyptians, e.g. under the name of Nechepso-Petosiris. It is a remarkable result of this tendency that Brugsch in 1883 gave to one of his editions of Egyptian inscriptions the title. "Astronomische und astrologische Inschriften altaegyptischer Denkmaeler" although none of the texts he published were astrological. And Borchardt, in 1920, suggested[2] an "astrological" significance for the diagonal star clocks of the Middle Kingdom (cf.

[1] Strasbourg D 521; cf. Neugebauer [1943], p. 121f.
[2] Borchardt, Zeitm., p. 55, note 1.

EAT I) after having rejected astrological interpretations by his predecessors of the Ramesside star tables (EAT II).

The earliest known genuine astrological document from Egypt is published in Parker, Vienna Pap., dealing with lunar and eclipse omina of clearly Babylonian origin, probably written in the fourth century B.C.

All other demotic astrological texts are from later periods. The earliest known horoscope from Egypt is cast for 38 B.C.,[3] to be followed by a long sequence of demotic and Greek horoscopes.[4] For demotic astrological doctrines cf. Hughes [1951] and the literature cited in EAT III, p. 217f.

Coptic literature is predominantly ecclesiastic in character, i.e. (monophysite) christian. The few fragmentary pieces which have some relation to astronomy are based on Greek methods and concepts. This holds for three Coptic shadow tables, first published by U. Bouriant [1898], which follow a pattern of similar Greek tables attested from the Ptolemaic period to deep into Byzantine times.[5] We find the same type in Ethiopic texts,[6] probably transmitted through Coptic treatises. It is still found in a treatise, written in Arabic in the 18th century, on christian calendaric matters[7] and ascribed to the Alexandrian patriarch Abba Demetrius who died about A.D. 230. His "Computus" is also preserved in Ethiopic.[8]

A short astrological treatise in Coptic (on the Ages of Life) is appended to a Greek horoscope cast for A.D. 95.[9] Some small astrological fragments have also been found in the Faiyum, in the Wadi Sarga[10] (south-west of Asyūt), and in a monastera near Sohāg[11] not far from Akmīm). Coptic magical texts often contain references to astral phenomena but mainly determined by biblical parallels.[12]

Ironically the only Egyptian eclipse record ever found concerns a solar eclipse of A.D. 601,[13] only four decades earlier than the Arab conquest which was destined to establish once more a center of learning in the country on the Nile.

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[3] Neugebauer-Parker [1968]. Correct in the commentary on p. 233 the solar longitude to ♉4 and change the subsequent data accordingly, in particular ♑25 as longitude of the moon.

[4] Cf. Neugebauer [1943] and Neugebauer-Van Hoesen, Gr. Hor.

[5] Cf. below IV D 2, 1 A.

[6] Cf. below p. 742.

[7] Published by Georgy Sobhy [1942]; cf. below p. 743.

[8] Cf. Cod. Aeth. Vat. No. 119 (p. 484, 10); unpublished. Demetrius is also known for an Easter cycle which was transmitted to Hippolytus in Rome (cf. below p. 944). This cycle was based on an octaeteris, in Roman usage expanded to a 16-year cycle and to a 112-year cycle ($=7 \cdot 16$). Cf. Richard [1966] and [1974]. Cf. also Georgy Sobhy [1942].

[9] From P. Lond. 98; cf. Neugebauer-Van Hoesen, Gr. Hor., p. 32ff.

[10] Cf. EAT III, p. 218.

[11] Published by P. Bouriant [1904].

[12] Cf. Stegemann [1935].

[13] From an ostracon from the west side of Thebes; cf. Ginzel [1883] and Allen [1947].

Book IV

Early Greek Astronomy

Cependant, au milieu des rêves philosophiques des Grecs, on voit percer sur l'astronomie des idées saines.

Laplace, Système du monde. Œuvres VI, p. 372.

Sehen Sie sich doch nur bei den heutigen Philosophen um, bei Schelling, Hegel ... und Consorten, stehen Ihnen nicht die Haare bei ihren Definitionen zu Berge? Lesen Sie in der Geschichte der alten Philosophie, was die damaligen Tagesmänner Plato und andere (Aristoteles will ich ausnehmen) für Erklärungen gegeben haben.

Gauss an Schumacher, 1. Nov. 1844.
Gauss, Werke 12, p. 62f.

Introduction

Überall im Studium mag man mit den Anfängen beginnen, nur bei der Geschichte nicht.

J. Burckhardt, Weltgeschichtliche Betrachtungen, p. 5.

It is the purpose of Book IV to present a survey of the fragmentary data from the early stages of Greek astronomy. About six centuries have to be covered by such an attempt, beginning with the calendaric cycles of Meton and his school in the fifth century B.C. to Ptolemy in the second A.D. Only in two areas is our information substantial enough to make a separate discussion desirable: early planetary and lunar theory on the evidence of tables preserved on papyri of the hellenistic and Roman period (cf. below V A) and the work of the direct predecessors of the Almagest, Apollonius and Hipparchus (above I D and I E, respectively). For the material left to be included in Book IV we must frequently operate with fragmentary data transmitted by authors of very limited technical competence. The little one can extract from these sources hardly deserves the name "history."

Fortunately we also have a few more or less complete treatises which concern our period: the elementary "Introduction" by Geminus (1st cent. A.D.), a little treatise by Theodosius of Bithynia (first half of first cent. B.C.) and Cleomedes[1] for the time of Posidonius (around 100 B.C.). Then treatises about spherical astronomy by Autolycus and Euclid (around 300 B.C.), by Hypsicles and again Theodosius (2nd cent. B.C.), Aristarchus' famous attempt to determine size and distance of sun and moon (3rd cent.) and the planetary data from the inscription of Keskinto on Rhodes. The latter makes us realize how little we actually know about early Greek mathematical astronomy.

To the period under consideration also belongs the contact with Babylonian astronomy. The available Greek sources, however, restrict us to little more than the tracing of elementary arithmetical methods in works of very limited astronomical interest — calendaric and meteorologic, or geographic. Again, the factual contribution to our knowledge of the origins and development of Greek mathematical astronomy remains rather meager. Nevertheless one may perhaps conjecture that it was due to Babylonian influence that early Greek astronomy was diverted towards significant numerical procedures, away from the preceding speculative approach.

The depth of our ignorance becomes once again evident as soon as one tries to supplement with concrete facts clichés as the emphasis on Alexandria as

[1] For the date of Cleomedes (4th cent. A.D.) cf. below V C 2.

a scientific center and its influence on astronomy. There are perhaps so astronomers in the four or five centuries before Ptolemy, who may or m have spent some time in Alexandria. Euclid and Autolycus around 30(then in the second half of the third century Conon and his pupil Dos Apollonius and Eratosthenes, Hypsicles and Hipparchus[2] in the seconc B.C., and Geminus in the first cent. A.D. We know practically nothing abc their conditions of work and sometimes equally little about their specific astronomical achievements.[3] Probably Conon would not be known to us were it not for the story about the constellation named "Berenice's Lock", told in the poems of Callimachus and Catullus.[4] What we know otherwise about his and Dositheus' astronomy is only their contribution to parapegmata, based, according to Ptolemy,[5] on observations in Italy and Sicily and Kos(?), not in Egypt.

I see no need for considering Greek philosophy as an early stage in the development of science; its role seems to me only comparable to the influence on science of the Babylonian creation myth or of Manichaean cosmogony. One need only read the gibberish of Proclus' introduction to his huge commentary on Book I of Euclid's "Elements" to get a vivid picture of what would have become of science in the hand of philosophers. The real "Greek miracle" is the fact that a scientific methodology was developed, and survived, in spite of the existence of a widely admired dogmatic philosophy. Obviously astronomy was in a much better position than all other natural sciences. Astronomy had at its basis simple observable facts and elementary mathematics sufficed to lead to significant results, leaving only little room for philosophical doctrines. Nevertheless, wherever philosophical speculation found a foothold, e.g. through the concept of nature admitting of no "useless space," the result was disastrous enough to forestall better insight for many centuries.

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[2] That Hipparchus observed in Alexandria is by no means certain: cf. above I E 1, p. 276.

[3] We are a little better informed about mathematics, thanks to the extant mathematical works of Archimedes and Apollonius.

[4] Wilamowitz in his brilliant lecture on "Die Locke der Berenike" (Reden und Vorträge I, 4th ed., Berlin 1925) states categorically (p. 213, note 1) that at the time of Conon "man arbeitete auf der Sternwarte Alexandreias an einem Fixsternkataloge." This is, of course, pure nonsense. Similarly it is only a modern invention to make Conon a "court-astronomer"; no such rank existed in Ptolemaic Egypt (cf., e.g., the list of officials discussed by Cumont, Ég. astr., Chap. 1 and 2). — What we really know about Conon is little enough. That he was a competent mathematician is evident from the way Archimedes mentions him — in spite of Apollonius' adverse remarks in the preface to Book IV of the "Conics." From Conon's parapegma we have only two data through Pliny (Nat. Hist. XVIII 74, Loeb V, p. 384/5) and seventeen in Ptolemy's "Phaseis." Seneca (first cent. A.D.) says (Quaest. nat. VII, III 3, Loeb II, p. 233) that Conon was a careful observer and that he "recorded solar eclipses observed by the Egyptians" — a story difficult to take seriously in view of what we know of Egyptian astronomy (cf. above p. 561). Probus (end of 1st cent. A.D.) ascribes seven books to him "*de astrologia*" (Thilo-Hagen, Servius, In Vergilii carmina comm., Vol. 3.2, p. 330, 14). — That Conon's name as astronomer was familiar to Virgil and Propertius (first cent. B.C.) may be due only to Callimachus' poem on Berenice's Lock.

[5] Ptolemy, Phaseis, Opera II, p. 67; cf. also below p. 929.

A. The Beginning of Greek Astronomy

§ 1. Chronological Summary

The majority of the dates given in the following are insecure. I have often rounded them to full decades to remind the user of the approximate character of such information. In case of more accurately given numbers not more is meant than customarily accepted dates. For the present purposes only a reasonably correct sequence of relative dates is required and that much, I hope, is achieved in the majority of cases.

1. The Early Period

The distinction of an "earlier" and a "later" period does not pretend to be very significant and well defined. Roughly speaking it separates the hellenistic age from the preceding centuries. The preservation of more or less complete astronomical treatises distinguishes the more recent period from the earlier one. — All dates are years B.C.

Hesiod	≈800 to 700	Eudoxus	≈400 to 347
Cleostratus	≈550 to 500	Polemarchus	≈370
Oenopides	≈450	Aristotle	382 to 322
Philolaus	≈430	Heraclides Ponticus	387 to 315/310
Meton	cycle: 432	Menaechmus	≈350
Euctemon	≈Meton	Philip of Opus	died after 347
Democritus	≈460 to 370	Autolycus	
Bion	≈400	Callippus	≈330
Archytas	≈400 to 350	Euclid	
		Eudemus	≈320

For continuation cf. below p. 574.

Notes. Autolycus: cf. for his date the edition by J. Mogenet (1950), Chap. 1.

Hesiod: mentioned here only for the interest in fixed star phases in relation to the agricultural calendar.

Democritus: cf. for his date Rehm, Parap., p. 6f.

Meton, Euctemon: these men form the nucleus of the first astronomical "school" which obtained results from observations that could be taken seriously by Hipparchus and Ptolemy (cf. the index of names in Ptolemy, Opera II, p. 271–282).

Eudoxus: he died in his 53rd year and later than Plato, i.e. after 347 B.C.; cf. Santillana [1940] and Cherniss [1969/60], p. 42, n. 1; Eudoxos, Fragm. (Lasserre), p. 137–147.

Philip of Opus or Medma: supposedly a pupil of Plato. What we factually know about him has been said by Böckh, Sonnenkr., p. 34–40 (1853). All sources agree that he had assembled data for a parapegma (he is, indeed, mentioned on a parapegma found in Miletus; cf. below p. 588) and Ptolemy in his "Phaseis" (Opera II, p. 67, 4f.; cf. below p. 929) gives the Peloponnesus, Locris and Phocis as his base of operation (similarly CCAG 1, p. 80, 21 = 5, 1, p. 205, 7 from "Palchus"; cf. also Hipparchus, Comm. Ar., p. 28, 13–18). With this line of activity agrees his interest in the 19-year cycle and the length of the year (Geminus, Isag. VIII, 50 [cf. below p. 623] and Maass, Aratea, p. 140 [cf. below p. 601]) as well as his association with shadow tables — as "King" Philip, most likely because of the Era Philip, commonly known through the Handy Tables (cf. below p. 739f. and p. 971).[1]

Plutarch (Loeb, Moralia XIV, p. 62–65) tells us that Philip wrote on lunar phases (σχῆμα) and from Stobaeus (≈500 A.D.) we know that he polemicized against the Philolaic doctrine of the "Counter-Earth" as cause of lunar eclipses (ed. Wachsmuth, p. 221, 6 = Diels, Doxogr., p. 360b, 3; the passage comes from Aetius (≈100 A.D.)). Suidas (ed. Adler IV, p. 733, 24–34) ascribes to Philip (among others) treatises On lunar eclipses, On the size of the sun, moon, and earth, On the distances of sun and moon, On the planets. This looks more like a list of works expected in late antiquity from an "astronomer" than plausible titles in the period before Eudoxus and Aristarchus.

2. More Recent Period

Continued from p. 573; for continuation cf. below p. 779f.

In general dates are given in years B.C., unless accurately known. Then the astronomical notation is used. In the right-hand column are listed original sources (papyri and inscriptions).

Eudemus	≈320		
Aristotherus Theophrastus Criton Aristyllus	≈300		
Aratus	315/310 to 275		
Berosus	350 to 270		
Timocharis	300 to 270	P. Hibeh 27 (calendar)	300
Aristarchus	310 to 230		
Cleanthes	310 to 230		
Dionysius	era: −284		
Epigenes	≈250		
Conon	≈250		
Dositheus	≈230		
Archimedes	286 to 212		
Eratosthenes	283 to 200		
Sudines	≈240		

[1] For a similar misunderstanding concerning the "Regula Philippi Aridaei" cf. Neugebauer [1959, 2].

Apollonius Perg.	240 to 170		
Apollonius Mynd.	≈ 200 (?)		
Crates	200 to 170		
Ps. Geminus parap.	≈ 200 (?)		
Leptines	≈ 190	P. Paris. 1 ("Ars Eudoxi")	≈ 180
Seleucus	≈ 150	P. Ryl. Inv. 666 (New Moons)	180
Hypsicles	≈ 150		
Hipparchus	190 to 120		
Sulpicius	died -148		
Parmeniscus	≈ 100	parapegmata Miletus	≈ 100
Theodosius	≈ 100	inscription of Keskinto (planets)	≈ 100(?)
Posidonius	≈ 130 to 50	Greek horoscope	-71
Cicero	-105 to -42	horosc. Antiochus of Commag.	-61
Serapion	≈ -50	demotic horoscope	-37
Sosigenes	≈ -50		
Antiochus	≈ -50 (?)		
Geminus	$\approx +50$		

Notes. Aratus: in an appendix to Schott, Aratos (p. 101–119) R. Böker proposed, among other untenable hypotheses, a date around 1000 B.C. for Aratus' globe and an adjustment for a latitude $\varphi = 33°$.

Epigenes: what is known, or conjectured, about Epigenes of Byzantium, his date and his relations to Babylonian astronomy, is summarized by Honigmann in Mich. Pap. III (1936), p. 310–311.

Greek horoscopes: the earliest preserved Greek horoscope, cast for -71 Jan. 21 or 16, comes from a collection of horoscopes by Balbillus and was probably recorded as late as -21 (cf. Neugebauer-Van Hoesen, Gr. Hor., p. 76–78). The next in time is the famous monument on the Nimrud Dagh which displays a horoscopic configuration for Antiochus of Commagene in -61 July 7 (Gr. Hor., p. 14–16). The earliest demotic horoscope, cast for -37 May 4, is published by Neugebauer-Parker [1968]. The earliest Greek horoscope on papyrus is cast for -9 Aug. 14 (Gr. Hor., p. 16).

Theodosius: cf. for his date below p. 749. He was a native of Bithynia, not of Tripolis; cf. Fecht, p. 3–5.

Serapion: cf. below p. 729, n. 15.

Sosigenes: according to tradition (e.g. Pliny NH 18, 211; Loeb V, p. 322/323) associated with Caesar in the reform of the Roman civil calendar and probably also in the preparation of the parapegma related to it (cf. Rehm in RE 3A, 1, col. 1154 and below p. 612). From him must be distinguished the Sosigenes who wrote around A.D. 170 on the homocentric spheres (cf. below p. 684, n. 1).

Geminus: cf. for his date below IV A 3, 1 and for the parapegma below p. 580f.

§ 2. Sphericity of the Earth; Celestial Sphere and Constellations

The existence of a highly sophisticated mathematical astronomy in Mesopotamia shows how far one can go without introducing any assumptions concerning the shape of the earth or of heaven. But it is the discovery of the

sphericity of the earth (and the related concept of the "celestial sphere" with its coordinates) that guided Greek astronomy into an entirely new direction. Now a cinematic description of the phenomena became possible which prepared the ground for a dynamical explanation.

We have no account of how the sphericity of the earth was discovered. But it seems plausible that it was the experience of travellers that suggested such an explanation for the variation in the observable altitude of the pole and the change in the area of circumpolar stars, a variation which is quite drastic between Greek settlements, e.g., in the Nile Delta and in the Crimea.[1] The interest which Greek geographers took in these phenomena is revealed in such a concept as the "arctic circle" which encloses the always visible stars and varies as one moves north or south.[2] On the other hand the star Canopus is said to be invisible north of Rhodes but is seen to rise and set in Egypt.[3] As corroborative evidence could be adduced the zenith position of the sun, reached twice each year in localities south of Syene,[4] and the variation in the length of the longest day, an element of basic importance in ancient geography.[5]

For the east-west curvature of the earth one has, of course, no such simple observational data at one's disposal but arguments of symmetry and successful explanation of the phenomena connected with risings and settings by means of a rotating celestial sphere must have strongly supported the concept of a spherical earth. Aristotle's pseudo-argument of a circular shadow on the moon seen at lunar eclipses[6] certainly seemed to demonstrate the reality of a spherical shape of the earth.

Tradition connects the discovery of the sphericity of the earth with the name of Parmenides of Elea (in southern Italy) around 500 B.C.[7] He not only declared the earth to be spherical but also assumed for it a central position. By the time of Aristotle the sphericity of the earth was obviously generally accepted among philosophers. Bion, a follower of Democritus, noted such mathematical consequences as the existence of locations at which day and night reach the length of six months.[8]

The first impact of the discovery of the sphericity of the earth misled early Greek astronomy into attempts at explaining planetary motion by means of strictly spherical models, a procedure which agreed well with the prejudices of the philosophers. And for centuries the fruitless discussions about the shape of

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[1] The corresponding latitudinal difference amounts to about 15°.

[2] Cf., e.g., Geminus, Isag. V, 2 (ed. Manitius, p. 44, 4-9) and below p. 582 and Fig. 2.

[3] E.g., Geminus, Isag. III, 15 (ed. Manitius, p. 42, 3-8); Vitruvius, Archit. IX V, 4 (ed Krohn, p. 213, 24-28; Loeb II, p. 244/245); Manilius I 216 and notes by Housman (Vol. I, p. 17); Theon Smyrn., Astron. II (Dupuis, p. 201). Cf. also Kepler, Werke II, p. 135, 15-136, 19.

[4] For references to the sun's zenith positions cf. below p. 937.

[5] Cf., e.g., below V B 8, 3.

[6] Aristotle, De caelo II, 14 (Budé, p. 100) and II, 11 (Budé, p. 80); for the error in this argument cf. below p. 1093.

[7] Diogenes Laertius IX, 21 (Loeb II, p. 430/431). Frank, Plato, p. 198-200, denies the validity of this statement in Diogenes and explains it as a misunderstanding of the original formulation by Theophrastus. Cf. also Heidel, Greek Maps, p. 70-72. Both scholars agree that Plato's Phaedo ($\approx$400 B.C.) must be very near to the time of discovery (Frank, p. 184 ff., Heidel, p. 79 ff.).

[8] Diogenes Laertius IV, 58 (Loeb I, p. 434/435).

the earth and of the universe – in part, no doubt, invented only for the sake of argument, e.g. the form of polyhedra – provided a cherished topic for cosmological introductions,[9] to be properly ignored in the text that mattered.

The concept of a celestial sphere is not at all "natural," i.e. suggested by the naive visual appearance. The Egyptians, e.g., visualized the heaven by no means as a hemispherical dome, sitting on the plane of the horizon, but as a flat roof, depicted by the hieroglyph . Innumerable monumental representations show the sky goddess Nut stretched out in all her length over the country.[10]

That the Babylonian astronomers assumed a spherical sky, or, to put it more cautiously, a spherical surface of reference, seems undeniable in view of the use of orthogonal ecliptic coordinates. However, we do not know how far this concept was considered to be a description of physical reality. We are also not sure whether these ecliptic coordinates were ever used much beyond the zone of the ecliptic and the "Normal Stars."[11] The coverage of the sky with picturesque configurations of stars[12] is not the equivalent of the use of mathematically defined spherical coordinates.

The first system of real spherical coordinates known to us is the Hipparchian coordinate system discussed at length in I E 2, 1 A. Nothing in this context points to Babylonian influence and we may safely assume that this extended usage of accurately defined coordinates – though not orthogonal – was the invention of Hipparchus, probably related to solving problems of spherical astronomy through stereographic projection. But we have also emphasized the lack of evidence for Hipparchus' use of orthogonal ecliptic coordinates for defining stellar positions, or for compiling a "catalogue of stars" based on any consistent set of coordinates. It seems to me a firmly established conclusion that the construction of a catalogue of stars in the modern sense of the term is actually Ptolemy's achievement.[13]

It is certain, on the other hand, that descriptive lists of constellations existed in Greek literature long before Hipparchus. Vitruvius ascribes[14] such a "catalogue" to Democritus; this would bring us back to the fifth century B.C. By the time of Aratus, a century or so later, this literary type was well established, obviously based on the work of Eudoxus. Descriptions of simultaneous risings and settings by Eudoxus are known to us from Hipparchus' commentary to Aratus as well as through fragments preserved in the Anthology of Vettius Valens.[15] A short catalogue of constellations, presumably by Hipparchus himself, has survived,[16] giving, however, only the number of stars in each configuration.

Great influence, at least in late-ancient and mediaeval literature, was exercised by a work that is known under the name of the "Catasterisms of Eratosthenes."

[9] Cf., e.g., Euclid, Phaen., Introd. (ed. Menge p. 6, 5); Cicero, De Nat. Deorum II, 47 (Loeb, p. 166/169); Theon Smyrn., Astr. I (Dupuis, p. 201); Achilles, Isag. 6 (Maass, Comm. Ar. rel., p. 37, 8); Almagest I, 4; Cleomedes I, 8.

[10] Cf., e.g., Neugebauer-Parker EAT I Pl. 8 (column V), Pl. 34, II Pl. 7, etc.

[11] For a list of Normal Stars with coordinates cf. Sachs [1952, 2]; also above p. 546.

[12] For Babylonian constellations cf. Weidner [1967].

[13] Cf. above I E 2, 1 A and 1 B.

[14] Vitruvius, Archit. IX V, 4 (ed. Krohn, p. 213, 15–17; Loeb II, p. 242/243).

Fragmentarily preserved as it is it seems to have described 42 constellations, giving the number of stars for the different parts of the pictures as well as the totals – but no coordinates. What gave the book its lasting influence are the mythological explanations of the names of the constellations.

Since not even the title of this work is certain [17] and since neither Hipparchus, nor Geminus, nor Ptolemy ever mention it, scholars proposed [18] dates between 250 B.C. (i.e. Eratosthenes) and A.D. 200. That the original composition was of hellenistic Alexandrian origin seems nevertheless well established.[19] The number of stars in a given constellation is usually smaller than in Hipparchus' list, which, in turn, remains below Ptolemy's catalogue.[20] This confirms the conclusion that no rigid scheme existed before the Ptolemaic catalogue for the size and the boundaries of constellations.

§ 3. Geminus

Under the title "Geminus' Introduction to the (celestial) Phenomena" (henceforth referred to as "Isagoge") is preserved a treatise which summarizes the contemporary knowledge of elementary astronomy. Though written after Hipparchus it carefully avoids the discussion of any mathematical details. Nevertheless it is a useful introduction to the basic concepts of Greek astronomy, even for the modern reader. The Greek text and a German translation with notes has been published by Manitius in 1898 (Teubner).

The arguments on which the dating of this work rests will be presented in the subsequent section. Whether the author wrote in Rhodes, Rome, or elsewhere, whether he was a Greek or Roman, cannot be decided from the extant material.[1] From a few fragments it is fairly certain that he had also written on mathematical subjects, meteorology, and optics,[2] while a "calendar," associated with the Isagoge in its present state, seems definitely to belong to another, earlier, author.[3]

A much discussed question is the dependence of Geminus on the famous stoic philosopher Posidonius of Rhodes (who died around 50 B.C.). The assumption of such a dependence mainly rests on close parallels between passages in the Isagoge and the writings of Cleomedes who repeatedly refers to Posidonius as his source.[4] In contrast, however, the Isagoge itself contains not a single reference to this philosopher nor does the frequent mention of Rhodes (or its

[17] The "accepted" title is only a re-translation by Rehm into Greek of a Latin version (cf., e.g., Keller, Erat., p. 23) which does not contain the word "Catasterism."
[18] Cf. for this literature Knaak in RE 6, 1, col. 277–381 (1909) and Keller, Erat. (1946), p. 18–28.
[19] Some always visible stars presuppose the latitude of Alexandria; cf. Rehm [1899], p. 268.
[20] Rehm [1899], p. 270.
[1] Nothing is gained by the countless speculations published on this subject. This includes also the hypotheses, hesitatingly proposed by Manitius (p. 247–252), about excerpts and commentaries supposedly made in Constantinople.
[2] Cf. Heiberg, Gesch. d. Math. u. Naturw. (1925), p. 2, p. 35; p. 74, p. 79.
[3] Cf. below p. 580f.
[4] Cf. below IV B 3, 3.

latitude) imply that Geminus was a pupil of Posidonius. Fortunately the whole question is of very little consequence. What we gain from Geminus is a quite detailed picture of the level of elementary astronomy at the beginning of our era. Cleomedes shows us that some of these topics were also of interest to Posidonius but without ever demonstrating any real astronomical competence. Geminus had nothing to learn from Posidonius.

Of greater interest would be Geminus' relation to Hipparchus. Unfortunately the Isagoge mentions Hipparchus explicitly only in connection with names of constellations[5] while the Hipparchian values for the length of the seasons are mentioned in Chap. I, the resulting solar model being described only in a qualitative way.[6] Of unique value in the Isagoge is the enumeration of numerical parameters of the lunar theory (in Chap. VIII) which are of Babylonian origin. The appearance of these data in an introductory treatise indicates how far Babylonian results penetrated early Greek mathematical astronomy.

1. Date

A passage in Chap. VIII of the Isagoge seems to lead directly to a determination of its date. Geminus, in describing the rotation of the Egyptian calendar through all seasons of the solar year criticizes the naive opinion "that the Isis festival (τὰ Ἴσια) according to the Egyptians and the winter solstice according to Eudoxus" would always coincide, simply because "120 years ago it so happened that the Isis festival was celebrated at the winter solstice."[1]

All would be fine if we knew (a) the Egyptian date of the Isis festival and (b) the julian date of the Eudoxan winter solstice. Unfortunately neither one is given us explicitly, (b) being the minor problem since an error of 2 or 3 days implies only an error of about one or two decades in chronology. The real problem is therefore the selection of the proper Isis festival among the several known ones from hellenistic Egypt. Hence one has to start from historically possible dates for Geminus and compute backwards the equivalents in the Egyptian calendar of winter solstices and hope to find a date attested as date for an Isis festival.

Fortunately it is only a narrow interval of time into which Geminus must be fitted. Since he is later than Hipparchus[2] he is certainly later than −150. On the other hand it would be strange for a Greek author writing, say, around +100 to say that "Greeks until our time" were not aware of the rotation of the Egyptian calendar,[3] a century after the Augustan reform in consequence of which double dates "according to the Egyptians" and "according to the Greeks" had become common place in papyrus documents.

Within the ample limits −150 and +150 for Geminus we can construct the following table of dates for the Winter Solstices:

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[5] Isagoge Chap. III, §8 and 13, Manitius, p. 38, 16 and p. 40, 15 and 19.
[6] Manitius, p. 8/9 and p. 14–19; cf. below p. 584.
[1] Geminus, ed. Manitius, p. 108, 4–10.
[2] Cf. above p. 579.
[3] Isagoge, p. 108, 3 Manitius.

Geminus	120 years earlier	W.S.	Calendar	
			Egyptian	Alexandrian
−150	−270	Dec. 24	Phaophi 26	
−100	−220	24	Athyr 9	
− 50	−170	24	21	
0	−120	23	Khoiak 3	
+ 50	− 70	23	15	
+100	− 20	22	27	Khoiak 26
+150	+ 30	22	Tybi 9	26

Consequently we must look out for festivals of the Isis-Osiris cycle which fall into the months Athyr or Khoiak.

For this we have two major sources. First a long hieroglyphic text in the East Osiris Chapel on the roof of the Temple of Hathor in Denderah.[4] This text, written in the late Ptolemaic period not later than 30 B.C., describes in great detail the rituals of an Osiris festival that last from Khoiak 12 to 30.[5] Also P. Hibeh 27 (around −300) mentions on Osiris festival for the 26th of Khoiak.[6]

Secondly Plutarch in De Iside et Osiride, written about A.D. 120,[7] describes similar rites for the days Athyr 17 to 20.[8] What calendar Plutarch had in mind follows from his remark (in Chap. 13) that the sun moves through Scorpio in the month Athyr. This is correct only in the Alexandrian calendar for which Athyr 17 (corresponding to Nov. 13/14) gives a solar longitude of about ♏21. In A.D. 116/120 Alexandrian Athyr 17 corresponds to Egyptian Khoiak 22/23. Thus we see that Plutarch did not commit an error (as has often been assumed[9]) in naming the month of the Isia but that he simply transposed the Egyptian date to the contemporary Alexandrian calendar.[10]

Hence it is clear that Geminus had in mind the Isis festivals which were celebrated in the Egyptian month Khoiak. This places him into the first century A.D. The duration of the festival and the possible insecurity of the date of the Winter Solstice prevent us from establishing a more accurate date, but the tenor of his remark would make the first half of the century more likely than the later part.

If we thus consider a date around 50 A.D. as fairly secure for the "Isagoge" we do not extend this result to the "calendar" or "parapegma" appended to the

[4] Porter-Moss VI, p. 94 and p. 97. – This is the same chapel which had the famous "round zodiac of Denderah" on the ceiling.

[5] A summary is given by Loret [1884], p. 99–102.

[6] Grenfell-Hunt, The Hibeh Papyri I, p. 144, p. 148 lines 58 to 62.

[7] For this date cf. Griffiths, Plutarch, p. 17.

[8] Chap. 13 (Griffiths, p. 138/139: Loeb, Moralia V, p. 36/37), Chap. 39 (Griffiths, p. 178/179; Loeb, p. 94–99); Chap. 42 (Griffiths, p. 184/185; Loeb, p. 100/101). Cf. also the commentaries Griffiths, p. 64f., p. 312, p. 448f.

[9] E.g. Loret [1884], p. 103.

[10] This has been seen by Grenfell-Hunt (p. 153 to line 60). – The Roman "Isia" were celebrated in November. Cf. for references Daremberg-Saglio III, p. 583; also Lydus, De mens. IV 148 (ed. Wuensch, p. 1666, 16f.) who gives November 2 and 3 as date for the Isis festival. The Egyptian Khoiak 12 corresponds in A.D. 120 to Alexandrian Athyr 6 = November 2, the date given by Lydus, and in A.D. 140 to Alexandrian Athyr 1 = October 28, the beginning date of the "Isia" in the Philocalus Calendar (cf. Stern, Cal. pl. XI, XII).

Isagoge.[11] Manitius, in his edition, has summarized the reasons which speak strongly against Geminus' authorship[12]: (a) in the Isagoge Geminus polemizes against weather prognostications from fixed star phases, a procedure which is the very essence of a parapegma; (b) the length of the seasons given in the calendar cannot be reconciled with the Hipparchian values used in the Isagoge; (c) neither Hipparchus nor Philip appear as authorities for the predictions in the calendar though both names appear in the Isagoge. Consequently the calendar seems to be of pre-Hipparchian date. On the other hand Dositheus, who flourished around 230 B.C., is named as an authority[13]. Hence the calendar may be about three centuries older than Geminus.

2. The Isagoge

The "Isagoge" is an elementary astronomical treatise that shows a very definite tendency to get rid of the speculations of the philosophical schools – e.g., the geographical doctrines of Cleanthes (who died around 230 B.C.) and Crates[1] (about 170 B.C.) – and to discredit popular beliefs, e.g., on the influence of the stars on the weather.[2] It is the same spirit which one senses in Hipparchus' polemics against the astronomy of Aratus – remaining largely unsuccessful in both cases.

Geographical data are scattered over the whole treatise, often in relation to sun dials.[3] Rhodes is given a longest daylight M of 14 1/2 hours, the standard value throughout antiquity.[4] A ratio $M:m=15:9$ is assigned to the area of Rome[5]; beyond the Propontis M reaches 16^h, 17^h and 18^h still further north.[6] The ratio $M:m=15:9$ appears once more in connection with Aratus[7] and again[8] referring to Greece in general,[9] a norm found indeed for Hellas also in Aratus and with Attalus.[10]

Geminus mentions two types of celestial globes, "solid" (στερεά) and "ringed" (κρικωτή),[11] the latter being a sort of armillary sphere. These spheres show the

[11] Cf. below IV A 3, 3.

[12] Manitius, p. 281. All these arguments come from Böckh, Sonnenkr., p. 22–26.

[13] The other authorities mentioned are Meton, Euctemon, Democritus from the 5th century and Eudoxus and Callippus from the 4th. – Incidentally: no "Egyptians" are mentioned; they appear first in a parapegma from Miletus about 100 B.C. (cf. below p. 588; also above p. 561 f.).

[1] Manitius, p. 173.

[2] Manitius, p. 181 ff.; cf. also p. 191 ff. on Sirius.

[3] Manitius, p. 32, 4; 32, 19; 36, 2; 80, 13–19; 84, 25, etc.

[4] Manitius, p. 6, 17; 8, 3; 70, 16.

[5] Manitius, p. 70, 17; cf. also below p. 711, n. 26.

[6] Manitius, p. 70, 19 f.; nowhere is recognition of a doctrine of seven climata discernable.

[7] Manitius, p. 90, 13–18.

[8] In a formulation also found in the "Enoptron" of Eudoxus; cf. Hipparchus, Aratus Comm., ed. Manitius, p. 22, 20–22.

[9] Manitius, p. 50, 12–52, 2. Manitius restores in the corrupt text "Hellespont," in spite of the fact that the Latin version, made from an Arabic version, has "regionis grecorum."

[10] Aratus, Phen., verses 497–499; Hipparchus, Aratus Comm., ed. Manitius, p. 26, 3–28, 8. Cf. also Ptolemy, Phaseis, Opera II, p. 67, 16–21 (ed. Heiberg) and Cleomedes I, 6 (ed. Ziegler, p. 52, 1–2).

[11] E.g. Manitius, p. 168, 17.

characteristic circles, equator, tropics, meridian and colures, etc. Arcs on these circles are often reckoned by Geminus in sexagesimal parts of the circumference,[12] besides the ordinary division in 360 degrees. Among the circles on the celestial sphere also appear an "arctic" and an "antarctic" circle, limiting the always visible and the always invisible stars from the stars which are seen to rise and to set.[13] This definition presupposes a given geographical latitude and Geminus tells us[14] that the globes were made under the assumption $\varphi = 36°$, hence with an "arctic circle" at $\bar{\varphi} = 54°$ (cf. Fig. 1). The obliquity of the ecliptic is always taken as the round value $\varepsilon = 24°$.

The geographically defined concept of "arctic and antarctic circle" disappears from later Greek spherical astronomy; but the "greatest always invisible circle" remains of practical importance in the construction of astrolabes.[15] The "ringed sphere" in combination with a terrestrial globe is discussed in Chap. 6 of Book VII of Ptolemy's Geography.[16]

The Isagoge contains very little mathematical geography. Only in passing is it mentioned that the meridian can divide the semicircle of the ecliptic above the horizon in very unequal parts, e.g. in 120° and 60°.[17] Terrestrial distances are expressed in stades such that $1° = 700$ stades[18] as is to be expected before Marinus and Ptolemy.[19]

In commenting on the names and the extension of zodiacal constellations Geminus emphasizes data which are similarly found in Hipparchus' Commentary to Aratus,[20] e.g. that the constellation Cancer covers less, Virgo more, than 30°. Also reminiscent of Hipparchus is the usage of the term "sign" (ζῴδιον) for any arc of 30° length,[21] or of calling 0° of a sign its "first degree" (πρώτη μοῖρα)[22]. In enumerating the constellations Geminus begins with the zodiac at Aries, to be followed by the northern, then by the southern constellations[23]. Hipparchus, however, puts the zodiacal signs, starting with Cancer, at the end[24].

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[12] Cf. below p. 590.

[13] E.g. Manitius, p. 168, 12. The same definition again in Hyginus (2nd cent. A.D.; Astron. I, 6 ed. Bunte, p. 24; Chatelain-Legendre, p. 5). in Achilles (3rd cent.; Isag. 26, Maass, Comm. Ar. rel., p. 59), and in Cleomedes (around 400; I, 2 ed. Ziegler, p. 20/22, et passim).

[14] Manitius, p. 168, 16–20. Emend in these lines the meaningless ἐν κλίμα to ἔγκλιμα (a term regularly used by Geminus — cf. the index p. 307/8).

[15] Cf. below p. 865f.; also p. 871.

[16] Cf. below and in general Schlachter, Globus, p. 46.

[17] Manitius, p. 28, 11–14. This happens at about $\varphi = 46;30°$ (between climates VI and VII).

[18] Manitius, p. 166, 1 implies that $6° = 4200$ stades.

[19] Cf. below p. 935.

[20] Geminus, p. 5, Hipparchus, p. 127 (ed. Manitius).

[21] Cf. above p. 278. The same terminology is found in Strabo II 5, 42 (Loeb I, p. 515), in Pliny N.H. II, 178 (Budé II, p. 78) and in Cleomedes I, 10 (ed. Ziegler, p. 94, 10) who explains the measure of "1/4 sign" for the altitude of the star Canopus as "1/48 of the zodiac"; similar in Geminus, Isag. III, 15 (Manitius, p. 42, 7). Hyginus (2nd cent. A.D.) says that all five important great circles on the sphere are "divided into 12 parts" (ed. Bunte I, 6, p. 25, 20f.; ed. Chatelain-Legendre, p. 6).

[22] E.g. Manitius, p. 26, 8ff.; p. 82, 20ff.; p. 92, 6ff., etc., incorrectly rendered as "1°" by Manitius. Cf. CCAG 1, p. 163, 21f.: "30° of Cancer or 1st degree of Leo." For Hipparchus cf. above p. 278f.

[23] Isagoge, Chap. III (Manitius, p. 36–43); for the text of this chapter see also Maass, Comm. Ar. rel., p. XXV to XXVIII. Cf. also Vitruvius, above p. 577.

[24] Cf. above p. 286.

To the zodiac is given a "width" (πλάτος)[25] of 12°, this being the earliest preserved instance of a tradition which can be followed through antiquity until Sacrobosco (13th cent.)[26]. This concept of a latitudinal expanse of 12° of the zodiac is usually motivated as corresponding to the space traversed by the moon and the planets in their latitudinal motion,[27] although it is admitted[28] that Venus might go 2° beyond the limits of $\pm 6°$.

Geminus' statement about the width of the zodiac (without any explanation) is found in the same chapter (V) in which he describes the position of the principal circles on the sphere in terms of sexagesimal "parts" (μέρη) of the circumference of a circle.[29] In these schematic terms one has for the obliquity of the ecliptic $\varepsilon = 4^p$, for the standard latitude $\varphi = 6^p$, etc. It could be simply a natural rounding within this pattern to consider 1^p $(=6°)$ as a sufficiently good description for the inclination of the lunar orbit. On the other hand one could also think of Babylonian influence since $\pm 6°$ were always assumed for the extremal lunar latitudes in Babylonian astronomy.[30]

In Chap. II Geminus discusses briefly the astrological doctrine of the "aspects." He discusses it mainly from the viewpoint of geometric relations in spherical astronomy, showing little belief in the postulated influence on human life or meteorological conditions. For us, however, it is of interest that he mentions in connection with the "trine" aspect (120° distance between signs) a doctrine of the "Chaldeans" according to which four wind directions (north, south, west, east) are related to four "triangles" (north to ♈ ♌ ♐, etc.). This is indeed a rule accepted in Babylonian astrology[31]; we shall see presently[32] more Babylonian data correctly reported in the Isagoge.

Geminus' dislike of astrological doctrines leads him to criticize them as astronomically incorrect, stubbornly ignoring the fact that he is dealing only with an empty play on symmetries.[33] Modern scholars argued that Geminus had no knowledge of the Eudoxan norm which places the cardinal points of the ecliptic in the midpoints of the signs[34] because this would restore the necessary symmetries. It seems to me, however, that this proves nothing for or against Geminus' knowledge of Eudoxan astronomy. He simply sees no justification for the astrological postulates, just as he has no confidence in the generally accepted doctrines of the "parapegmata"[35].

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25 Isagoge V, 53 (Manitius, p. 62, 8 f.).

26 Pliny N.H. 66 (Budé II, p. 29; Loeb I, p. 215); Hyginus (2nd cent.) ed. Bunte, p. 104, 20–23; Chalcidius (4th cent.) ed. Wrobel, p. 136, 17–20; Martianus Capella (5th cent.) ed. Dick, p. 438, 15/16, p. 456, 22; Remigius of Auxerre (9th cent.) ed. Lutz, p. 258 (ad 434, 14); Sacrobosco, De spera, ed. Thorndike, p. 88, trsl. p. 128. A passage in Manilius (Astron. I, 682) is doubtful; cf. ed. Housman I, p. 61.

27 Chalcidius, e.g., says (ed. Wrobel, p. 136, 17–20) that "according to the opinion of the old ones" the latitudinal amplitude for the moon and for Venus is 12°. Cf. also below IV B 1, 3 p.

28 E.g. Pliny; cf. note 26.

29 Cf. above p. 582 and below p. 590.

30 Cf. above p. 515; also Neugebauer-Sachs [1968/1969], p. 203 and note 27 there.

31 Schott-Schaumberger [1941], p. 109, note 1.

32 Cf. below IV A 4, 3 A.

33 Cf. for this type of astrological doctrines Bouché-Leclercq AG, p. 159 ff.

34 Manitius. Geminus Isag., p. 255, note 6; for the Eudoxan norm cf. below p. 599.

35 Cf. above p. 581.

Looking for mathematical astronomy in the "Isagoge" one finds practically nothing about the planets. In Chap. I a few generalities about circular motion are given,[36] followed by remarks on arrangement in depth and round values for the sidereal periods (from Saturn's 30 years to 27 1/3 days for the moon)[37]. Interestingly Geminus denies the existence of a single sphere for the fixed stars, which he assumes to be distributed in depth[37a].

Although the existence of stationary points in the motion of the planets is mentioned, nothing is said about methods to explain such a phenomenon. The same tendency to avoid any mathematical discussion is also evident in the solar theory. Though the Hipparchian values for the lengths of the seasons are given[38] nothing more is deduced from them than a qualitative description of the eccentricity of the solar orbit. Similarly the "equation of time" was known to Geminus but what he considers within the grasp of his readers is only a very vague description of its causes[39].

In the lunar theory our author begins to warm up to some technical details. Here he relies on two different types of sources: in the main body of his treatise (particularly in Chap. VIII) he reports data from Greek attempts to construct calendaric luni-solar cycles. In the concluding chapter (XVIII), however, he displays parameters which we know not only from Ptolemy (who considers them Hipparchian) but also from cuneiform texts. Geminus mentions neither Hipparchus nor the Babylonians but embellishes his report with historical details which have no factual basis whatsoever, as we can say thanks to the evidence obtainable from the Babylonian texts.

For the length of the synodic month is assumed[40] the time of approximately[41] 29 1/2 1/33 days, i.e.

$$1^{m} \approx 29\ 1/2\ 1/33 = 29;31,49,5,27,\ldots^{d} \tag{1}$$

an otherwise unknown parameter. The length of the year is simply taken to be $365\ 1/4^{d}$ which is also the total of the Hipparchian seasons.[42] The "lunar year" of 12 months is schematically rounded to 12 times 29 1/2 days, i.e. to 354 days.[43]

These crude roundings are then taken as basis for the calendaric eight-year cycle, the "octaeteris."[44] Reckoning in this case "months" schematically as 30 days and calling

$$365;15-354=11;15=e \tag{2}$$

[36] Manitius, p. 10/11.

[37] Manitius, p. 12/13. At a later occasion (Manitius, p. 194, 24) the three outer planets are called "the greatest (μέγιστοι) of the planets."

[37a] The same opinion is also held by Proclus (Comm. Rep., trsl. Festugière III, p. 170, 15f.).

[38] Manitius, p. 8/9.

[39] Manitius, p. 69/71. Theon in his Great Commentary to the Handy Tables, mentions Serapion as being concerned with the equation of time. If this Serapion is the well-known contemporary of Cicero we would have evidence from the first century B.C. for the recognition of such a correction (in tables based on the era Philip). Cf. Rome [1939] and CA III, introd., p. CXXXIII, note (1). Text in Monum. 13, 3, p. 360.

[40] Manitius, p. 100, 9 and 17; p. 200, 9.

[41] Manitius, p. 200, 9: ὡς ἔγγιστα.

[42] Manitius, p. 8, 23; p. 102, 2 et passim.

[43] Manitius, p. 102, 4 et passim.

[44] Manitius, p. 110, 22; p. 114, 15 et passim.

(our "epact"), one has $8e = 90^d = 3^m$. Consequently one makes

$$8^y = (8 \cdot 12 + 3)^m = 99^m. \tag{3}$$

This determines the intercalation pattern of the octaeteris. On the other hand

$$8^y = 8 \cdot 365;15^d = 2922^d = (8 \cdot 354 + 90)^d \tag{4}$$

whereas, according to (1), these 8 years should contain

$$99^m = (99 \cdot 29\ 1/2\ 1/33)^d = 2923\ 1/2^d. \tag{5}$$

Thus (4) shows an accumulated error of 3^d in 16 years.[45] Because $29\ 1/2 \cdot 99 = 2920\ 1/2$ Eq. (4) shows that the three intercalary months in 8 years should be full while (5) shows that three hollow months in 16 years should also be made full. All this is, of course, pure arithmetical juggling without relation to the actual Greek civil calendars. No mention is made of the improved Hipparchian cycle or the Hipparchian value for the tropical year.[46]

Geminus realizes that (1) is too short and he gives

$$1^m = 29;31,50,8,20^d \tag{6}$$

as the correct value, well-known to us as a basic parameter in the Babylonian lunar theory of System B.[47] He also knows about the superior quality of the 19-year cycle[48] which is based on the following relations:

$$19^y = (19 \cdot 12 + 7)^m = 235^m = 125 \text{ full} + 110 \text{ hollow m.} = 6940^d. \tag{7}$$

Geminus ascribes these relations to the school of Euctemon, Philip, and Callippus (without mentioning Meton[49]). Apparently under the same heading goes also the slightly modified cycle of $4 \cdot 19 = 76$ years

$$76^y = 76 \cdot 365\ 1/4^d = 27759^d = (4 \cdot 6940 - 1)^d = 4 \cdot 235^m = 940^m. \tag{8}$$

Geminus does not mention the consequences of these relations, i.e.

from (7): $1^m = 29;31,54,53,36, \ldots^d$ $1^y = 365;15,47,22, \ldots^d$
from (8): $1^m = 29;31,51,3,49,46, \ldots^d$ $1^y = 365;15^d$

excepting the very last one — which is trivial because of the initial construction of the cycle.

Chap. XVIII begins by assuming again $29\ 1/2\ 1/33^d$ as approximate length of the synodic month,[50] ignoring the better Babylonian value (6)[51] from Chap. VIII.

[45] From (4) and (3) it would follow that $1^m = 2922:99 = 48,42:1,39 = 29;30,54,32,43,38, \ldots^d$.
[46] Cf. above I E 2, 2 C.
[47] Manitius, p. 116, 20–23; cf. also above p. 69 (1), p. 310, and p. 483 (3).
[48] Manitius, p. 118 to 123.
[49] Rome CA III, p. 839, note (1) suggests an emendation of the text on the basis of Theon's version of the history of the 19-year cycle. Note, however, below p. 623, note 12.
[50] Above p. 584 (1).
[51] Above note 47.

The next parameter,[52] however, the length of the anomalistic month

$$1 \text{ anom.m.} = 27\ 1/2\ 1/18 = 27;33,20^d \qquad (9)$$

is of Babylonian origin: the variable daily velocity of the moon has exactly this period according to System B.[53]

Next comes the fundamental period relation, called the "exeligmos," well-known from the Almagest[54]

$$669 \text{ syn.m.} = 19756^d = 717 \text{ anom.m.} = 723 \text{ sid.rot.} + 32^\circ = 260312^\circ \qquad (10)$$

[or sexagesimally

$$11,9 \text{ syn.m.} = 5,29,16^d = 11,57 \text{ anom.m.} = 12,3 \cdot 6,0 + 32^\circ = 1,12,18,32^\circ].$$

Geminus explains[55] that the "Chaldeans" found from (10) for the mean velocity of the moon

$$\frac{260312^\circ}{19756^d} = 13;10,35^{\circ/d} \qquad (11)$$

and for the length of the anomalistic month the value (9) from

$$\frac{19756^d}{717} = 27;33,20^d. \qquad (12)$$

In fact neither one of these equations is accurate. Instead of (11) one finds

$$1,12,18,32/5,29,16 \approx 13;10,34,51,55,17, \ldots^{\circ/d}$$

and instead of (12)

$$5,29,16/11,57 \approx 27;33,13,18, \ldots^d.$$

But both results claimed in (11) and (12) represent exactly mean value and period of the lunar velocity in System B.[56]

The concluding discussion[57] fully confirms the ultimate Babylonian origin of these data by giving all parameters of the linear zigzag function for the daily lunar velocity in System B[58]:

$$\begin{aligned} \text{extrema:}\quad & m = 13;10,35 - 2;4 = 11;\ 6,35^{\circ/d} \\ & M = 13;10,35 + 2;4 = 15;14,35^{\circ/d} \\ \text{difference:}\quad & d = \ \ 0;18 \end{aligned} \qquad (13)$$

[52] Manitius, p. 200, 10; p. 204, 23/24.
[53] Denoted above II B 2, 3 as F*; cf. also below p. 602.
[54] Almagest IV, 2 (Manitius, p. 195f.), obtained by multiplication with 3 of the famous "Saros" relation (cf. above p. 502 (1) and p. 310 (5)). Surprisingly Geminus ignores the further equivalence with 726 = 3 · 242 draconitic months which is the key to the theory of eclipses.
[55] Manitius, p. 205.
[56] Cf. above p. 480 (1) and (2a). This fictitious derivation of standard Babylonian parameters is reminiscent of Ptolemy's procedure in motivating the value (6) of p. 585 (cf. above p. 310).
[57] Manitius, p. 205–211.
[58] Cf. above p. 480 (1) or below p. 602.

with mean value (11) and period (12) as arithmetical consequences. The derivation which Geminus gives for (13) is again purely fictitious. This is evident not only on historical grounds – Geminus could hardly know how these parameters were obtained in Mesopotamia many centuries earlier – but his arithmetical arguments[59] are not conclusive since, e.g., $d = 0;16$ and $0;18$ would be equally admissible. Here, what Geminus has to say about lunar theory ends abruptly.

3. The Parapegma

As we have mentioned before[1] the Isagoge is followed by a calendaric text which is probably two centuries or so older than Geminus, but which we nevertheless simply call the "Geminus parapegma."

The term "parapegma" originally means an inscription, exhibited, like a sundial, for public use. It contains a list of fixed star phases and associated predictions for the weather for a whole year. Because of the irregular fluctuations of the Greek lunar calendars the dates of the phases in the civil calendar of the current year were shown on pegs which were inserted in little holes[2] drilled into the stone beside the lines of the text.[3] Fragments of such inscriptions were excavated in Miletus,[4] one exactly dated to the year $-109/8$, the other about two decades younger.[5]

Beside the stone inscriptions a literary form of "parapegmata" developed of which the Geminus parapegma is the earliest extant example. The text gives, exactly as the inscriptions, a list of consecutive fixed star phases and weather prognostications. The role of the holes which represent consecutive days is now taken over by a day-by-day progress of the sun. Thus the "Geminus parapegma"

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[59] The arithmetical rule that in a linear progression the mean value is half the total of the extrema is reminiscent of (less trivial) statements by Hypsicles (cf. below IV D 1, 2 A).

[1] Above p. 580f.

[2] Greek κυκλίσκοι; cf. Diels-Rehm [1904], p. 102.

[3] The Greek term παράπηγμα belongs to a verb meaning to "fix beside, or near." The German term for these inscriptions is "Steckkalender" (Diels-Rehm [1904], p. 100).

[4] Photographs in Diels-Rehm [1904], Pl. II (also Diels, AT pl. I). As an example may serve the following lines from the earlier Milesian parapegma (l.c. p. 104):

30 (days remain)
O the sun in Aquarius.
O [Leo] begins morning setting, and Lyra sets.
O O
O Cygnus begins acronychal setting.
O O O O O O O O O O
O Andromeda begins morning rising.

The holes between the lines represent so many days without predictions. Hence the setting of Cygnus belongs to day 5 in Aquarius, the rising of Andromeda to day 15.

[5] Cf. Rehm R. E. Par. col. 1300, 5 and 42. For two other small fragments, one from Athens, the other from Pozzuoli, cf. l.c. col. 1301/2 A 3 and A 4. Cf. also Degrassi, Inscr., p. 299, p. 306–311.

has the title[6]

> "Time intervals in which the sun traverses the single signs
> with prognostications as recorded.[7]
> We begin with the summer-solstice.
> The sun traverses Cancer in 31 days."

Similarly for the other signs, e.g. "The sun traverses Sagittarius in 29 days" or "... Taurus in 32 days." Such a scheme seems to imply the recognition of a solar anomaly but we shall see that this is too hasty a conclusion.[8]

In all probability the construction of parapegmata was an invention of Meton and Euctemon, in the fifth century B.C.[9] Subsequently many of the well-known astronomers must have compiled such meteorological calendars in relation to fixed star phases; six such authorities are quoted in the Geminus parapegma,[10] twelve in Ptolemy's calendar.[11] The earliest authorities are usually Meton and Euctemon,[12] the latest Dositheus (3rd cent.) in Geminus, Caesar in Ptolemy. Eudoxus appears prominently in all sources. The older Milesian parapegma does not mention its sources though it has been shown that it follows Euctemon and Callippus[13]. The same monument also carried an introduction to the use of the calendar[14] and gives a definite date for the summer solstice of −108 June 26 in the form of Skirophorion 14 = Payni 11 of the year of the Athenian archon Polykleitos[15] after having mentioned the summer solstice of −431 June 27 (Skirophorion 13 = Phamenoth 21), being the starting point for Meton's 19-year cycle[16].

The second parapegma from Miletus, however, mentions its sources. Preserved are the names of Euctemon and Philip, of Eudoxus and the "Egyptians" (prominent in the same anonymous fashion in Ptolemy's parapegma[17]) and a "Kallaneos from India" who appears in no other parapegma. Diels made the plausible suggestion that this Kallaneos is identical with the gymnosophist Kalanos, known from the Alexander-history, playing a role similar to Nechepso-Petosiris, Zoroaster, etc.[18]

The earlier Milesian parapegma is peculiar also in so far as it ignores the meteorological purpose of such a calendar. Beside the holes for the pegs it gives

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[6] For reasons unknown Manitius (p. 211 ff.) translated neither the title nor the headings but gave only a paraphrase.
[7] The term "parapegma" does not occur in the title of the literary versions. Geminus, however, in the Isagoge, uses the term as freely as the moderns (e.g. Manitius, p. 182, etc.). Manitius' rendering "calendar" is quite appropriate.
[8] Cf. below p. 628.
[9] Cf., e.g., Rehm Parap., p. 7.
[10] Cf. above p. 581, n. 13.
[11] In Book II of the "Phaseis"; cf. below V B 8, 1 B.
[12] In the Geminus parapegma Meton is only mentioned once (cf. below p. 629, n. 12).
[13] Rehm RE Par. col. 1300, 23.
[14] Diels-Rehm [1904], p. 102/3.
[15] In Diels-Rehm [1904] the julian date is incorrectly given as June 27. Merritt, Ath. Cal., p. 88 gives by mistake Payni 14 instead of 11. All dates are correct in Dinsmoor, Archons, p. 312.
[16] Cf. below p. 622.
[17] Cf. below p. 929 and above p. 562 f.
[18] Diels-Rehm [1904], p. 108/9, note 1.

only stellar phases but nothing about expected climatic conditions. It almost looks as if Geminus was not alone in his skepticism about predictions from stellar phases[19]. The mediaeval development, however, went in the exactly opposite direction. When Sinān b. Thabit b. Qurra around 900 A.D. made an excerpt from Ptolemy's "Phaseis" he omitted all stellar phases and gave only the meteorological predictions, now arranged according to the julian dates of the Syrian calendar,[20] obtained without change from the Alexandrian dates given by Ptolemy some seven centuries earlier.

Bibliographical Note. The following references should serve as a first guide to the extensive literature on the parapegmata (also called "calendars," but not to be confused with Greek civil calendars).

Boekh, Sonnenkr. (1863) is the basic study in this area. For the texts cf. Lydus, De ost. (ed. Wachsmuth, 1897) which includes the calendars from Geminus and from Ptolemy's Phaseis, reedited by Manitius[21] (1898) and Heiberg[22] (1907), respectively. The discovery of fragments of parapegmata on stone provided the starting point for the modern discussion: Diels-Rehm [1904], Dessau [1904], Rehm [1904]. Then follow the essays collected in Boll, Griech. Kal. I–V (1910–1920); finally Rehm [1927] and his articles RE Ep. (1940), RE Par. (1949), with Parap. (1941).

§ 4. Babylonian Influences

The mere fact itself of Babylonian influences on hellenistic astronomy is obvious. It suffices to mention the all-pervading use of the sexagesimal system, e.g. in the reckoning of time or in the division of the circle,[1] the presence of Babylonian parameters in Hipparchus' lunar theory, or the frequent use of Babylonian arithmetical patterns in various fields, e.g. mathematical geography, gnomonics, etc.

It is much more difficult, however, to determine with reasonable accuracy the time of transmission or the mode of contact and to evaluate correctly the degree and importance of the influence of Babylonian astronomy on the nascent Greek science. Without insight into specific technical details one can easily overemphasize influences which in fact do not require more than the transmission of a few basic

[19] Cf. above p. 581 and p. 583. Also Columella, De re rustica XI 1, 31 (Loeb III, p. 68/69), opposes the doctrine of fixed definite dates for the changes of air. Similarly Ptolemy has his doubts about the prognostications associated with the stellar phases (cf. below p. 926, n. 4).

[20] Sinān's "parapegma" is preserved through Bīrūnī in Chap. XIII of his "Chronology" (trsl. Sachau, p. 233–267). Cf. JAOS 91 (1971), p. 506.

[21] Geminus, Elementa Astronomiae, p. 210–233 with German translation and notes.

[22] Ptolem. Opera II, p. 14–67; p. III–V; p. CL–CLXV. No translation.

[1] Contrary to a widespread belief the sexagesimal system did not originate from any astronomical concept. Its beginnings go back to the earliest Mesopotamian civilization, more than a millennium before any computational astronomy existed. Its origin can be found in the norms for weights and measures in combination with palaeographical processes which lead to the place value notation which is the most characteristic element of this number system: cf. Neugebauer [1927] and Thureau-Dangin SS.

concepts. One has to face still greater difficulties in the evaluation of secondary transmissions, e.g. into India where the problem of hellenistic and Iranian intermediaries may seem unsolvable.

1. The Sexagesimal System

The earliest evidence among Greek authors of sexagesimal units seems to be with Eratosthenes, i.e. around 250 B.C., in his division of the circumference of a circle in 60 parts.[2] Such a division of the equator has its equivalent in the Babylonian sexagesimal division of the day in the ephemerides of the Seleucid period.

Aristarchus, some 50 years earlier, gives angular measurements in fractions of quadrants only; e.g. for an elongation of $87^\circ = 90^\circ - 3^\circ$ he would say "less than a quadrant by one-thirtieth of a quadrant."[2a]

Perhaps about a century after Eratosthenes we find in the inscription of Keskinto in Rhodes[3] a division of the circle in 360 degrees or in 720 half-degrees called "points." The probably somewhat later "Anaphorikos" of Hypsicles[4] uses degrees[5] as we do.

All sources agree that Hipparchus divided the quadrant of the meridian in 90° while his lunar parameters also exactly follow the Babylonian sexagesimal norm. Hence one may say that the available evidence suggests the century from about 300 to 200 B.C. as the period of reception of the Babylonian sexagesimal system by Greek astronomy. Some data in the Almagest (VII, 3) would concern the very beginning of this interval when Ptolemy credits the school of Timocharis[6] with a list of declinations of fixed stars,[7] measured in degrees and fractions of degrees.[8] One must, however, admit the possibility that Ptolemy, or Hipparchus, adapted these earlier observations to the units they were using themselves.

In the Almagest the results of arithmetical operations with sexagesimal numbers are always given without further explanation. Even such an elementary

[2] Strabo, Geogr. II 5, 7 (Loeb I, p. 438/9; with different reading: Budé I, 2, p. 85). The same norm is still used in Geminus (Isag., Manitius, p. 58, 23ff.; p. 183, 3ff. etc.; 1st cent. A.D.), in Manilius (Astron. I, 561ff.; Housman I, p. 53; Breiter, p. 21/22; 1st cent. A.D.), in Plutarch (Moralia 590 F, Loeb VII, p. 464/465; ≈A.D. 100), by Galen (2nd cent., cf. Rehm [1916], p. 82), in Hyginus (Astron., I, 6, ed. Bunte, p. 24; Chatelain-Legendre, p. 5; 2nd cent.), in Achilles (Isag. Maass, Comm. Ar. rel., p. 59, 5; p. 70, 12; 3rd cent.), by Macrobius (≈400: Comm. II 6, 2–5, ed. Eyssenh., p. 606, 24–607, 16; ed. Willis, p. 116, 15–117, 3; trsl. Stahl, p. 207), and by Severus Sebokht (A.D. 660: Nau, Const., p. 93).

Achilles (Maass, p. 59, 24ff.) adds the remark that "some" divide the circle not in 60 but in 360 degrees (*μοίρας*) "because the year has 365 days." In the subsequent description of the angles shown in Fig. 1 (below p. 1351) he makes several mistakes. Martianus Capella (De nupt. VIII 837, ed. Dick, p. 439; 5th cent.) makes 1 quadrant = 18 parts thus $1^p = 5^\circ$. This is obviously absurd since it implies $\varepsilon = 4^p = 20^\circ$. Thus one must emend the $(8+6+4)^p = 90^\circ$ of the text to the same norm $(6+5+4)^p = 90^\circ$ found in the above mentioned sources.

[2a] Heath, Arist., p. 352/3 (Hypot. 4); cf. also below p. 773, notes 6 to 9.

[3] Cf. below p. 699.

[4] Huxley [1963], p. 103 suggests the middle of the second century B.C.

[5] De Falco-Krause-Neugebauer, Hypsikles, passim.

[6] Observations by Timocharis mentioned in the Almagest range between −294 and −271.

[7] Alm. VII, 3, Heiberg II, p. 19–23; Manitius II, p. 18–20.

[8] The translation of Manitius is misleading in so far as he gives always minutes of arc where the text has only unit-fractions of degrees.

treatise as the "Isagoge" of Geminus[9] considers sexagesimal computations within the grasp of its readers. But the further one moves away from the productive period of Greek astronomy towards the didactic phase the more one finds long and clumsy explanations of sexagesimal operations.[10] Byzantine treatises are full of such trivia,[11] e.g. in connection with linear interpolation.

Unfortunately the Greek astronomers never adopted a strictly sexagesimal notation. Ptolemy, e.g., will write 125;15° instead of 2,5;15°. In later periods one also finds ordinary fractions after sexagesimal fractions, e.g. 43;48,51 1/2 in Pappus,[12] thus anticipating the mixed notation of modern astronomy. In part it is no doubt due to the influence of the Egyptian arithmetical habits when results of sexagesimal computations are finally given in the form of unit fractions.

Babylonian influence on units of measurement is not limited to the sexagesimal division of the circle in degrees and its fractions but it is also visible in the use of "cubits" (*πήχεις*) and "fingers" (*δάκτυλοι*). These units are, of course, basically measures of lengths, only secondarily adapted to angular measurements – perhaps through divisions on observational devices. In Babylonian astronomy two norms for these units are attested[13]: an older one (from the Persian period and before) according to which

$$1 \text{ cubit} = 30 \text{ fingers} = 2;30^\circ \tag{1}$$

and a younger one, in use during the hellenistic period

$$1 \text{ cubit} = 24 \text{ fingers} = 2^\circ. \tag{2}$$

Notice that in both cases

$$1 \text{ finger} = 0;5^\circ, \qquad 1^\circ = 12 \text{ fingers}. \tag{3}$$

Which one of the norms (1) or (2) is followed in a Greek text is usually impossible to say. Ptolemy discarded these units in favor of degrees; nevertheless we owe to his references the two earliest observational records which use half-cubits[14] for distances of Mercury from bright stars, based on observations made in the years 67 and 75 of the "Chaldean era," i.e. in −244 and −236, respectively.[15] Comparison with modern data seems to speak in favor of the norm (2).[16]

For Hipparchus we have a variety of sources: Strabo quotes him for solar altitudes, expressed in cubits[17] which undoubtedly follow (2). Through Ptolemy

[9] First century A.D.; cf. above (p. 579f.).
[10] Such is the case already in the Commentaries to the Almagest by Pappus and Theon; cf. Mogenet [1951] or Rome CA II, p. 452–462.
[11] Cf., e.g., Diophantus, Opera II, ed. Tannery, p. 3–15, or Pachymeres, Quadrivium, ed. Tannery, p. 331–363 (written about A.D. 1300).
[12] Rome, CA I, p. 186, 13.
[13] Cf., e.g., ACT I, p. 39 and Neugebauer-Sachs [1967/1969] I, p. 204/205.
[14] Alm. IX, 7 Heiberg II, p. 267, 14 and 268, 2 (*πήχεως ἥμισυ*).
[15] Cf. above p. 159f.
[16] In both cases one finds that the longitude of Mercury (as morning star) was almost exactly 1° greater than the longitude of the star (the latitudinal intervals are 1;35° and 1;5°, respectively). Since Ptolemy is only interested in the longitudinal component of the distance between the planet and the mean sun, the term *ἐπάνω*, literally "above," seems here to mean "ahead (in longitude)." This is reminiscent of the terminology of Theodosius, where, however, *ἀνώτερον* denotes the point ahead in the direction of the daily rotation (cf. below p. 758).
[17] Strabo, Geogr. II 1, 18; cf. above p. 304.

we know of four instances which involve fingers and one distance between fixed stars counted in cubits.[18] Finally we have direct data from the second part of the Commentary to Aratus[19] where Hipparchus expresses distances of fixed stars from the meridian to the east or to the west as "about 1/2 cubit" (ὡς ἡμιπήχυον)[20] or "about 2/3 cubits" (ὡς δύο μέρη πήχεως)[21]; the directional preference is perhaps the result of some specific instrumental arrangement. By comparison with modern data Vogt concluded[22] that the underlying norm is the older one (1).

Fingers as angular measurements do not occur in the Aratus Commentary.[23] A relatively late occurrence is found in a horoscope cast for A.D. 81.[24]

Peculiar metrological data appear in a papyrus fragment from the first or second century A.D., P. Oslo 73,[25] where we find the following statements: first, in measuring the apparent diameter of the sun the following units are used:

$$1 \text{ solar-cubit} = 7\,1/2 \text{ palms} = 30 \text{ fingers}, \qquad (4)$$

second: 720 solar diameters cover the whole cosmic circle, third: the solar diameter is divided into "palms, fingers, barley-corns, and hairs." The relation (4) implies 1 palm = 4 fingers and is well-known in Egypt,[26] whereas the "barley-corn" is a Mesopotamian unit.[27] The "hair" must be a fraction of the barley-corn but seems otherwise unknown. It is clear that the text assumes $1/2^\circ$ for the solar diameter but it is not sure that the "solar-cubit" (πῆχυς ἡλίου) is meant to be the same as the apparent diameter of the sun — otherwise we would have here a division of the latter into 30 digits.

Additional problems arise from the customary division of the apparent diameters of sun and moon in 12 "digits" (δάκτυλοι, i.e. the same term as "fingers"). According to the norm (3) this would mean an apparent diameter of 1°, twice the actual value. It is precisely this relation which we find expressly accepted in a cuneiform text from the pre-Seleucid period.[28] One can hardly escape the conclusion that this incorrect evaluation is the basis of the later norm for the "digits" of eclipse magnitudes — by then completely separated from the distance measurements for which (3) remains valid.

Perhaps the astrological literature has preserved a trace of the early estimate of the size of the luminaries. Thrasyllus (≈ A.D. 30, thus contemporary with Geminus) says[29]: "the degree is the size of the moon or also the sun's [diameter]."[30] Of course one could also assume a simple mistake by interpreting the 12 diameter-

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[18] Alm. VII, 1 Heiberg II, p. 4, 16; 5, 1; 7, 14; 8, 2 and p. 6, 11, respectively.
[19] Manitius, p. 186–280; cf. above p. 279.
[20] Manitius, p. 186, 11 etc. (50 cases), perversely translated by Manitius by "Mondbreite."
[21] Manitius, p. 206, 4 etc. (9 cases). Otherwise one finds only two more passages which mention cubits, and this only in a loose fashion (Manitius, p. 190, 10 and 272, 1).
[22] Vogt [1925], col. 30.
[23] The only passage, Manitius, p. 272, 2 is an arbitrary emendation; cf., however, above note 18.
[24] P. Lond. 130; cf. Neugebauer-Van Hoesen, Gr. Hor., p. 26.
[25] Pap. Oslo III, p. 30.
[26] Cf., e.g., Gardiner, Eg. Grammar, § 266.
[27] Cf., e.g., ACT I, p. 39: 1 finger = 6 barley-corns.
[28] Cf. Neugebauer-Sachs [1967/1969] I, p. 203; also above II C 2, p. 551.
[29] CCAG 8, 3, p. 99, 7f.
[30] Text by mistake "circumference."

digits as 12 distant-fingers and hence equating them with 1°. This type of error could also be assumed for a statement of Geminus[31] in which he assigns to the earth's shadow a width of 2°. If, according to (3), the diameter of the shadow is taken to be 24 "digits" as the lunar diameter has 12, one obtains the conventional relation[32] $u = 2\,r_{☾}$.

2. The Ecliptic and its Coordinates

The Almagest and all ancient and medieval mathematical astronomy uses orthogonal ecliptic coordinates for its coordinate system. That this system is of Babylonian origin became clear as soon as one gained access to the ephemerides in cuneiform texts of the Seleucid period. Here it will suffice to remark that the zodiacal signs of fixed equal length (30° each) are already attested in the Achaemenid period.[1] Hence the change from the older irregular zodiacal constellations to the familiar coordinates of longitudes seems to have taken place around 500 B.C. This does not mean, however, that older notations disappeared at once;[2] lunar positions, e.g., with respect to "Normal-Stars" occur in a horoscope as late as S.E. 169 (= −141)[3] and a similar procedure is followed regularly in the "Normal Star Almanacs" during the whole hellenistic period.[4]

The question when and how the Greeks assimilated the Babylonian zodiac and when they recognized the ecliptic as a definite great circle produced an extensive literature of little consequence for our understanding of the working of early Greek astronomy. Starting from a passage in Pliny,[5] ambiguous and unclear as usual, it has become customary to associate the introduction of the zodiac into Greek science with the name of Cleostratus (probably about 550 to 500 B.C.[6]). For us the only interesting information is the fact that the earliest extant treatises on spherical astronomy, by Autolycus and Euclid[7] around 300 B.C., deal with the zodiac and the ecliptic as well-known concepts. Archaeological evidence begins still later, in Egypt, with the temple of Esna A[8] around 200 B.C. In short, reliable documentation begins, as usual, only with the hellenistic age.

The reckoning in the Almagest of the 360 degrees of longitude, beginning at the vernal equinox, called Aries 0°, is, of course, related to the discovery of pre-

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[31] Isagoge XI, 7 Manitius, p. 134/135; p. 271, note 23.

[32] Cf., e.g., below p. 635 (4) and (5); p. 654 (8); p. 667 (1).

[1] In a list of solar eclipses, preserved for the years from −474 to −456; cf. Aaboe-Sachs [1969], p. 17. Other early evidence: Neugebauer-Sachs [1967], p. 197f. (≈ −430); Aaboe-Sachs [1969], p. 3ff. (≈ −400).

[2] For an example of the earlier terminology (in −418/17) cf. Sachs in Neugebauer, Ex. Sci.[(2)], p. 140.

[3] Cf. Sachs [1952, 3], p. 62.

[4] Cf. Sachs [1948], p. 281; also above II C 1, p. 545.

[5] Pliny, NH II, 31 (Loeb I, p. 188/9; Budé II, p. 17).

[6] Cf., e.g., W. Kroll in RE Suppl. 4 (1924), col. 912f.

[7] Cf. below IV D 3, 1.

[8] Cf. Neugebauer-Parker, EAT III, p. 204f. Since zodiacs belong naturally to the ceiling decorations the chance of destruction of these monuments is particularly great.

cession and the resulting decision to define the solar "year" as the tropical year.[9] In accepting this definition one completely severs all relations between the zodiacal signs determining longitudes and the zodiacal constellations. In Babylonian astronomy no distinction was made between tropical and sidereal year. Longitudes were not counted from the vernal point but from the siderеally fixed endpoints of the zodiacal signs, i.e. in terms of signs and degrees from 0 to 30, not in degrees from 0 to 360.[10] In this fixed ecliptic the equinoxes and solstices were located at certain points, in "System A" at the 10th degree, in "System B" at the 8th degree of their respective signs.[11] Kugler first noticed[12] that both norms, in particular the second one, appear also in Greek and Roman sources, thus providing another striking example of the impact of Mesopotamian astronomy on the west.

If we assume that around 300 B.C. the vernal point was given a sidereal longitude (i.e. reckoned with respect to a certain fixed star) of about 8° or 10° then its sidereal longitude around the beginning of our era would be, because of precession, in the neighbourhood of 5°. This is in fact the order of magnitude of the deviation between modern (i.e. tropical) longitudes and longitudes given in Greek horoscopes.[13] In other words the astrological literature of the hellenistic-Roman period still preserves the norm of Babylonian astronomy.

A. Aries 8° as Vernal Point

Pliny in Book XVIII of his Natural History relates the work of the farmer to the seasonal conditions as they change in the course of the year and he considers the phases of the fixed stars as "indicative" for these changes, accepting the well-known doctrine of the "parapegmata."[1] In this context it happens that he places the cardinal points of the astronomical seasons at the 8th degree of their respective signs,[2] e.g. the summer solstice (on June 24) at ♋ 8°.[3] Since Pliny expressly follows Caesar ("dictator")[4] we can consider his testimony as being valid as well for the early half of the first century B.C. Probably because of the same norm the catalogue of stars in the Liber Hermetis[5] begins with a small star[6] at ♋ 8°; this again

[9] Cf. above I B 1, 1.
[10] The enumeration of the signs, however, always begins with Aries and in this sense one can say that longitudes are counted from 0° to 360°. Incidentally, the Babylonian name for the first sign does not mean "the Ram" but "the Hireling" (lú-ḫun-gà).
[11] Cf. above II Intr. 4, 1 etc.
[12] Kugler, BMR, p. 104ff.
[13] Neugebauer-Van Hoesen, Greek Horosc., p. 180ff.
[1] Cf. above IV A 3, 3 and below p. 929.
[2] Pliny NH XVIII 58, 221 (Loeb V, p. 328/329; ed. Jan-Mayhoff III, p. 204, 12).
[3] Pliny NH XVIII 58, 264 (Loeb V, p. 356/357; ed. Jan-Mayhoff III, p. 216, 5).
[4] Pliny NH XVIII 57, 214 (Loeb V, p. 324/325; ed. Jan-Mayhoff III, p. 202, 1 and 17). On the other hand Lydus, De mensibus IV, 18 (ed. Wuensch, p. 79, 13f. = Caesar, Comm. III, ed. Klotz, p. 219) says – about A.D. 550 – that according to Caesar the sun enters Aquarius on Jan. 22. The Philocalus Calendar of A.D. 354 has even Jan. 23 (cf. Stern, Cal., p. 58f.); but the comparison with the other dates in this calendar shows clearly that Jan. 23 falls outside the scheme of the remaining dates and should be emended to Jan. 17. Obviously Jan. 22 or 23 is a later correction, made in order to obtain a solar longitude in agreement with the norm of Aries 0° for the vernal point. Indeed one has for -50 Jan. 22 for the sun $\lambda \approx 301°$.
[5] Gundel HT, p. 148.
[6] Can γ.

would be a witness from the beginning of the first century B.C.[7] Toward the end of the century Vitruvius states that the vernal equinox occurs when the sun traverses the 8th degree of Aries.[8] The "Fasti Venusii"[9] concerning the two decades after 15 B.C. place the entry of the sun into Cancer on June 19, the summer solstice 7 days later on June 26.[10] This is obviously the calendaric equivalence for the sun's travel between the first degree of Cancer and the solstice at Cancer 8°. Hence we have ample evidence for Aries 8° as vernal point for the two centuries which straddle the beginning of our era.

The earlier history of the appearance of this norm in Greek astronomy depends, unfortunately, on two Roman works on agriculture, one by Varro (about 40 B.C.), the other by Columella (around 65 A.D.[11]). Varro says that the 23rd "day" of Aquarius, Taurus, Leo, and Scorpio marks the boundaries of the seasons spring, summer, autumn, winter, respectively.[12] The use of "days" within zodiacal signs is, of course, the equivalent of degrees, based on an average solar motion of 1° per day. The "seasons" thus schematically defined by quadrants of the ecliptic must be distinguished as "civil seasons" from the "astronomical seasons" which are limited by the equinoxes and solstices and thus determine a second set of quadrants on the ecliptic. Assuming that the astronomical seasons exactly bisect the civil seasons one obtains for the equinoxes and solstices the 8th degree of Aries, Libra, and Cancer, Capricorn, respectively ($23+45=60+8$). It is these points which are expressly mentioned in Columella's treatise on agriculture.[13] The combined system of eight arcs of 45° each, based on ♈8° as vernal point, was called by Mommsen the "Bauernkalender" (= rustic calendar)[14]; he also gave a table of the corresponding dates in the Julian calendar as established by Caesar.[15] As we have seen Caesar's parapegma operated with the same norm.[16]

A whole literature has grown up around the problem of the prehistory of this system.[17] Columella himself remarked[18] that the entry of the sun into Capricorn on Dec. 17 defines the winter solstice according to Hipparchus, while December 24

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[7] Cf. for this date Neugebauer, Ex. Sci.[(2)], p. 68f.

[8] Vitruvius, De archit. IX, 3 (Loeb II, p. 232–235; ed. Krohn, p. 209; also Loeb II, p. 266/267; Krohn, p. 222).

[9] From Venosa in Apulia (east of Melfi). Cf. Degrassi, Inscr., p. 55–62 and Tab. IX.

[10] Degrassi, Inscr., p. 58f. Since the entry of the sun into Gemini is given for May 18 the date of entry into Cancer should perhaps be emended to June 18. Such variations by one day are, of course, explicable by the ambiguity of the use of the term "first degree." The Philocalus Calendar of A.D. 354 gives June 15 for the entry, June 24 for the solstice (as Pliny) but the differences suggest an emendation to 16 or 17; cf. Stern, Cal., p. 58f. and Degrassi, Inscr., p. 248/249.

[11] Cf. for this date Rehm, RE Par. col. 1309, 50.

[12] Varro, De re rust. I, XXVIII (Loeb, p. 248–251).

[13] Columella, De re rust. IX, XIV 10–12 (Loeb II, p. 480–489; also Wachsmuth, Lydus, De ost., p. 303).

[14] Mommsen, Chron.[(2)], p. 58, p. 60. The term is misleading: no peasant constructed this calendar which is simply the Roman version of a Greek parapegma. Mommsen only intended to underline the usefulness and need for agricultural work of a calendar based on a solar year, recognizable by fixed star phases.

[15] Mommsen, Chron.[(2)], p. 62 or Ginzel, Hdb. II, p. 282.

[16] Cf., however, above p. 594, note 4.

[17] For additional references cf. Rehm [1927]; also Rehm RE Par. col. 1324 (B 18), col. 1352, 57–1353, 45 and Rehm Parap. Chap. III. I never succeeded separating facts from mere hypotheses in this vast literature.

[18] Columella, De re rust. XI, II 94 (Loeb III, p. 124/125).

is the winter solstice according to the Chaldeans. What obviously underlies this statement is the fact that the "1st degree" of Capricorn (meaning ♑0°[19]) is the solstice in Hipparchus' norm whereas the Chaldean norm gives ♑1+7=♑8° (without understanding, of course, the change of ecliptic coordinates from Babylonian to Greek astronomy). One could also consider the symmetric four-division of the year as a reflection of the similar pattern in the "Uruk-scheme"[20] but such a division seems natural at a primitive stage of astronomy and therefore cannot be used as evidence for direct borrowing.[21]

In IX, XIV 12 Columella once more states that Hipparchus located the cardinal points at the "first degrees" of their signs, in contrast to the 8th degree according to Eudoxus and Meton. What we know about the reception of the sexagesimal system,[22] however, makes it very doubtful that Eudoxus (in the fourth century B.C.) and Meton (in the fifth) would have operated with a degree-division of the ecliptic. I suspect that the famous names Eudoxus and Meton were only added, some four centuries later, to give credit to an antiquated norm which had survived among Roman calendar makers[23] and astrologers. In the case of Eudoxus we have excellent evidence that he had placed the cardinal points at the midpoints of the signs,[24] i.e. at 15°, not at 8° as Columella would have it.[25] Admission of our ignorance about Meton's vernal point[26] seems to me required, and the situation is hardly different for Callippus, a century later.[27]

Proof for the continued use of the Babylonian norm of "System B" for the vernal point during the first five centuries A.D. need not to be discussed in great detail; the subsequent table will give the essential information.

A.D. 15	Manilius	Astron. III 257 and 680[28]
30	Thrasyllus	CCAG 8, 3, p. 99, 7
50	Geminus	Isag. I, 9, Latin version, Manitius, p. 286, 4 and 11–17
65	Columella	De re rust. IX 14, 12; XI 2, 31[29]

[19] Cf., e.g., Rhetorius (≈ A.D. 500) who says "at the 30th degree of Cancer, that is at the 1st degree of Leo" (CCAG 1, p. 163, 12f.). The traditional mixup between "first degree" and 0° or 1° mars ancient as well as modern interpretations; cf. also above p. 278 and below p. 600, n. 23.

[20] Cf. above p. 360.

[21] Rehm, Parap., p. 33f. and RE Par. col. 1343, 48ff. thought that Eudoxus' parapegma could be more accurately dated to 370 B.C. because the symmetry of the seasons should have been adopted under Plato's influence who was opposed to an anomaly of the solar motion.

[22] Cf. above IV A 4, 1.

[23] Columella (IX, XIV 12): "antiquorum fastus astrologorum" (Wachsmuth, Lydus De ost., p. 303, 26f.; Loeb II, p. 488/489).

[24] Cf. below p. 599.

[25] Rehm, for reasons that escape me, follows Columella (cf. below p. 599, note 10).

[26] Rehm (RE Par. col. 1310, 47–49) does not follow Columella in the case of Meton (why?) but postulates ♈10° for Meton's vernal point (cf. below p. 598, note 1).

[27] I am not convinced of the customary association of the Roman "rustic calendar" with Callippus (Rehm [1927], p. 216; Parap., p. 44 and references given there).

[28] Ed. Housman III, p. 23, p. 68; ed. Breiter, p. 73, p. 88 and p. 87, p. 106. Manilius does not follow, however, a consistent system; in I 622 and 625, e.g., he assumes the beginnings of the signs (cf. Breiter, p. 106).

[29] Loeb II, p. 488/489; Loeb III, p. 86/87; etc.

A.D. 75	Pliny	NH II 17, 81; NH XVIII 59, 221 [30]
120	Manetho	Apotel., ed. Koechly, p. 5/6, v. 72–74 [31]
150	P. Mich. 149	XI, 27–31 [32]
160	Vettius Valens	Anthol. IX 11 and tables in Book VIII [33]
200 [34]	Fragm. Censorini	De die nat., ed. Jahn, p. 76, 14–19; ed. Hultsch, p. 56, 9–14
200	Hyginus	Astron. IV, 2, ed. Bunte, p. 101, 14–17
250 [35]	Achilles	Isag. 23 [36]
350 [37]	Firmicus Maternus	Math. VIII 12 [38]
354	Philocalus Calendar	Degrassi, Inscr., p. 248/249 [39]
420	Martianus Capella	De nupt. VIII 824 and 828–833 [40]
714	George, Bishop Arab.	Ryssel [1893], p. 45

The references in the Latin literature to an 8°-position of the cardinal points were handed down through the Middle Ages in the sad compendia and commentaries which constituted the "scientific" literature of these times. One finds this norm accepted, e.g., in Isidore of Seville's writings [41] (around 600 A.D.) or in the commentaries to Martianus Capella which go under the names [42] of Johannes Scotus Erigena [43] and Remigius [44] (9th century). Around 1120 Walcher of Malvern learned about the vernal point in Aries 8° from oriental sources [45] and the same position is mentioned as late as about 1400 in a "computus." [46]

[30] NH II: Loeb I, p. 224/225; Budé II, p. 35 (with antiquated notes on p. 169f.); NH XVIII: Loeb V, p. 328/329.
[31] The summer solstice is located at ♋ 8°.
[32] Mich. Pap. III, p. 76; transl., p. 114.
[33] Anthol. IX, 11: ed. Kroll, p. 354, 7 = CCAG 5, 2, p. 128, 17. The tables in Book VIII (ed. Kroll, p. 321–328) are based on ♈ 8° as vernal point (thus System B) in combination with oblique ascensions which follow System A; cf. Neugebauer-Van Hoesen, Greek Hor., p. 174f. In Anthol. I 2 (ed. Kroll. p. 6, 5 = CCAG 2, p. 93, 2) it is implied that the vernal point lies beyond the beginning of Aries.
[34] Associated with the manuscripts of Censorinus, De die natali (written in 238/9) but probably much older since a similar text appears as scholion to the Aratea of Germanicus who died in A.D. 19. Cf. RE 3, 2 col. 1910, 13–19 and RE 10, 1 col. 461, 67; cf. also C. Robert, Eratosthenis catasterismorum reliquiae (Berlin 1878, reprinted 1963), p. 203.
[35] Date uncertain: 3rd century or before Firmicus.
[36] Maass, Comm. Ar. rel., p. 54, 18. Achilles mentions the opinion of "some" (οἱ μὲν ... οἱ δὲ) authors who place the summer solstice at Cancer 0°, 8°, 12°, or 15°. Cf. also the scholion No. 499 (l.c. 438, 10–17) which mentions only Cancer 8°.
[37] For the date of the completion of the Mathesis (about A.D. 355) cf. Thorndike [1913], p. 419, note 2.
[38] Ed. Kroll-Skutsch II, p. 306f.; cf. also Boll, Sphaera, p. 246f.
[39] Cf. also above p. 595, note 10.
[40] Ed. Dick, p. 434–438; the essential passage also occurs at the end of a manuscript of Hyginus, Astron. (ed. Hasper, p. 31f., corresponding to ed. Dick, p. 437, 11–438, 9).
[41] Etym. V 34; Nat. rer. VIII 1 (ed. Fontaine, p. 204/205). In both works the cardinal points are placed at the 8th calends of April, July, October, January, respectively. The same dates are found in two parallel inscriptions from Rome, known as "calendarium Colotianum" (first cent A.D.) and "calendarium Vallense," the latter (now lost) combined with sun dials. Cf. for these texts Degrassi, Inscr., p. 284–287 and Pl. 81–86; also Wissowa [1903].
[42] For the intricate questions of authorship and sources of these commentaries cf. Stahl [1965], p. 107ff.
[43] Ed. Lutz, p. 174, p. 176.
[44] Ed. Lutz II, p. 260–262, p. 268.
[45] "Scientia Petri Ebrei, cognomento Anphus, de dracone, quam dominus Walcerus prior Maluernensis ecclesie in latinam transtulit linguam"; cf. Millás Vallicrosa [1943], p. 88. On Petrus Alphonsi (≈ 1100) cf. Cutler [1966], p. 190, n. 16.
[46] Cf. Kaltenbrunner [1876], p. 294 on a version, composed in 1396, of a computus of 1200.

Bīrūnī in his "Chronology" (A.D. 1000) remarks[47] that "the Chaldeans are said to have commenced the seasons 8 degrees after the equinoxes and solstices" and he conjectures that this may have to do with the amplitude of 8° assumed for the trepidation of the equinoxes — a conjecture which has much in its favor.[48]

B. Other Norms for the Vernal Point

Aries 10°. The only reference to this position, which is the norm in "System A" of the Babylonian ephemerides, is found in Manilius' poem where he says (III 681) that "some" people use 8°, some 10°.[1]

Aries 12°. Achilles (Isag. 23[2]) makes the statement, without further explanations, that ♋0°, or 8°, or 12°, or 15° had been assumed for the summer solstice. Since nowhere else in ancient sources is a position of 12° found for the cardinal points, it was suggested that one read 10° instead of 12°.[3] I prefer to admit the existence of an unsolved problem.

Aries 15°. A true lunar calendar is lacking a fixed relation to the agricultural seasons unless intercalary months bring back the ordinary sequence of 12 months to more or less the same place within the solar year. We know that intercalary months were used very early in Mesopotamia but we can only conjecture on what basis the insertion of such a month was decided. Actual harvest conditions were probably one major reason. Of astronomical criteria equinoxes and solstices are not well defined for primitive means of observation; obviously the heliacal risings and settings of bright stars or conspicuous constellations provide a much better guide for the steady progress of the solar year. And we have indeed textual evidence that a development of this kind took place in Babylonian astronomy.

A small "series" of texts, consisting of two "tablets," called ^{mul}Apin ("The Plow Star"[4]), operates with a schematic year of 12 months of 30 days each. Within this idealized chronological pattern the dates of the rising of some 15 bright stars and constellations are listed, together with the dates of the solstices (on the 15th of the months[5] IV and X) and of the autumnal equinox (on VII 15). A list of intervals between these dates agrees with the schematic total of 360 days.[6] It is clear that we have here the ideal sequence of events about which the actual civil lunar calendar fluctuates back and forth. It is also evident that the schematic rounding of the "year" to 360 days will not impair the usefulness of these texts

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[47] Transl. Sachau, p. 322.
[48] Cf. below IV B 2, 3.
[1] Ed. Housman III, p. 68. Rehm, Parap., p. 30, n. 1 suggests Meton for the 10° norm, considering the rhetorical question "who else ...?" as a proof. Cf. also above p. 496.
[2] Cf. above p. 597, n. 36.
[3] E.g. Rehm, Parap., p. 30, n. 1.
[4] The name of a canonical collection of texts, arranged in a definite "series," is taken from its initial words (as one refers to papal bulls) and mentioned in the colophon of each tablet in the series (cf. the colophon given in Bezold-Kopff-Boll [1913], p. 36/37).
[5] The names of the months are the usual ones of the civil calendar, i.e. I = Nisan, etc.
[6] Cf. Pritchett-van der Waerden [1961], p. 43f.; also Bezold-Kopff-Boll [1913]. The date of this collection is about 700 B.C., based on observations made in Babylon about 1000 B.C., according to van der Waerden [1949], p. 20f.

for the control of the calendar. By paying attention to the rising of stars one obtains a fixed norm for the year, regardless of the schematic day numbers.

In another and older "series," called "Enūma-Anu-Enlil," one finds different dates for the solstices and equinoxes: the solstices are located at III 15 and IX 15, the equinoxes at XII 15 and VI 15.[7] In the present context it is only essential to note that the cardinal points are equidistantly placed at the midpoints of the respective schematic months.

The preceding description of early Babylonian schemes does not imply the suggestion that the Greek parapegmata or the placing of the cardinal points at the 15th degree indicate Babylonian influence. Such devices are too simple and too natural to exclude independent origin. All we can say is that early calendaric astronomy runs through approximately the same stages in Mesopotamia and in Greece, with some centuries of priority in Mesopotamia. This difference in time could well be considered as an argument in favor of independent developments.

Whatever the case may be we have clear evidence from early Greek astronomy of locating the cardinal points into the midpoints of their signs (probably only later on denoted as "15°"). The main witness is Hipparchus who, in his Commentary to Aratus, ascribes this norm to Eudoxus[8] (4th cent. B.C.) and confirms it by explicit quotations from Eudoxus' writings[9] and by specific examples.[10]

P. Hibeh 27[11] is a text, written in the Saite Nome of Lower Egypt around 300 B.C. This date rests on the date Tybi 20 for the vernal equinox.[12] For the years from -300 to -297 Tybi 20 corresponds to March 25 when the sun is at the vernal equinox. But the text is strongly schematic (assuming, e.g., linear variation of the length of daylight[13]) and cannot be taken as representing accurate observations. Nevertheless the rapid change in the julian equivalents of Egyptian dates restricts us to a few decades around -300.

After the equinox on Tybi (V) 20 the sun enters Taurus, it is said, on Mekheir (VI) 6. This implies for the equinox, 16 (or rather 15) days earlier, a location in the middle of the preceding sign. Hence we see that this text, written only a few decades after Eudoxus, still follows his norm.

On the basis of some passages which agree verbatim with P. Hibeh 27 one can say that another papyrus of the Ptolemaic period, P. Par, 1, is closely related to

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[7] Cf., e.g., van der Waerden [1949], p. 19; [1951], p. 22.

[8] Hipparchus, ed. Manitius, p. 128, 21–27.

[9] Manitius, p. 20, 4–17; p. 22, 1–9; p. 132, 20–134, 2.

[10] Manitius. p. 48, 7–10; p. 56, 15f. It is amazing to see that the ample testimony of Hipparchus, who still had the writings of Eudoxus at his disposal, is explained away in favor of one sentence in a Roman work on agriculture (Columella; cf. above p. 596), four centuries after Eudoxus. In order to rescue the norm Aries 8° for a "genuine" Eudoxus Rehm postulates a "false" Eudoxus, or at least a "aegyptisierende Überarbeitung" of his parapegma in which the vernal point was moved (why?) to Aries 15° (Rehm RE Par. col. 1308, 35–39; col. 1343, 20–24; Parap., p. 18, p. 35–37). Why Hipparchus used such a version remains unexplained. Rehm also speaks about a "anderweitig erschlossene" false Eudoxus by mentioning doubts cast on the genuineness of a work on the octaeteris, although it has probably nothing to do with the parapegma or the work used by Hipparchus. Böckh (Sonnenkr., p. 192ff.) tried to reconcile our sources by postulating different norms for Eudoxus' calendaric and "astrognostic" writings.

[11] Grenfell-Hunt, Hibeh Papyri I, p. 138–157.

[12] Lines 62, 63, and 209.

[13] Cf. below p. 706.

the Hibeh papyrus.[14] This is the famous "Eudoxus Papyrus," so named because of an acrostic saying "Art of Eudoxus".[15] The date of its writing is determined by the remark[16] that the winter solstice according to Democritus and Eudoxus falls on Athyr (III) 20 or 19. If we allow the interval for the solstice to range between Dec. 28 and 24 we obtain for Athyr 19/20 a date between −196 and −173. As we shall discuss presently[17] this text is only a rather careless compilation of an earlier version. Among the most conspicuous elements of textual disorder are the illustrations which in part have no relation to the extant text, e.g. the drawing of a scorpion.[18]

Fortunately a contemporary papyrus, P. Ryl. Inv. 666, which refers to the first year of Philometor, i.e. to −180/179, associates the month Thoth with the sign of Scorpio.[19] From −180 to −177 Thoth 1 corresponds to October 7, hence to a solar longitude of about ♎ 11°. Obviously this association of Scorpio with Thoth again presupposes the Eudoxan norm and we thus may assume that the scorpion in P. Par. 1 also refers to the month of Thoth as would be quite natural in a text which begins and ends with excerpts from a parapegma.[20]

The Eudoxan norm did not remain restricted to the calendaric literature. Astrological arrangements of zodiacal signs repeatedly presuppose symmetry with respect to their midpoints – it suffices to recall the polemics of Geminus in such cases.[21] The same symmetry still holds, e.g., in the "thema mundi" of Firmicus Maternus, Math. III, 1[22] (4th century A.D.).

Aries 0°. Usually called "first degree," or "day 1," of Aries, a terminology which upset ancient and modern arithmetic alike.[23] Since this norm has been used consistently by Hipparchus and by Ptolemy it remained the basis of reckoning in mathematical astronomy, Aries 0° being the vernal point, i.e. the intersection of equator and ecliptic.

We know from Hipparchus[24] that the majority of the "old" mathematicians divided the ecliptic in this form. This statement agrees with sources still available to us; Euctemon (about −430) placed all four cardinal points on the "first day" of their respective signs.[25] The same norm holds for Callippus[26] (about −330) and is underlying the era of Dionysius (beginning −284/3).[27] As far as we know this norm is attested nowhere in Babylonian astronomy.

[14] Cf. below p. 688.
[15] Cf. below p. 686.
[16] Column XXII, 21. Blass, p. 25; Tannery HAA, p. 294, No. 54.
[17] Cf. below p. 688.
[18] Cf. below p. 1453, Fig. 12 on Pl. VII (col. X).
[19] Turner-Neugebauer [1949], p. 7.
[20] Cf. below p. 688.
[21] Geminus, Isag. II, 27ff. (Manitius, p. 31); cf. above p. 583.
[22] Cf. Bouché-Leclercq, AG, p. 187, Fig. 23.
[23] Pliny, e.g., NH II 184 (ed. Jan-Mayhoff I, p. 198, 1–5; Loeb I, p. 316/317) says that at Meroe the sun reaches the zenith twice, at ♉ 18 and at ♌ 14, i.e. 43° distant from ♋ 1°, instead of ♋ 0°.
[24] Hipparchus, ed. Manitius, p. 132, 7–9.
[25] Cf. Rehm [1913]; Pritchett-van der Waerden [1961], p. 32–36.
[26] Böckh, Sonnenkr., p. 25.
[27] Cf. below VI A 2, 4 and above p. 159.

3. Mathematical Astronomy

Babylonian influence on the nascent Greek mathematical astronomy of the hellenistic period is made evident by the appearance of numerical parameters which are so specific that independent discovery is excluded. With these parameters must have been transmitted a number of basic concepts (e.g. lunar anomaly, nodal motion, planetary phases, etc.) to which these parameters relate.

It is difficult to say how much farther Babylonian influence reached. This influence is evident in problems involving oblique ascensions, determined by means of arithmetical methods (cf. below IV D 1, 2). But within the lunar and planetary theory the Greek cinematic procedures completely dominate the extant literature. Only a few papyrus fragments have survived, just enough to show that at least some Babylonian techniques in computing ephemerides had become known to Greek astronomers. We shall discuss these texts in another context later on (cf. V A 1 and 2); at present we will deal mainly with basic parameters and period relations.

A. Lunar and Solar Theory

In two obviously related versions[1] we have a list of parameters for the length of the year as supposedly assumed by different astronomers

A		B	
Meton, Euctemon, Philip	365 5/19	Euctemon, Philip, Apollinarius[2]	365 5/19
Aristarchus of Samos	[365] 1/4 κ' ξβ'	Aristarchus of Samos	365 1/4 ι' δ'
Chaldeans	365 1/4 ε' ζ'	Babylonians	365 1/4 ε' ζ'
Babylonians	365 1/4 1/144	Sudines[3]	365 1/4 γ' ε'
		[ ]	365 1/4 ρ' σ'[

The first number is the traditional value for the Greek version of the Metonic cycle.[4] The remaining numbers are obviously corrupt[5]; the number of Aristarchus

[1] Version A: from Vat. gr. 191 fol. 170' (CCAG 5, 2, p. 127, 17–19 = Vettius Valens, Anthol. IX 11, ed. Kroll, p. 353, 10–13). Version B: from Vat. gr. 381 fol. 163', published Maass, Aratea, p. 140. For the relationship between these two codices cf. also Maass [1881].

[2] Apollinarios (listed in RE 1, 2 col. 2845 as Apollinaris No. 12) is mentioned by Vettius Valens (p. 250, 26 Kroll = CCAG 5, 2, p. 38, 17); hence he cannot be much later than about A.D. 150. Hephaestion (about A.D. 380) seems to associate him (CCAG 8, 2, p. 61, 16; p. 63, 21) with Antiochus (of Athens) who lived in the first or second century A.D. Hence a date around A.D. 100 could be assumed for Apollinarios. The "Anonymus of 379" (and, following him, "Palchus": CCAG 5, 1, p. 205, 5 and CCAG 1, p. 80, 19) treat Apollinarios as parapegmatist like Meton and Euctemon, observing in Athens (suggested by his association with Antiochus of Athens?) Honigmann's (SK, p. 42) "von Laodikeia" is a simple mistake (mixup with a christian author).

Achilles (Maass, Comm. Ar. rel., p. 47, 13f. or Aratea, p. 143, n. 52) mentions Apollinarios as having written on solar eclipses in the seven climates. A long excerpt in an anonymous fragment (CCAG 8, 2, p. 132, 4 to perhaps 133, 28) shows him as being familiar with the technical terminology of lunar theory. Porphyri in his Introduction to the Tetrabiblos (CCAG 5, 4, p. 212, 14 = Riess [1891], p. 334, frgm. 3) refers to him in connection with arithmetical methods of computing oblique ascensions (cf. also CCAG 8, 4, p. 50, fol. 46).

In astrological context the name of Apollinarios appears beside the above mentioned passages in Vettius Valens and Hephaestion (also CCAG 8, 2, p. 62, 1) in the preface to the Isagogika of Paulus Alexandrinus (p. 1, 13, ed. Boer) and in CCAG 6, p. 15 (fol. 341').

[3] Distorted to Σωδίνων.

[4] Cf. below p. 623.

[5] It is not difficult to replace the numbers in the text by others which are not astronomically excluded but this kind of simply rewriting a text carries little conviction.

in A could perhaps be rescued by interpreting ξβ' as ἑξηκοστὰ δευτέρα i.e. "seconds," but the resulting 365;15,20 would still leave B unexplained, nor is there any relation to another supposedly Aristarchean value 365;15,2,13, ...[6] The ε'ζ' in the next line makes no sense at all. The value 365;15,25 in A of the "Babylonians" is at least possible[7] but it is neither attested in cuneiform sources nor is it supported by version B. Nevertheless we cannot simply dismiss these data as fanciful inventions because they must be seen in a wider context. In a famous passage a few lines later[8] Vettius Valens claims having made use of eclipse tables of Hipparchus, Apollonius, and "Kidynas and Sudines."[9] Needless to say we cannot tell how such tables — Greek as well as Babylonian (translated to Greek?) — may have looked, but this is no sufficient reason to deny Vettius Valens knowledge of material no longer available to us.

There remains the question of the origin of the basic Greek value of 365 1/4 days for the year. Theon's story is obvious nonsense: Callippus obtained this value, as well as 29 1/2 days for the synodic month, by comparing Chaldean observations with his own.[10] No comparison of real observations could possibly result in these trivial round numbers, not to mention the difficulties which such a comparison would have to overcome, caused by the Babylonian lunar calendar or the schematic dates for the equinoxes and solstices.[11] Thus I see no justification for assuming Babylonian influence in the choice of a parameter which itself is attested nowhere in Babylonian astronomy.

In the Greek milieu, however, the construction of parapegmata could have easily led within a few decades to the discovery that fixed star phases do not remain fixed in the Egyptian calendar while a correction of one day every four years repairs the damage. In this way one obtains easily a sidereal year of 365 1/4 days. Exactly like the parapegmata themselves it has no relation to the civil calendar.[12]

We mentioned before[13] that Geminus knew the "exeligmos," although omitting the nodal months, and that he derived from it — historically as well as arithmetically incorrect — the parameters of a linear zigzag function:

$$m=11;6,35^{\circ/d}, \quad M=15;14,35^{\circ/d}, \quad \mu=13;10,35^{\circ/d}, \quad d=0;18. \tag{1}$$

This is exactly the Babylonian form of describing the daily motion of the moon, i.e. in our terminology[14] the zigzag function F*. Its period

$$P=27;33,20^{d} \tag{2}$$

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[6] Cf. below p. 603.

[7] This parameter appears, e.g., with Ulugh Beg (Kennedy, Survey, p. 167 sub P), i.e. about 1440. Cf. also for Hipparchus above p. 293.

[8] Cf. Cumont [1910]; Anthol., ed. Kroll, p. 354, 2–7.

[9] Cf. for details below IV A 4, 4 A.

[10] Rome [1926], p. 9, translating a passage published in Rome CA III, p. 838, 26–839, 10.

[11] Cf. above II Intr. 3, 2.

[12] The latter seeks a balance between solar years and lunar months by introducing some new forms of "years" which fit some convenient pattern of intercalations. Cf., e.g., for the 19-year cycle above p. 601 (first line in A and B) or below IV B 1, 2.

[13] Above p. 586.

[14] Cf. above p. 480.

represents the length of the anomalistic month. Since

$$27;33,20=\frac{4,8}{9} \tag{3}$$

we have here implicitly also the relation

$$9 \text{ anom. m.}=248^d \tag{4}$$

which is utilized in a variety of ancient sources.[15]

The above listed parameters and the parameters which are embedded in the "exeligmos" constitute the total of Babylonian lunar data explicitly attested to be known by the time of Hipparchus in Greek astronomy. This material represents only a very small segment of Babylonian lunar theory. In particular there are no Greek traces of the Babylonian methods of dealing with the solar anomaly.[16] Only in the norm of the vernal point and in the schemes for the rising times of the ecliptic[17] do we meet Babylonian patterns related to the solar theory.

Tannery argued convincingly[18] that the exeligmos was already known to Aristarchus, i.e. about a century before Hipparchus. Censorinus, in Chap. 19 of De die natali, says[19] that Aristarchus agreed with Callippus about the length of the year but that he added a correction of 1/1623 of one day to it (i.e. to 365 1/4[20]). It was this peculiar number 1623 (=27,3) which Tannery was able to derive from one of the relations within the exeligmos:

$$19\,756^d=669 \text{ syn. m.}=54 \text{ rot. of the sun}+32°. \tag{5}$$

Instead of (5) one can say

$$19\,756^d=54+\frac{32}{360}\text{ years} \quad \text{or} \quad 5,29,16^d=54;5,20^y$$

and therefore

$$\begin{aligned} 1\text{ year}&=\frac{5,29,16}{54;5,20}=\frac{4,6,57,0}{40,34}=365;15+\frac{1,30}{40,34} \\ &=365;15+\frac{3}{1,21,8}\approx 365;15+\frac{3}{1,21,9}=365;15+\frac{1}{27,3} \end{aligned} \tag{6}$$

which explains the correction mentioned by Censorinus.[21]

The meaning of this procedure would be that Aristarchus accepted the validity of the exeligmos (5) but then pointed out that this would imply a small modification of the traditional length of 365 1/4^d for the year. Unfortunately we do not know

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[15] Cf. below p. 809f.

[16] Cf. for this problem below p. 626.

[17] Above IV A 4, 2 A and below IV D 1, 2, respectively.

[18] Tannery, Mém. Sci. II, p. 345f.; repeated in Heath, Aristarch., p. 314f.

[19] Censorinus, ed. Jahn, p. 57, 6–8

[20] The text has by mistake only 365 days; cf., however, below p. 623.

[21] It follows from (6) that $40,34^y=2434^y=4,6,57,0^d=889020^d$ gives the smallest number of years corresponding to an integer number of days on the basis of (5). Censorinus (Chap. 18, ed. Jahn, p. 55, 14f.) says that Aristarchus assumed 2484^y for the return of all planets to the same position. This is certainly a mistake since no such small common planetary period exists. Tannery suggested emending 80 to 30 (π for λ) and assumes that the exact completion of solar years and of days caused a misinterpretation of (6) as a planetary "great year."

whether he left it at that; we also obtain no new information about the parameters ascribed to Aristarchus, mentioned at the beginning (p. 601).

As a concession to the innumerable discussions concerning the prediction of a solar eclipse in -584 (May 28) by Thales a few remarks may be made here though I have no doubt that they will remain without effect.

In the early days of classical studies one did not assume that in the sixth century B.C. a Greek philosopher had at his disposal the astronomical and mathematical tools necessary to predict a solar eclipse. But then one could invoke the astronomy of the "Chaldeans" from whom Thales could have received whatever information was required. This hazy but convenient theory collapsed in view of the present knowledge about the chronology of Babylonian astronomy in general and the lunar theory in particular. It is now evident that even three centuries after Thales no solar eclipse could be predicted to be visible in Asia Minor — in fact not even for Babylon.

There remains another vague hypothesis: the prediction by means of cycles (if need be again available upon request from Babylon). But unfortunately there exists no historically manageable cycle of solar eclipses visible at a given locality and any attempt to establish a cycle would require access to centuries of local records.

Hence there is no justification for considering the story of the "Thales eclipse" as a piece of evidence for Babylonian influence on earliest Greek astronomy. All available sources point to no such contacts until three centuries later.

B. Planetary Theory

For the Greek lunar theory we came to the conclusion that the Babylonian influence did not reach much farther than the communication of some basic concepts and related parameters. In the planetary theory the impact from Babylon seems to be limited even more to the transmission of fundamental period relations. Since the Babylonian theory is based upon the determination of the planetary phases[1] it contained no elements directly useful for a cinematic theory. Furthermore the Babylonian ephemerides do not consider planetary latitudes,[2] again an element of importance in a geometric-cinematic model. Even the Greek order, from Saturn to Mercury, has no predecessor in Mesopotamia where an arrangement prevails for which we do not know the underlying reason[3] but which is certainly different from the natural order by sidereal periods in Greek astronomy.[4]

The only direct borrowing in the Almagest from Babylonian planetary theory is found in the introduction to Book IX where Ptolemy enumerates the basic period relations. In this context he mentions the following parameters[5] (N = num-

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[1] Cf. above p. 386f.

[2] For an early text considering latitudes cf. Neugebauer-Sachs [1968/1969] I, p. 209; cf. also above II C 3, p. 554.

[3] Cf. below p. 690.

[4] The Greek order differs also from the Egyptian sequence; the Indian order, however, is derived from the Greek one since it is the order of the days in the planetary week. Cf. below p. 690.

[5] Alm. IX, 3; omitting here Ptolemy's refinements expressed as corrections beyond or below exact returns; cf. above p. 151.

ber of years, R = sidereal rotations, A = synodic periods):

Saturn:	$N=59$	$R=2$	$A=57$
Jupiter:	$N=71$	$R=6$	$A=65$
Mars:	$N=79$	$R=42$	$A=37$
Venus:	$N=R=8$		$A=5$
Mercury:	$N=R=46$		$A=145$.

(1)

These are the well-known Babylonian "Goal-year-periods"[6] which belong to the earlier phase of Babylonian mathematical astronomy.[7] Ptolemy's refinements, however, operating with tropical years, are no longer depending on the Babylonian tradition.

We would be at the end of our enumeration of contacts in the planetary theory were it not for Greek astrology which, in doctrines involving the planets, preserved for us many numerical data taken from the advanced Babylonian theory of the Seleucid period. These parameters often appear in a context which has no astronomical meaning at all, e.g. in predictions concerning the duration of life. In the same fashion the sun is associated with the number 1461 (from the Sothic period), or with 19 (from the "Metonic" cycle), the moon with 25 (from the Egyptian lunisolar cycle[8]).

The whole set of periods from the Babylonian planetary ephemerides was known to Psellus (died in 1081) who mentions the following relations[9]:

Saturn:	265 years = 9 sid. rot. = 256 synod. per.
Jupiter:	427 years = 36 sid. rot. = 391 synod. per.
Mars:	284 years = 151 sid. rot. = 133 synod. per.
Venus:	1151 years = [720 synod. per.]
Mercury:	480 years = 1513 synod. per.

(2)

These are exactly the Babylonian parameters[10] with the exception of a scribal error in the case of Venus.[11] The numbers for the years are repeatedly attested in the astrological literature, e.g. in Rhetorius (6th cent.) who excerpts Antiochus (1st cent.),[12] or in Lydus (6th cent.),[13] or in Vat. gr. 1056 (CCAG 5, 2 p. 132) where the above listed numbers of years are called "supercolossal" (ἔτη ὑπερμεγέθη). This name is given in contrast to another triple of periods, associated with each planet, called "greatest," "mean," and "smallest" (or similar). This second group

[6] Cf. above p. 351. In the astrological literature these parameters are rarely mentioned: an example is CCAG 7, p. 120f. which mentions the values of N listed in (1), with the exception of $N=83$ for Jupiter, which is, however, also a Babylonian goal-year parameter (cf. p. 391 (12)). The text in question might be from Heliodorus (around A.D. 500); cf. l.c. p. 119, n. 27. The same set of parameters, again with Jupiter's 83, is also used in the "Almanac" of Azarquiel (epoch 1088 Sept. 1) which is based, however, on much older sources; cf. Boutelle [1967].

[7] Cf. above p. 554.

[8] Cf. above III 2.

[9] Cf. Tannery, Mém. Sci. 4, p. 265, from Cod. Scor. III. Y. 12 = CCAG 11,1 cod. 7, fol. 71 = Catálogo ... Biblioteca de el Escorial II, p. 160, No. 282, 3.

[10] Cf. above p. 390 (10a) and (10b).

[11] The text has 309 instead of 720 (error for 309 syn. months = 25 Eg. years).

[12] CCAG 1, p. 163, 17–20.

[13] Lydus, De mensibus, p. 56f. ed. Wuensch (error for Mars: 294 instead 284).

of periods, again attested in many versions (and with various errors) throughout the astrological literature[14] displays the following "periods":

	max.	mean	min.
Saturn:	57	43 1/2	30
Jupiter:	79	45 1/2	12
Mars:	66	40 1/2	15
Venus:	82	45	8
Mercury:	76	48	20

(3)

The mean values (often given without the fractions) are of no interest since they are mere arithmetical consequences of the extrema. The "minimum" periods for Saturn, Jupiter, and Venus are the universally known round values and need no explanation. The 15 years for Mars seem not to be attested in cuneiform sources and could be motivated by the mean motion of about 351° easily obtainable for 15 years, e.g. from the goal year period in (1). A similar situation holds for Mercury since 20 Egyptian years correspond very nearly to 63 rotations, as Ptolemy remarks.[15] The maxima, however, have no astronomical meaning whatsoever. Their total is 360 and each number is taken from degrees of arc, not from years as one would expect for planetary periods. The number associated with each planet is the total of degrees in the astrological "terms" assigned to its influence in the zodiac, both in the "Egyptian" system and in Ptolemy's "ancient manuscript."[16] These numbers (taken as years) supposedly control the length of life.[17] Their ultimate origin is unknown but probably a hellenistic creation, not Babylonian.

Also of hellenistic origin is the construction of huge common periods, based on the above mentioned planetary periods but with the addition of the Egyptian cycles of 25 years for the moon and 1461 years for the sun.

A common multiple of the shortest periods in (3) and of 25 and 1461 is 175 3200 (i.e. 1200 · 1461) years, called a "cosmic period" (κοσμικὴ ἀποκατάστασις).[18] A much larger common period is ascribed to Sosigenes (2nd cent. A.D.), called "perfect year" (τέλειος ἐνιαυτός); it is obtained as product of the Babylonian sidereal periods listed in (2), again with the addition of the Egyptian parameters 25 and 1461. The result is

$$64\ 8483\ 4167\ 3864\ 000\dot{0} \text{ years}$$

(grouping the digits into myriads). This, no doubt, is the number referred to by Proclus in his Commentary to Plato's Republic, sadly corrupted in the manuscript tradition.[19]

[14] Cf., e.g., CCAG 5, 3, p. 132; Lydus, De mens., p. 56f. ed. Wuensch; Firmicus II, 25, ed. Kroll-Skutsch I, p. 73/74; Paulus Alex., ed. Boer, p. 12f.; Bīrūnī, Astrol., p. 255, ed. Wright, etc.
[15] Alm. IX, 10 Manitius II, p. 154, 20.
[16] Cf. Paulus Alex., ed. Boer, p. 12, 15 and p. 14, 15–18; also Ptolemy, Tetrab., Loeb, p. 97 and p. 107.
[17] Cf. Bouché-Leclercq AG, p. 408ff.
[18] CCAG 1, p. 163 (cf. apparatus for the correct number); Lydus, De mens., p. 57, 6–8 (ed. Wuensch); Psellus, Omnif. doctr. §161, 9f. (ed. Westerink, p. 82), also Tannery, Mém. Sci. 4, p. 261f. and Boll [1898]. The smallest common multiple of these numbers would be 1461 · 200 = 292 200.
[19] Proclus, Comm. Rep., ed. Kroll II, p. 23; trsl. Festugière II, p. 128f.

Greek astrological treatises contain still other types of planetary periods which have, however, no recognizable historical or astronomical background. For example Rhetorius (6th century) assigns to the seven planets the consecutive integers from 7 to 13 (years) as periods[20] or Vettius Valens (2nd cent.) derives from the minimum periods in (3) new numbers of days as "periods" which have no longer any astronomical meaning.[21] Unfortunately we seem to have no good reason to blame these products of Greek wisdom on superstitious orientals. Babylonian astrology is far more primitive than its hellenistic successor.

That hellenistic Egypt is the cradle of ancient astrology in the form it spread to all regions of the world is abundantly documented in our sources. It is therefore a plausible assumption that Egypt also played a role in the transmission of Babylonian mathematical astronomy. From the early Roman imperial period exist planetary ephemerides in demotic and in Greek but without a key to their method of computation except that they are not arranged like Babylonian ephemerides of the hellenistic period. Nevertheless, van der Waerden in a series of publications tried to demonstrate that Babylonian methods were underlying these computations. We shall discuss this problem later on (in V A 1, 2); in the present context it will suffice to say that the texts from Egypt do not provide any specific parameters or concepts to shed light on the early phases of Greek planetary theory.

While the ways of contact with Babylonian astronomy remain obscure, even for hellenistic Egypt, we obtain still less help from stories like the foundation of a "school" in Kos by the Babylonian Berosos, around 280 B.C. Scattered through many works of ancient literature are more or less secure "fragments" from Berosos' writings[22] but these contain very little of astronomical interest and it seems to me one may doubt that between Babylonian cosmogony and mythological history much room was left for technical astronomical information.

4. Ancient Tradition; Summary

Innumerable passages in ancient literature refer to the "Chaldeans" or Babylonians in connection with astronomy or astrology or other sciences. But these references are usually very general and vague and thus of little historical value. The haziness of the tradition concerning Mesopotamian science remained, of course, unchanged during the Islamic and Byzantine period and the same is quite frequently characteristic of modern references to the "Babylonians." It is

[20] Cf. CCAG 5, 4, p. 136–147; p. 179; CCAG 5, 3, p. 132, 22–25.

[21] Vettius Valens, Anthol. IV, Chap. 1, 3, and 30 (ed. Kroll, p. 158f. and p. 205f.). The total of the minimum periods in (3) is 129: multiplication with the factor 2;50 changes it (exactly) to 365;30. Consequently these new periods are now called "days"; then they are subjected to new arithmetical modifications which are supposed to represent the combined influence of the planets, etc. Similarly Firmicus Maternus II, 25 (Kroll-Skutsch, p. 73f.) with some errors which can easily be corrected on the basis of the numbers in (3).

[22] Collected in Schnabel, Ber., p. 250–275. Schnabel's own contributions must be taken with great caution as far as Babylonian astronomy is concerned.

only in a very few exceptional cases — among which the Almagest takes first place — that Greek statements about Babylonian astronomy are of real historical interest.

Finally there is another group of ancient references to Mesopotamia which are quite specific but obvious nonsense. Modern scholars have dealt with these reports in a fashion comparable to the attitude of certain readers of the Bible: they do not believe in the miracles but replace them to their hearts' content by possible natural events.[1] I think it better simply to discard such sources for the reconstruction of historical data.

As an example a remark by Proclus (5th century) may serve where he tells us[2] that Iamblichus (4th cent.) said, on the authority of Hipparchus, that the Assyrians had made observations for 270000 years and had kept records of the return of all seven planets to the same position. The only point of interest in this story seems to me the fact that we have evidence here from a Greek source for playing with huge sexagesimal numbers (270000 = 1,15,0,0) which became predominant in Indian astronomy, beginning in about the same period.

Another story[3] rests on the authority of Porphyry (around 300) who tells us — six centuries after the supposed event — that Callisthenes, a companion of Alexander in the Persian war, dispatched Babylonian records to Aristotle in Athens of eclipses covering 31000 years.[4] On the other hand Georgius Syncellus (around 800) reports[5] on the authority of Berosus (around 300 B.C.) and Alexander Polyhistor (first cent. B.C.) that King Nabonassar destroyed the records of his predecessors in order to have people count years from his reign. Modern scholars like the first story (without the 31000 years) and ignore the second which is not much more implausible.

An often repeated statement by Cicero (made around 45 B.C.[6]) seems to establish that Eudoxus disbelieved Babylonian horoscopic practices. This would be by far the earliest evidence (from the fourth century B.C.) for Greek contact with Babylonian astrology and therefore requires a more detailed discussion.

The earliest direct evidence in Greek sources of astrological concepts and techniques is furnished by Greek and demotic horoscopes from the first century B.C.[7] and shortly later, in the next century, by some sections in the "Isagoge" of Geminus.[8] From a demotic ostracon we can conclude[9] the existence in Egypt of the concept of the twelve zodiacal signs around 250 B.C. Eclipse and lunar omina, from a demotic papyrus, concerning Egypt and adjacent countries can be securely

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[1] Explaining the star of Bethlehem by planetary conjunctions, comets, novae, etc., is a classical example.

[2] Commentary to Plato's Timaeus. ed. Diehl I, p. 100, 29–101, 2; transl. Festugière I. p. 143. Cf. also the discussion by Martin [1864].

[3] From Simplicius, commentary to Aristotle's De caelo II, 12 (Comm. in Arist. gr. VII, p. 506, 8–16, ed. Heiberg).

[4] One need hardly to point out that no trace of such Babylonian data can be found in Aristotle's work. All this has been said long ago by Martin [1864], of course, with very little effect.

[5] Chronogr. 207 (ed. Dindorf, p. 390, 1–5 = Schnabel, Ber., p. 268, 28–33), written A.D. 794 (Dindorf, p. 389, 20).

[6] Cicero, De divinatione II 42, 87 (Loeb, p. 468/471).

[7] Cf. Neugebauer-Van Hoesen [1964], p. 66 and Neugebauer-Parker [1968]; also above p. 575.

[8] Cf. above p. 583.

[9] Neugebauer [1943], p. 121 f.

dated to the Persian occupation of Egypt around 500 B.C.[10] This text clearly depends on Babylonian models, e.g. by its parallel use of Babylonian and Egyptian months, but it shows no element that points to personal horoscopy in the latter hellenistic-Roman sense; the absence of any reference to the zodiac underlines this fact.[11]

What Eudoxus did say, according to Cicero (or his source) is only that one should not trust the Chaldean predictions concerning a man's life "from his birthday."[12] Cicero then naively describes what he thinks to be Chaldean astrology and what is indeed a fairly accurate description of astrological doctrine at his time.[13] In fact, however, the existence of purely calendaric omina is attested in Babylonian[13a] as well as in Egyptian[13b] texts. These omina do not at all involve the celestial bodies and nothing compels us to see in the Eudoxan passage a reference to horoscopic astrology and the astronomical techniques of later centuries.

Another early, but rather unspecific reference to Babylonian astrology comes from Proclus' Commentary to the Timaeus,[13c] where he tells us that Theophrastus in his book *Περὶ σημείων*[13d] admired the Chaldeans for predicting "from the heavens" not only the weather but also private and public affairs. That Theophrastus ($\approx$ 300 B.C.) should have known of the existence of Babylonian astrology is not surprising but a parapegma-literature[13e] has so far not been found in cuneiform sources.

Ancient authors whom we constantly consult (Pliny, Censorinus, Lydus, etc.) can concur in statements which are demonstrably wrong. This is the case, e.g., in the question of the epoch of the day in Mesopotamia and in Egypt. Bilfinger, by collecting all data he could find in Latin and Greek authors came to the conclusion that the Babylonian day began at sunrise, the Egyptian at sunset[14] — both flatly contradictory to the evidence from the original sources, cuneiform as well as Egyptian texts and papyri. Before the decipherment of the Egyptian and cuneiform

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[10] Parker, Vienna Pap.

[11] In contrast the astrological geography in Manilius, which reflects conditions in Ptolemaic Egypt at the end of the third century B.C. (cf. Bartalucci [1961]), operates with the association of countries and zodiacal signs; cf. also Cumont [1909].

[12] "Chaldaeis in predictione et in notatione cuiusque vitae ex natali die minimum esse credendum."

[13] What he says, however, about the geocentric distances of the planets (De divin. II 43, 91) is not part of any astrological theory. Modern scholars act much in the same way by reading into the text words which are not there; cf., e.g., the "translation" in Loeb, p. 471: "from *the positions of the stars on* the day of his birth" (italics mine).

[13a] Labat, Calendr. Babyl., § 64 (p. 132–135) where the future of a child is predicted from the month in which it was born. For a Hittite translation of a Babylonian text of this type (from the second half of the second millennium B.C.) cf. Sachs [1952, 3], p. 52, note 17a.

[13b] An Egyptian papyrus from the Ramessite period, concerning lucky and unlucky days (published: Bakir, Cairo Cal.) contains also entries of the type "someone born on this day will die by a crocodile," etc. There is, of course, no trace of astrology in this text of the 12th century B.C. and the only element of prediction or advice is the calendar date with its associated religious feasts or mythological events. By the time of Cicero a text of this type could easily be ascribed to the "Chaldeans."

[13c] Ed. Diehl III, p. 151, 1–9; transl. Festugière IV, p. 192. This passage has been discussed many times, e.g. by Kroll [1901], p. 561 or by Cumont [1911], p. 5.

[13d] Rehm, Parap., p. 122–140 argued that much in "De signis" belonged to Euctemon. There is no reason, however, to assume that the passage in question belongs to this "Grundschrift."

[13e] Cf. also below IV A 4, 4 B.

[14] Cf. texts and tabulation in Bilfinger, Bürgerl. Tag, p. 1–16.

writings reliance on classical sources alone had produced a mere caricature of oriental civilizations.

A. "*Schools*" *and Astronomers*

At the beginning of this century Kugler's discoveries in Babylonian astronomy and the simultaneous work on the Greek astrological literature opened entirely new aspects in the understanding of ancient references – a famous paper by Cumont[15] is characteristic of this period. Here one finally had references in ancient literature to the "Chaldeans" that made sense and which could be confirmed by the newly opened cuneiform sources. But the authors were all late compilators – Strabo, Pliny, Vettius Valens – and the fascination of the resulting "Quellenforschung"[16] made one forget that the factual information thus obtained did not lead much beyond what was now known from cuneiform texts. In several details the original optimism about established Graeco-Babylonian parallels had to be considerably dampened.

There is first the list of Babylonian cities where astronomical "schools" or "sects" existed: Babylon, Borsippa, Sippar (if "oppida Hipparenum" is correctly equated, as is customary, with Sippar), and Uruk.[17] All texts known presently come from Babylon or from Uruk; of course this does not exclude the existence of archives in other places still untouched. It was once assumed that Sippar appeared in the colophon of an ephemeris but this turned out to be a misreading.[18] It also seemed tempting to identify the "Systems" A and B with the "doctrinae" or δόγματα in Pliny or Strabo, but in fact both Systems are used in both cities.[19]

That Vettius Valens seems to make a distinction between Chaldeans and Babylonians we have seen before.[20] His astronomical competence, however, is not very impressive as will be clear to every student of his "Anthology." His famous statement[21] that for eclipses he used "Hipparchus for the sun, Sudines and Kidinnu and Apollonius for the moon, and again Apollonius for both types (of eclipses, solar and lunar)" is difficult to interpret when one wishes to associate with these words definite methods known to us from ancient eclipse computations.[22]

Secondly, the above quoted passages from Strabo and Pliny also mention Chaldean astronomers: Kidenas, Naburianus, Sudines, Seleucus. The last mentioned, Seleucus from Seleucia[23] or from the Erythrean Sea[24] (i.e. the Persian Gulf) is the best known to us. Hipparchus refers to him as an authority on a theory

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[15] Cumont [1910]; also Fotheringham [1928].

[16] A beautiful example is the diagram in Schnabel, Ber., p. 110. His derivation of all these works from two single sources, Berosus and Posidonius, is about as well established as our descent from Adam and Eve.

[17] Strabo XVI 1, 6 (Loeb VII, p. 203); repeated in the Tribiblos of Theodoros Meliteniotes (about 1370): cf. CCAG 5, 3, p. 140, 30–141, 1 = Migne PG 149 col. 997/8: Pliny NH VI 30, 121–123 (Loeb II, p. 431); these passages also in Schnabel, Ber., p. 9. – Cf. also above p. 352.

[18] ACT, p. 5, note 14 and p. 22 (Zo line 2).

[19] ACT, p. 10.

[20] Cf. above p. 601; from Anthol., p. 353, 12 (Kroll) = CCAG 5. 2, p. 127, 19.

[21] Anthol., p. 354, 4-6 = CCAG 5, 2, p. 128, 14–16; cf. Cumont [1910], p. 161–163.

[22] One also would like to know which Apollonius is meant; cf. above p. 263.

[23] Strabo XVI 1, 6 (Loeb VII, p. 203).

[24] Strabo III 5, 9 (Loeb II, p. 153). For a Seleucia on the Persian Gulf cf. Cumont [1927]; also Tarn-Griffith, Hellenistic Civilization (3rd ed., 1952), p. 158.

of tides,[25] some fragments of which have survived.[26] Since Seleucus knew Crates, around 170 B.C., he could be an older contemporary of Hipparchus.[27]

In modern times Seleucus has become famous as supporting Aristarchus in the assumption of an axial rotation of the earth, relating it to his theory of the tides,[28] and by his assertion that the cosmos is infinite.[29] Less credit he earned by his telling us that Hermes wrote 20000 books.[30]

A century earlier, around 240 B.C., we find a Sudines at the court of Attalus I at Pergamon. About his astronomy even less is known than for Seleucus. Some meager fragments of his work concern the qualities of precious stones.[31]

The name Nabūrimannu probably occurred in the colophon of a lunar ephemeris of System A from Babylon for the year −48/47.[32] This has been taken as evidence for Naburianus being the inventor of System A of the lunar theory. Unfortunately we are not able to translate the relevant expression "tersitu of N" by a definite term[33] nor is there any compelling reason to see in the person mentioned the inventor of the underlying method, centuries earlier.

The same term also occurs in relation to Kidin or Kidinnu, i.e. the Greek Kidenas (or similar), again in lunar ephemerides from Babylon, but now of System B (for the years −103/100 and two undetermined years).[34] As before it is not at all evident that the colophons in question mean that Kidinnu is the architect of System B. In the Greek tradition,[35] however, he is explicitly credited with the discovery of the relation

$$251 \text{ synodic months} = 269 \text{ anomalistic months}$$

which is indeed embedded in the computations of System B.[36] It cannot be doubted that much more in the lunar theory belongs to him, or was at least known to him. Here the Greek sources seem comparatively well informed.

[25] Strabo I 1, 9 (Loeb I, p. 19).
[26] Cf., e.g., Susemihl. Griech. Litt. I, p. 764, n. 265; RE Suppl. 5 col. 962f. (Kroll).
[27] Bergk, Abh. p. 170 suggests, with no trace of a proof, that observations mentioned in the Almagest using the "Chaldean era" were therefore made by Seleucus. This era is nothing but the Syrian form of the Seleucid Era; cf. above p. 159.
[28] Cf. Heath, Aristarchus, p. 305.
[29] Pseudo-Plutarch, De plac. II, 1 (ed. Bernardakis V, p. 297, 10) and Stobaeus, Ecl. phys. I, 21 (ed. Wachsmuth I, p. 182, 20f.). Pines [1963] discovered in Arabic sources (Rāzī, died about 925) arguments in support of this theory, probably belonging to Seleucus.
[30] Iamblichus, The Egyptian Mysteries VIII, 1 (Budé, p. 195). Manetho reported 36525 books (a number which represents 25 Sothic periods of 1461 years; cf. Manetho, Loeb, p. 227, p. 231). Of no interest is the enumeration of 7 phases of the moon (Clemens Alex., Stromata VI 16, 143, 3, ed. Stählin, p. 505, 1–5).
[31] Cf. Susemihl, Griech. Litt. I, p. 861f.; Kroll in RE 4 A, 1 col. 563; Esther V. Hansen, The Attalids of Pergamon (Cornell Studies in Classical Philology 29, 1947), p. 370; Cumont [1910], p. 162. Also Wellmann [1935], p. 427, 433, 438; Bidez-Cumont, Mages I, p. 193. The name Sudines poses some problems; an Old-Babylonian name Suddānu is attested (Ranke PN, p. 166). One also could think of a name ending in -idin or -idina but the first half should then contain more than su (or shu?). In our material of astronomical texts the name does not occur.
[32] Cf. ACT (No. 18), p. 100 and p. 23 (Zq).
[33] Cf. for tersitu ACT, p. 12/13.
[34] ACT No. 122 and No. 123a. For the name cf. ACT, p. 25, note 49.
[35] CCAG 8, 2, p. 126, 15.
[36] Cf. ACT, p. 76 (2) and above p. 482 (3).

Pliny tells us that according to Cidenas and Sosigenes (Caesar's calendaric advisor) Mercury never deviates from the sun by more than 22°.[37] The same parameter is found in several authors (Pliny probably being the earliest)[38] and it can be considered as representing a round mean value of actual observational data.[39] It does not fit, however, the Babylonian theory for Mercury as we know it [40] which operates only with the phases of first and last visibility, ignoring maximum elongations and stations. Hence the reference to Cidenas is much more suspect than to Sosigenes who (in the first century B.C.) was surely familiar with Greek cinematic models which necessarily are based on maximum elongations.

B. Parapegmata

Sosigenes appears a second time in Pliny, again in connection with Babylonian doctrines of which we have no evidence in the original sources. Pliny in Book XVIII 57, 207ff. discusses parapegmata, i.e. the climatic significance of fixed star phases.[41] He says that there were three main schools (*sectae*), the Chaldean, the Egyptian, and the Greek to which a fourth was added by Caesar with the assistance of Sosigenes who wrote treatises (*commentationes*) on the subject.[42] This seems the only occasion that "Chaldeans" are named as authorities for parapegmata but it is clear that Pliny (or rather his source) had very specific data in mind. Pliny also gives a list of localities to which his data about fixed star phases refer, e.g. by saying that "Egypt" includes Phoenicia, Cyprus, and Cilicia. Similarly he defines "Chaldean" data as concerning Babylonia and Assyria[43] and in fact he mentions in the subsequent text, which concerns farm work in different seasons, ten phases for which he quotes twice the Chaldeans (or Chaldea) and for the rest "Assyria." It is, of course, possible that "Assyria" does not mean ancient Assyria but western Syria to the north of Palestine.[44]

A second mention of parapegmata in association with the Chaldeans again comes from the first century. Columella[45] says that he had polemicized in his writings against the doctrine of the Chaldeans that definite dates could be associated with the changes of weather.[46] There can be no doubt that Columella had seen parapegmata which were ascribed, rightly or wrongly, to the Chaldeans.[47]

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[37] Pliny NH II 6, 39 (Loeb I, p. 193, Budé II, p. 19). One MS has 23° instead of 22° (cf. ed. Jan-Mayhoff I, p. 139, 1).
[38] E.g. P. Mich. 149 (2nd cent. A.D.) X, 31 (Mich. Pap. III, p. 75 and p. 102) and Maass, Comm. Ar. rel., p. 601 (from Anonymus Sangallensis).
[39] Cf. Ptolemy's discussion in Alm. XII, 10 (above I C 3, 1).
[40] Cf. above II A 5, 1 C.
[41] Cf. above p. 587.
[42] Pliny NH XVIII 57, 211 f. (Loeb V, p. 322–325). The relevant passages are collected by Wachsmuth in Lydus, De ost., p. 321–331.
[43] NH XVIII 57, 215 f. (Loeb V, p. 325/7); cf. also above p. 562.
[44] This terminology is attested, e.g., in Parker, Vienna Pap., p. 6, a text belonging to the Persian period, around 500 B.C.
[45] Columella, De re rustica XI 1, 31 (Loeb III, p. 69).
[46] For another reference to Chaldeans by Columella cf. above p. 595 f. (concerning the winter solstice).
[47] That parapegmata could have been abstracted from Normal Star Almanacs (cf. above p. 553) is not impossible but not very plausible.

C. Summary

The areas in which Babylonian influence on Greek science is most visible are easily enumerated: sexagesimal computation and arithmetical methods in elementary astronomy, e.g., with oblique ascensions; orthogonal ecliptic coordinates; general mathematical background as reflected in many of Heron's writings (first cent. A.D.) and in Diophantus; recognition of the main components of lunar motion, parameters and period relations, the latter for planets as well (goal year periods); finally the basic principles and astronomical techniques of astrology.

Astrology is in some sense typical of the changes which result from a transmission of Babylonian material to the Greeks. Astrology is the latest form of a segment of Babylonian omen literature which had gathered forebodings from all seemingly strange events in human life, mostly for the community as a whole, or the king as its personification, as well as occasionally, for individuals.[48] In all of its manifestations this doctrine concerns luck or misfortune in store for Mesopotamian society which had developed the whole theory on the basis of its own experiences, fears and expectations.

Before the fifth century B.C. celestial omina probably did not include predictions for individuals, based on planetary positions in the signs of the zodiac and on their mutual configurations. In this latest and most significant modification astrology became known to the Greeks in the hellenistic period. But with the exception of some typical Mesopotamian relics the doctrine was changed in Greek hands to a universal system in which form alone it could spread all over the world. Hence astrology in the modem sense of the term, with its vastly expanded set of "methods" is a truly Greek creation, in many respects parallel to the development of Christian theology a few centuries later.

An analogous process took place in the transmission of mathematical astronomy, chronologically, it seems, more or less contemporary with the transmission of astrology, probably not much before 200 B.C. The extremely clever and efficient methods for computing ephemerides, developed in Mesopotamia probably in the Persian period, contain no element of adjustment for other areas. They are made and used for predictions concerning the lunar calendar, eclipses, and planetary phases for the meridian and the horizon of Babylon.[49] From the time that the Greeks took over the geographical element had to be taken into account. It appears in the arithmetical schemes for the length of daylight, based on the Babylonian "Systems A or B" but endowed now with a new parameter, the greatest length of daylight for the region in question.[50] Similarly in the lunar theory: the local problem of first and last visibility vanishes from the theory which becomes a real theory of lunar motion, however much indebted in its basic parameters to Babylonian astronomy. The same happened in the planetary theory. The visibility problems, of central importance in the Babylonian procedures, are relegated to an appendix of the cinematic theory of planetary motion.[51] Everywhere the theory

[48] Cf. on these origins Sachs [1952, 3], introduction.

[49] This should not be taken too literally; no element in the theory is sufficiently sensitive to require a distinction, e.g., between Babylon and Uruk.

[50] Cf. IV D 1, 2.

[51] Cf. above p. 230.

starts from Babylonian discoveries and concepts but rapidly takes on a new aspect which has very little or nothing in common with its Mesopotamian origin. It is my guess that it was Hipparchus who played the decisive role in this transformation.

In mathematics the process of transmission follows a chronologically very different pattern. The Mesopotamian origins precede the beginnings of astronomy by a millennium or more. By the hellenistic time a long established mathematical tradition existed which naturally was not restricted to local use only as was the case for the omen literature and eventually for mathematical astronomy. Babylonian mathematics — sexagesimal arithmetic, algebraic problems, geometric rules (e.g. the "Pythagorean theorem" for right triangles) had spread early over a far greater area than Babylonian astronomy ever reached. This type of mathematics was easily accessible and remained useful for centuries to come and it is not surprising to see it in continued existence during antiquity and deep into the Islamic and Byzantine period. Nevertheless an entirely new form of Greek mathematics was developed since the discovery, in the fifth century, of the existence of "irrational" quantities. This was followed by the consequent geometrization of mathematical proofs and the mastery of problems involving processes of integration. This development reached its climax with Archimedes and Apollonius, ending at the very moment when astronomy begins to become a mathematical science, a step to which Apollonius contributed so much by his geometric-cinematic discoveries. Whether he had any knowledge of Babylonian astronomy we unfortunately cannot tell.

B. Early Lunar and Solar Theory

§ 1. Luni-Solar Cycles; Lunar Theory

Nature provides us with three units of time measurement: the solar day, the lunar month, and the solar year. Unfortunately these units are not commensurable in a simple way. Consequently the problem arose to find integers a, b, c, which satisfy with a reasonable accuracy over a long span of years a relation

$$a \text{ years} = b \text{ months} = c \text{ days}. \tag{1}$$

In the course of history many solutions of this problem have become of practical importance—every luni-solar calendar has a relation of the form (1) as its ultimate foundation. One may speak in this context of "cycles" because (1) can be interpreted in terms of complete revolutions of sun and moon. At the same time, however, (1) can also be read in the form

$$1 \text{ year} = \frac{c}{a} \text{ days}, \quad 1 \text{ month} = \frac{c}{b} \text{ days}, \quad 1 \text{ year} = \frac{b}{a} \text{ months}, \tag{2}$$

i.e. as a statement concerning the (mean) length of a month or of the year. In spite of the fact that (1) and (2) are arithmetically equivalent they represent very different historical aspects of the problem. A relation (1) can be established in practice by the simple counting of completed years and months without any requirement of high accuracy; the result may nevertheless be valid for several, if not many, cycles. A relation (2), however, can imply the use of accurate observations and sophisticated arguments to establish, say, the moments of solstices in order to find the length of the year, or the derivation of parameters for the lunar motion from eclipses. And it is not at all necessary that an accurate value of the form (2) is translated into a cycle, just as a convenient cycle need not to be considered the exact equivalent of a statement (2).

It is of importance for the proper evaluation of historical developments to keep these two facets of the same arithmetical relation strictly separated. It is, e.g., historically misleading to say that Callippus greatly improved the lunar theory after having found by diligent observations discrepancies with earlier parameters – only because a value for the length of the mean synodic month deviates by only 10 seconds from the modern value.[1] Actually the Callippic 76-year cycle is based on two assumptions, neither one having anything to do with observations by Callippus: (a) the length of the year is 365 1/4 days, and (b) the ("Metonic") 19-year cycle is correct, i.e. 19 years contain 7 intercalary months (hence a total of $19 \cdot 12 + 7 = 235$ months). Consequently the shortest cycle with integer coefficients is

$$4 \cdot 19 = 76 \text{ years} = 4 \cdot 235 = 940 \text{ months} = 76 \cdot 365\,1/4 = 27759 \text{ days}. \tag{3}$$

[1] Schiaparelli, Scritti II, p. 85f. (=[1877], p. 172f.).

Obviously this has nothing to do with observations or increased accuracy. Accepting the two assumptions (a) and (b) the length of the month, expressed in days, is no longer a free parameter and it is purely accidental if a convenient calendaric arrangement leads to a higher accuracy for the mean synodic months. We may, of course, say that (3) implies

$$1 \text{ month} = 27759:940 = 7,42,39:15,40 = 29;31,51,3,\ldots^{d} \qquad (4)$$

and be pleased by the accuracy of this value but one may well ask whether Callippus was ever interested (or able) to carry out the division in (4).

It is quite obvious that it is of very little interest for early Greek astronomy to derive relations (2) from cycles (1); there is simply no useful place for a parameter like (4). Such data are of importance only at a much later stage of development when one begins to construct tables of mean motions and equations or for the numerical control of eclipses. Such problems make sense beginning with the time of Apollonius and Hipparchus but not in a period of a still strongly speculative astronomy (as represented by the homocentric spheres of Eudoxus-Callippus-Aristotle[2]) or in the framework of elementary arithmetical methods.[3]

Although we can safely consider (2) as only secondary consequences of cycles (1) we must try to understand the motive underlying the construction of cycles. To the modern historian the obvious answer would be that the purpose of luni-solar cycles is the establishment of regular intercalation patterns in the lunar calendars. This explanation finds support in the role of the 19-year cycle in the Babylonian calendar since the 5th century B.C.[4] On the other hand all available evidence from Greek calendars drastically contradicts the concept of controlling the (sometimes chaotic) civil calendar by astronomical considerations. One easily gets the impression that scientifically minded reformers were simply ignored by the politicians who were entrusted with the handling of the calendar, its intercalations and relations to public office and festivals.

There is no evidence in ancient literature for the assumption that cycles were made known in order to reform the civil calendar. Since all interference with the calendar would be a highly political affair in Greek city government one should expect to find mentioned in our sources attempts of reform and change. On the other hand one has ample references to the role of cycles in relation to the weather calendars, i.e. the "parapegmata." These are by their nature associated with the solar year and with the fixed-star phases,[5] phenomena which can only be effectively listed within a secure framework of time reckoning.

The well-attested lack of influence of calendaric cycles on the civil calendar[6] on the one hand and the equally well-known relationship of cycles to the

[2] Cf. below IV C 1, 2.

[3] Cf. below IV D 1 and IV D 2.

[4] Cf., e.g., II Intr. 3, 1.

[5] Cf. above IV A 3, 3.

[6] That the Athenian calendar (the only one about which we have ample information) shows no relation to the Metonic 19-year cycle has been stated repeatedly, e.g. in Meritt, Ath. Cal., p. 4/5. From a tabulation made by Pritchett (Ch. M. Tables 8 and p. 62) I reproduce here in Fig. 2 A the epigraphically secure evidence for intercalary (•) and ordinary (o) years, in B the corresponding numismatic results (from M. Thompson, Coinage, p. 612/613) for the last cycles in the same period (cycle 6, 18 = −319 to cycle 18, 9 = −100).

parapegmata leads to the conclusion that the primary purpose of the known cycles from the 5th and 4th centuries was the construction of reliable parapegmata, not the reform of civil calendars. When Diodorus says[7] that the Metonic cycle was still in use in his time (i.e. around 50 B.C.) "with the majority of Greeks" he obviously did not expect his readers to think of the civil calendars and he refers explicitly to Meton's success in predicting the "significant" days[8] for the weather prediction from the stars.

Furthermore a myth should be dispelled that had been introduced by Fotheringham: the "astronomical calendar" of the Greeks.[9] Obviously observations separated by any length of time cannot be composed on the basis of dates expressed only in a Greek civil calendar. Since nevertheless astronomers were able to say how many days had elapsed between contemporary and past observations it is necessary to assume the existence of a fixed time scale, independent of the fluctuations of the local calendars. The obvious solution to this problem is the Egyptian calendar which permeates the whole astronomical literature. As a typical example one can mention the dates in a parapegma from Miletus.[10] There we find not only the contemporary civil date of a summer solstice equated with an Egyptian date (Archon Polykleitos, Skirophorion 14 = Payni 11 [= −108 June 26] but also Meton's initial solstice of Skirophorion 13 is equated with the Egyptian Phamenoth 21 [= −431 June 27], exactly as in the Almagest some 250 years later.[10a]

This solution is so simple and so well documented that it was overlooked by Fotheringham who constructed instead a hypothetical calendar, blindly accepting one of these oversimplified stories found in the elementary astronomical literature written ad usum Delphini. In this case it was Geminus who explained a 19-year cycle in which every 64th day-number is omitted in order to obtain a scheme of 30-day months.[11] That such a scheme never could have worked in practice is obvious. The poor astronomer should have known that in some year perhaps Boedromion 5 and 7 were consecutive days, in another year in another month, e.g., 17 and 19. Needless to say there is no trace in the literature of any observation dated in this hypothetical calendar although one should expect to find with Hipparchus or Ptolemy such dates mentioned instead of Egyptian dates and the era Nabonassar.[12] One can safely dismiss as completely unhistorical the concept

[7] Diodorus XII 36, 3 (Loeb IV, p. 448/449).

[8] The ἐπισημασίαι; cf. below p. .

[9] Fotheringham [1924]; revived by van der Waerden [1960].

[10] Cf. above p. 588 and below p. 622, n. 2. The text of the parapegma is given also in Meritt, Ath. Cal. p. 88 but should be corrected following Dinsmoor, Archons, p. 312, n. 1.

[10a] Added in proofs. My denial of evidence for a calendar expressly constructed for astronomical purposes was based on the implicit assumption that the Athenian dates which are given as the equivalents of Egyptian dates by Ptolemy and in the parapegma in Miletus are dates in the civil calendar, thus useless for astronomical purposes. To this my colleague G. J. Toomer objects that no motive can be seen for the use of Athenian dates in Miletus (which has a calendar of its own) or by Timocharis in Alexandria. He therefore assumes that the cycles of Meton, and then of Callippus, contained definite schematic rules for the lengths of the months and for intercalations, of course independent of the local calendar, although using the Athenian names. Cf. for details the article "Meton" in DSB, vol. 9, p. 337–340.

[11] Geminus, Isag. VIII, 50–56 (Manitius, p. 120/123).

[12] For the number of years any era (e.g. Olympiads, or lists of archons, etc.) would suffice to establish the correct distance.

of a calendar supposedly made for the astronomers alone but nowhere attested in astronomical records.

Remotely related to the concept of cycle is the grouping of years into larger units called "great years." Strictly speaking the concept "cycle" is of interest only if at least two units are set into a relation of the form (1). But occasionally such a relation can also be implicitly understood with a single number, e.g., with the "Sothic cycle" of 1460 years.

The difficulty with the term "great year" lies in its ambiguity. Almost any period can be found sometime or somewhere honored with this name. One has "great years" ranging from one or two years to huge sexagesimal or decimal round numbers – usually without any astronomical significance. Some examples may illustrate this situation:

P. Par. 1: in the acrostic[13] the μέγας χρόνος means 365^d, i.e. the Egyptian year.

Censorinus 18. 10–11 ed. Hultsch, p. 39, 3f.): *annus* ἡλιακός or θεοῦ ἐνιαυτός = 1461 years i.e. *annus canicularis* = κυνικός (p. 38, 23f.) i.e. the Sothic Period.

Antiochus (CCAG 1, p. 163, 15–23): κοσμικὴ ἀποκατάστασις = cosmic return of 1 753 200 (= 1461 · 1200) years.[14]

Or huge sexagesimal numbers:

Censorinus 11 (ed. Hultsch, p. 39, 14f.): Heraclitus and Linus 10800^y (= 3,0,0); Cassandrus 3600000^y (= 1000 · 1,0,0).

Aetius (Diels, Doxogr. p. 363): Heraclitus 18000^y (= 5,0,0); Diogenes $360 \cdot 1800^y$ (= 30,0,0,0).

Or pure numerology:

Hippolytus, Contra heres. IV, 7 (ed. Wendland, p. 39, 23f.; trsl. Preysing, p. 50): 7777 years.

CCAG 4 p. 114–119: 1000-year period for each of the 7 planets.

Macrobius, Somnium II, Chap. 11, 11f. (trsl. Stahl, p. 221): "world year" of 15000^y for the return of all planets.

Some of the practically important cycles have also been named "great year," e.g. the 19 years of the Metonic cycle[15] or the expanded Metonic cycle of $28 \cdot 19 = 532$ years.[16] We have also mentioned before[17] meaningful planetary periods combined with numbers which are only a numerological game. Obviously the first step in dealing with such material should consist in a classification according to origin and purpose as well as to time and geographical provenance. Unfortunately such indispensable preliminary work has not been done. On the contrary far reaching new hypotheses have been added to an indiscriminate use of most diverse sources,[18] the results of which are bound to be generally accepted and to obscure the issue beyond recognition.

[13] Cf. below IV C 1.3 A.
[14] Cf. above p. 606.
[15] Censorinus 18, 8 (ed. Hultsch, p. 38, 9–12).
[16] Remigius, Comm. in Mart. VIII, ed. Lutz, p. 284, 2f. Cf. for this cycle below p. 624.
[17] Above p. 606f.
[18] E.g. by van der Waerden [1952, 2] and [1970].

1. Early Greek Cycles

What we know about calendaric cycles, supposedly proposed in the 5th century B.C., comes from late antiquity (mainly Censorinus in the 3rd century A.D.) and is, of course, based entirely on secondary sources. Optimistically one could perhaps say that this late tradition shows at least the existence of attempts to establish luni-solar cycles as early as the 5th century B.C. But details are usually lacking or make no sense. Only in one instance, in the case of Philolaos (of the 2nd half of the 5th century), do we have sufficient data to see how he constructed his 59-year cycle. This sheds some interesting light on the "astronomy" of the period.

The cycle of Philolaos, the Pythagorean, is said to consist of 59 years with 21 intercalations[1] (thus obviously lunar years) while his "annus naturalis" (i.e. solar year) contained 364 1/2 days.[2] These numbers are consistent. The number of lunar months in the cycle is

$$59 \cdot 12 + 21 = 729. \tag{1}$$

Reckoning a lunar month as 29 1/2 days[3] the number of days will be

$$729 \cdot 29\,1/2 = 21505\,1/2 \text{ days} \tag{2}$$

and division by 59 gives for one "year" exactly the 364 1/2 days. It has long been observed[4] that the number 729 of months in the cycle has special relevance for the pythagoreans as a number associated with the sun, and being simultaneously the square of 27, the number of the moon,[5] and the cube of 9, the number of the earth.[6] Thus the length of the year of 364 1/2 days is merely the result of numerological speculation combined with the commonly accepted round value of 29 1/2 days for the lunar month.

A 59-year cycle is also attributed[7] to Oinopides (middle of 5th cent., thus a little earlier than Philolaos), with the additional statement that he reckoned the year as 365 22/59 days. It has been suggested[8] to connect this parameter with the smallest multiple of Egyptian years and lunar months of 29 1/2 days

$$59^{y} = 730^{m} = 59 \cdot 12 + 22^{m} (= 21535^{d})$$

by making the 22 intercalary months 30 days long, but we have no textual evidence for such a procedure.[8a] A cycle supposedly proposed by Democritus (2nd half of

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[1] Censorinus, De die nat. 18, 8, ed. Hultsch, p. 38. 13–15 (also Diels, VS$^{(5)}$, p. 404, 18 f.).
[2] Censorinus 19, 2, p. 40, 14 f. (also Diels, VS$^{(5)}$, p. 404, 19 f.).
[3] Cf., e.g., the Eudoxus Papyrus, below p. 624.
[4] Cf. Ideler, Chronol. I, p. 309 and [1810] p. 410.
[5] Cf., e.g. the Eudoxus Papyrus, below p. 624.
[6] Cf. the sequence of the powers of 3 mentioned by Plutarch, De animae procreatione (1028 B), ed. Hubert, p. 183, 23–25.
[7] Aelianus, Varia historia X. 7 (ed. Herscher, p. 109, 15–18; also Diels, VS$^{(5)}$, p. 394, 15–17). Censorinus 19, 2 (Hultsch, p. 40, 19 f.).
[8] Aaboe-Price [1964], p. 5. For another reconstruction cf. Tannery, Mém. Sci. II, p. 358 f.
[8a] G. J. Toomer considers the reference to a specific number of days in the cycle to be a later addition. Cf. DSB, vol. 10, p. 180.

5th cent.) of 82 years with 28 intercalations[9] makes no sense since 28 intercalary months belong to a 76-year cycle.[10]

We are on safe ground, at least astronomically, with the 8-year cycle, the so-called "octaeteris." The construction of this cycle does not require any special observations, in fact the less observations the better. One has only to assume that the difference between solar year (actually Egyptian year) and lunar year amounts to $e=11$ days, the lunar month being again 29 1/2 days.[11] Then it is clear that $3e$ and $6e$ come relatively near to the length of one or two months, respectively, while $8e=88$ days are almost accurately 3 months (i.e. 88 1/2 days). Hence an 8-year cycle with 3 intercalary months (in the years 3, 6, and 8) seems a plausible way of balancing lunar and solar years. It is indeed this rule which we find in the Eudoxus Papyrus[12] as well as in Geminus.[13]

There remains the question of how full and hollow months should be distributed. An explicit rule in the Eudoxus Papyrus is so badly garbled[14] that nothing can be safely said except that the "years" are Egyptian years. Hence the total of days in the cycle should be $8 \cdot 365 = 2920$ which would be obtainable from $8 \cdot 354 + 2 \cdot 29 + 30$. This would mean that the cycle contained $8 \cdot 6 + 1 = 49$ full months and $8 \cdot 6 + 2 = 50$ hollow months. Since the text repeatedly refers to the Egyptian norm of the "year" it seems certain that the above arrangement represents the original version of the octaeteris.[15]

In its later form, as described by Geminus, the "year" is assumed to contain 365 1/4 days. Consequently the number of days in 8 years is now 2922 and 51 months become full, 48 hollow. It is this form which is usually meant when one speaks about the Greek octaeteris.

It is quite clear from our sources that even in antiquity one had no certain information about the authorship of the octaeteris. Censorinus reconstructs an obviously absurd sequence of events according to which a 2-year and a 4-year cycle preceded the 8-year cycle.[16] Then he says that it is "commonly believed" that the octaeteris was instituted by Eudoxus, while others consider Cleostratus to be its inventor; additional authors are mentioned for variants in the intercalation pattern (without giving details) while Dositheus "is most generally identified with the octaeteris of Eudoxus."[17] Eratosthenes who himself had written a treatise on the 8-year cycle[18] apparently doubted Eudoxus' authorship and also

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[9] Censorinus 18, 8 (ed. Hultsch, p. 38, 16f.).
[10] The error is perhaps caused by contamination with the directly preceding Callippic cycle of 76 years with 28 intercalations.
[11] Cf., e.g., the Eudoxus Papyrus; below p. 624.
[12] Tannery, HAA, p. 290, No. 36.
[13] Geminus, Isag. VII, 25 (Manitius, p. 110-113).
[14] Most numbers given by Tannery are restored (without warning); cf. for the text Blass, p. 20 col. XIII, 12-XIV, 6.
[15] This rule implies synodic months slightly shorter than 29 1/2 days, since $48,40:1,39 \approx 29;29,42$.
[16] Censorinus, De die nat. 18 (ed. Hultsch, p. 36-40).
[17] Translation by Heath (Aristarchus p. 291) of "cuius maxime octaeteris Eudoxi inscribitur", whatever this should mean. Dositheus was a friend of Archimedes and is mentioned for his observations by Ptolemy in the "Phaseis" (cf. below p. 929); also above p. 581.
[18] This seems to follow from Geminus, Isag. VIII, 24 (Manitius, p. 110, 2).

Diogenes Laertius expresses himself very cautiously.[19] It seems best to leave the question of the origin of the octaeteris unanswered.

In Censorinus one also finds data ascribed to Aristarchus: (a) a cosmic period of 2484 years, (b) the statement that 1/1623 of one day should be added to the year of Callippus (i.e. to 365 1/4).[20] We have already mentioned[21] that Tannery suggested emending the first number from 2484 to 2434 in order to reconcile it to the second; this led him to the further conclusion that Aristarchus knew at least that part of the "exeligmos" which implies that

$$54 \text{ rotations of the sun} + 32° \approx 19756 \text{ days}.$$

If Tannery's reconstruction is correct, as seems to be the case, we have arrived at the threshold of Babylonian influence and thus the beginning of real mathematical astronomy.

Beginning with the Roman rule over the mediterranean world the concept of a "reform" of the civil calendar took concrete shape. Thus the Egyptian calendar fell victim (under Augustus) to Caesar's idea that the civil calendar should be adjusted to the correct length of the tropical year. By that time it had long been an established fact that an accurate solution of this condition would require cycles extending over centuries unless one agreed on the 4-year cycle. Fortunately at least the Egyptian schematic months of constant length were kept untouched. The result was the Alexandrian calendar, which froze the Egyptian calendar in the position valid for the years from -25 to -22.[22]

Any change in a civil calendar is a highly political matter, as experience shows down to the present day. It is a testimony to Caesar's superior political skill that his arrangement of the Roman calendar was put into permanent operation, even after his death. Obviously he tampered as little as possible with the traditional order to which we still owe the absurd months of the "Julian" calendar. The astronomical component of this calendar is, of course, utterly trivial and Caesar needed for it no technical "advisor," be it Sosigenes or anyone else. Only the combination of the new calendar with a parapegma seems to furnish a plausible explanation of the association in the calendar reform with an astronomical assistant.[23]

With the victory of Christianity the 19-year cycle of the Babylonian lunar calendar experienced an unexpected revival in the Easter computus. Henceforth this fragment from Late-Babylonian and hellenistic astronomy became the core or what was left of astronomical knowledge in the western Middle Ages.

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[19] "Some say" (VIII, 87; Loeb II, p. 403).

[20] Censorinus, De die nat., p. 39, 12f. and p. 40, 16f., ed. Hultsch.

[21] Above p. 603, n. 21.

[22] That is to say: during these 4 years the Egyptian Thoth 1 is the same as the Alexandrian Thoth 1 (i.e. August 30 in -25, August 29 in -24 to -22). In -21 Alex. Thoth 1 = Egypt. Thoth 2 (= August 30). Cf. for the "Era Augustus" below p. 1066.

[23] Cf. above p. 575.

2. The Metonic and Callippic Cycle

The 19-year cycle assumes that

$$19 \text{ years} = 19 \cdot 12 + 7 = 235 \text{ syn. months.} \qquad (1)$$

A corresponding intercalation pattern, using six times a month XII_2 and once a month VI_2, is well-known not only from the Babylonian ephemerides but also as the regulating principle of the Babylonian civil calendar. Its use is attested from about 490 B.C. (except one disturbance in $-384/3$) into the first century of our era.[1]

Again on the basis of the relation (1) Meton in Athens introduced a 19-year cycle, beginning at the summer solstice on Skirophorion 13 of the year of the archon Apseudes (-431 June 27).[2] The date Skirophorion 13 shows that the civil lunar month did not begin near the solstice,[3] a fact that has caused much discussion concerning the "beginning" of Meton's cycle. This problem vanishes, however, when one realizes that Meton did not attempt to introduce a new lunar calendar but intended to establish a definite starting point in the solar year for the construction of parapegmata.[4]

It is, of course, perfectly possible that the "Metonic cycle" of 432 B.C. is an independent discovery of the relation (1), known a generation earlier in Mesopotamia. It is only the advanced state of astronomical techniques and the ease of communication of a rule as simple as expressed in (1) that suggests Babylonian priority.

On the other hand, however, one should notice a difference in approach between the Babylonian and the Greek 19-year cycle. The Babylonian cycle is used in conjunction with real lunar months. Hence there is no schematic rule connected with the cycle that concerns the order, or even the relative frequency, of full and hollow months. Essential for the cycle is only the pattern of the position of the intercalary months and the assumption that

$$1^y = 12;22,6,20^m \qquad (2)$$

which is, but for a very close rounding, the equivalent of the basic relation (1).[5] No assumption is made about the length of the solar year.

The emphasis of the Greek versions, however, lies in the number of days which make up the solar year (or the total of years in the cycle). This attitude is easy to understand if the purpose of the Metonic (or Callippic) cycle was the construction of a simple pattern of schematic lunar-months that could be related to the given Egyptian calendar. Again it is the connection with the parapegmata,

[1] Cf. above p. 354f.

[2] Diodorus XII 36, 2 (Loeb IV, p. 446/447). Ptolemy, Alm. III, 1 (Manitius, p. 143) says that the summer solstice in this year was observed, but only superficially recorded, by the school of Meton and Euctemon for the morning of Phamenoth 21 (of the year Nabonassar 316, i.e. -431 June 27; cf. above p. 294 and p. 617).

[3] The "Uruk scheme" (above II Intr. 3, 2) would give as date of the summer solstice III 11 (not 13); from Parker-Dubberstein BC one obtains III 10 for -431 June 27. Apparently Skirophorion was about 2 or 3 days ahead of the real lunar month.

[4] Cf. above p. 616.

[5] Cf. above p. 355 (3) and p. 358 (5) and (6).

i.e. the solar year, that determines the character of the cycle, not the improvement of any extant lunar calendar. Thus the basic assumptions for the Greek cycles are either (a) that 19 years of 235 months contain an integer number of days, or (b) that the true length of the solar year is 365 1/4 days.[6]

The first-mentioned assumption (a) is in fact only a convenient modification of (b); it introduces merely a formal definition of a new type of "year" by giving it the length[7]

$$1^{y}=365+5/19 \text{ days} \tag{3}$$

which is the nearest approximation of (b) compatible with (a). Consequently one has according to (a)

$$19^{y}=19\cdot 365+5=6940^{d}. \tag{4}$$

This total can be obtained as the combination of 125 full and 110 hollow months.[8]

According to the assumption (b), however, one would find

$$19^{y}=19\cdot(365+1/4)=6939+3/4^{d} \tag{5}$$

such that only $4\cdot 19=76$ years give an integer number of days

$$76^{y}=27\,759^{d}. \tag{6}$$

The relations (3) or (4) are ascribed by Hipparchus and Theodosius[9] to Meton and Euctemon, by Geminus[10] to Euctemon, Philip, and Callippus, by Censorinus[11] to Meton. In modern literature this is the "Metonic cycle."[12]

About a century after Meton, Callippus (around 330 B.C.) again insisted on the validity and importance of the 365 1/4 days as the length of the year. Also accepting the validity of the 19-year cycle he had no other choice than to operate with a 76-year cycle.[13] Neither the Metonic nor the Callippic cycle have anything to do with an accurate determination of the length of the mean synodic month.[14]

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[6] We know from Hipparchus that this was before his time the commonly accepted value (Alm. III, 1, Manitius I, p. 145).

[7] Geminus, Isag. VIII, 58 (Manitius, p. 122, 13–15). In view of the later 76-year cycle the fraction 5/19 is also put in the form 1/4+1/76 (e.g. Alm. III, 1 Heiberg, p. 207, 10; also Geminus, Isag. VIII, 58 Manitius, p. 122, 15f.), and Theon in his Commentary to Alm. III, 1, quoting Hipparchus (cf. Rome CA III, p. 838 or [1926], p. 9f.).

[8] Geminus, Isag. VIII, 52 (Manitius, p. 121).

[9] Alm. III, 1 (Manitius I, p. 145, 14f.). Theodosius, De diebus II, 18 (ed. Fecht, p. 152, 2) in a passage for which cf. below p. 754, n. 19.

[10] Geminus, Isag. VIII, 50 (Manitius, p. 121).

[11] Censorinus, De die nat. 18, 8 (ed. Hultsch, p. 38, 9–11).

[12] The role of Meton seems not too well defined in the ancient tradition. Ptolemy (Alm. III, 1, Manitius I, p. 143) in discussing the summer solstice of −431 speaks first about observations by the "school of Meton and Euctemon," later on (p. 144) only about the "school of Euctemon." In the "Geminus"-parapegma Meton is mentioned only once (in contrast to Euctemon "passim"). The "Eudoxus Papyrus" (P. Par. 1) mentions for the length of the seasons only Eudoxus, Democritus, Euctemon, Callippus (Tannery HAA, p. 294, No. 55). Meton's name does not appear in Geminus' Isagoge and is also omitted in Maass, Aratea p. 140, list "B" for the length of the year (in contrast to "A"; cf. above p. 601). Modern scholars have avoided the problem by "emending" versions they did not like; cf. Rome [1926], p. 8 = CA III, p. 839, note (1).

[13] Alm. III, 1 (Manitius I, p. 145); Geminus, Isag. VIII, 59 (Manitius, p. 123).

[14] For the Metonic cycle one obtains from (4) $1^{m}=29;31,54,53,37,\ldots^{d}$; for the Callippic cycle $29;31,51,3,49,\ldots^{d}$ (cf. above p. 616). Note that neither one of these numbers appears among Babylonian parameters.

The 76-year cycle had obvious advantages since it combined the best known luni-solar cycle with the Alexandrian calendar. But the christian rules for the celebration of Easter not only involve the vernal equinox and the synodic month (that much is supposedly taken care of by the 19-year cycle) but also the 7-day week. Consequently a cycle of $7 \cdot 76 = 532$ years was adopted, e.g., in the monophysite church, and still rules the Ethiopic calendar.[15]

An astronomically nontrivial modification of the Callippic cycle is due to Hipparchus as we know from Censorinus.[16] Hipparchus found that the tropical year was about 1/300 of a day shorter than 365 1/4; hence about 300 years should contain one day less than according to Callippus. But 300 years are very nearly 4 Callippic cycles, i.e. 304 years. Hipparchus therefore assumed that

$$4 \text{ Callippic cycles} = 4 \cdot 76 = 304 \text{ years} = 304 \cdot 365;15 - 1 = 111\,035 \text{ days.} \quad (7)$$

The number of synodic months is 16 times the number in one 19-year cycle, i.e. $16 \cdot 235 = 3760$ months, $16 \cdot 7 = 112$ of which are intercalary. There is no reason to assume that this Hipparchian cycle ever found calendaric application.[17]

3. Lunar Theory

Some very elementary data concerning the lunar motion are given in the "Eudoxus Papyrus," a text which belongs undoubtedly to a period not much later than about -300 though it is not characteristic for the astronomical teaching of Eudoxus.[1] In this treatise the synodic month is simply taken to be 29 1/2 days long; hence 2 months are 59 days and the lunar year is 354 days, 11 days shorter than the solar year which is conveniently identified with the Egyptian year.[2] Although slightly garbled in the extant text (which is a secondary composition) it is clear that the sidereal month is taken to be 27 days long since the moon is said to remain 2 1/4 days, i.e. 27/12, in each sign.[3] This could be either a residue of the pythagorean doctrine which associates the consecutive powers of 3 with the celestial bodies, 27 being the number of the moon,[4] or it is based on some simple arithmetical approximation of the lunar motion.[5] The text shows no recognition either of the lunar or of the solar anomaly, hence such crude estimates would be quite fitting for this early period.

That Eudoxus ignored these anomalies is clear from his cinematic models. We know from Simplicius[6] that he used only three (concentric) spheres for the

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[15] Cf., e.g., the "years of grace" (Chaine, Chron., p. 111).

[16] Censorinus, De die nat. 18, 9 (ed. Hultsch, p. 38, 18f.); cf. also above I E 2, 2 C.

[17] Cf. above p. 297.

[1] Cf. below p. 687.

[2] Cf. Tannery HAA, p. 285ff. (Nos. 7, 34, 35, 40).

[3] Tannery Nos. 9 and 41.

[4] Cf. above p. 619.

[5] The sidereal month is 2 1/2 days shorter than the synodic month because $2;30^d \cdot 12^{\circ/d} = 30^\circ$ where $12^{\circ/d}$ is the daily elongation of the moon from the sun (because $12^{\circ/d} \cdot 30^d = 360^\circ$) and 30° is the progress of the sun in one schematic month of 30^d.

[6] Simplicius, Comm., ed. Heiberg, p. 494, 23–495, 16; Schiaparelli, Scritti II, p. 97f.; [1877], p. 184.

moon[7]: the outermost one produces the daily rotation common to all celestial objects, the next one represents the ecliptic, and the third (innermost) one carries on its equator the moon at a fixed inclination toward the ecliptic, corresponding to the maximum lunar latitude. The second of these three spheres rotates with a slow retrograde motion that produces the recession of the nodes.[8] Since all rotations are assumed to proceed with uniform angular velocity a single sphere which produces the motion of the moon along its orbit cannot account for the anomaly.

Callippus, apparently on the basis of the inequality of the seasons observed by Euctemon and Meton,[9] found it desirable to take into account the variability of the solar velocity; the same seems to have been the case with the lunar motion, although the formulation of the problem is not without ambiguity in our sources.[10] Both for sun and moon Callippus added two spheres to the original three assumed by Eudoxus; Schiaparelli made the plausible conjecture that these two spheres were used to move the luminary on a "hippopede" — an 8-shaped curve on the sphere — in order to obtain a periodic change of velocity.

What we know about the hippopede comes from the Eudoxan planetary theory and therefore shall be discussed in a later context.[11] At the moment it suffices to say that in the arrangement assumed for Callippus by Schiaparelli the double point of the 8-shaped curve takes the place of the (mean) moon in the Eudoxan model. The true moon, by its motion on the curve, obviously adds to the mean motion when traversing one half of the hippopede and subtracts from it during the other half of the anomalistic period. The length of each loop, above and below the double point, would then correspond to the observed maximum equation. This could be used to determine the parameters of the hippopede.[12]

As we shall see the theory of the homocentric spheres cannot be considered astronomically successful for the planetary motions, excepting perhaps in a very modest way, Saturn and Jupiter.[13] For the moon, however, a quite satisfactory explanation for the anomaly as well as for the motion in latitude could have been achieved. Unfortunately none of the extant sources gives any numerical details, nor can we say for sure that Callippus actually used the hippopede for representing the lunar anomaly.

Whatever form the Callippic modifications of the Eudoxan lunar theory took, they left no trace in the subsequent development. For this the contact with

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[7] Also for the sun; cf. below IV B 2, 2 for the solar latitude.

[8] For a mistake in Simplicius' formulation concerning the nodal motion cf. Schiaparelli, Scritti II, p. 21; [1877], p. 118 (or Tannery, Mém. Sci. I, p. 328–332). Text: Simplicius, Comm., p. 494, 23–495, 16; Schiaparelli, Scritti II, p. 97 (No. 3); [1877], p. 184; Tannery, Mém. Sci I, p. 330.

[9] Simplicius, Comm., p. 497, 18–22; Schiaparelli, Scritti II, p. 100/101; [1877], p. 187. For the inequality of the seasons cf. above I B 1, 3.

[10] Eudemus in Simplicius, Comm., p. 497, 12–21; Schiaparelli, Scritti II, p. 100/101 and p. 85; [1877], pp. 187 and p. 172.

[11] Cf. below IV C 1, 2 A.

[12] Schiaparelli suggested (Scritti II. p. 85 f.; [1877], p. 172 f.) about 6° for the maximum equation, thus $\gamma=6°$ for the angle between the axes of the two spheres which generate the hippopede (cf. below Fig. 27). From this results a latitudinal width of the curve of about 0;9° ($r \approx 0;0,10$) and for the additive or subtractive velocity a maximum of about $1;20^{\circ/d}$ (cf. below IV C 1, 2 B (2) and (6) with $\Delta t \approx 27;30^{d}$) — a very reasonable estimate for the equation.

[13] Cf. below p. 683.

Babylonian astronomy became of primary importance insofar as the different components of the lunar motion had been correctly established and numerical data of comparatively very high accuracy were introduced and became inseparably connected with theoretical models.

One aspect of this development has been discussed before: the acceptance of the Babylonian scheme for the variation of the lunar velocity.[14] Another element which may ultimately go back to Babylonian influences is the assumption of a maximum lunar latitude of 6°.[15] In the Greek milieu this parameter is embedded in the description of the zodiac as a belt of 12° width which we meet first in the Isagoge of Geminus.[16] Martianus Capella says that the moon oscillates within the full width of these 12° and that Hipparchus accepted this estimate.[17] In this, however, Martianus is certainly mistaken. Theon of Smyrna ascribes[18] to Hipparchus the determination of the inclination of the lunar orbit as 5°, in contrast "to the majority of mathematicians" who assume $i=6°$. Indeed the Almagest leaves no doubt that Hipparchus, exactly as Ptolemy, took i to be 5°, e.g. in the discussion of parallaxes.[19]

§ 2. Solar Theory

Early Greek solar theory in all its characteristic aspects seems to be independent of Babylonian influence. The only parallel consists in the early recognition of the existence of a solar anomaly. The Greek discovery of this important phenomenon is in all probability based on the observation of the unequal lengths of the seasons.[1] We do not know how Babylonian astronomers became aware of the variability of the solar velocity but it seems pretty certain that it was not through the inequality of the seasons.[2]

There is no room in Babylonian procedures for another feature of early Greek solar theory, the assumption of a motion in latitude.[3] And finally, the hypothesis of a slow oscillation of the equinoxes[4] could have been influenced by Babylonian astronomy only to the extent that it apparently uses the Babylonian norm which locates the equinoxes and solstices at 8° of their respective signs.[5] The theory itself, however, has no Babylonian counterpart.

[14] Cf. above p. 586 and p. 602f.
[15] For the Babylonian evidence cf., e.g., Neugebauer-Sachs [1968/9] I, p. 203 and ACT I, p. 190f. (No. 200 obv. I, 20); above p. 515 and p. 520.
[16] Cf. above p. 583; also below p. 782.
[17] Martianus Capella VIII, 867 (ed. Dick, p. 456/7).
[18] Dupuis, p. 313/315.
[19] Alm. V, 7 (Man. I, p. 285, 6–17); also Theon, Comm. to Alm. IV, 9 (Rome CA III, p. 1068, 2) or Proclus, Hypot. IV, 63 (Man., p. 116, 22). For the parallaxes cf. above p. 101 and p. 324.
[1] Cf. below IV B 2, 1.
[2] Cf. above II Intr. 5.
[3] Below IV B 2, 2.
[4] Below IV, B 2, 3.
[5] Above IV A 4, 2 A.

1. Solar Anomaly

One cannot doubt that in the cinematic models designed by Eudoxus, Callippus, and Aristotle the motions of the sun and of the moon were considered similar in all essential points; this includes, e.g., the assumption of a solar latitude and a motion of the corresponding nodes.[1] Consequently Eudoxus postulated for the sun, exactly as for the moon, three concentric spheres[2] (daily rotation, longitudinal mean motion, nodal motion). This shows, as in the case of the moon,[3] that the Eudoxan theory had made no provision for a solar anomaly.

Callippus, however, modified the arrangement for the sun, again exactly as for the moon, by adding two new spheres. We are told, on the authority of Eudemus,[4] that this happened because of the solar anomaly, the existence of which was revealed through the inequality of the seasons observed by Euctemon and Meton. This lends support to Schiaparelli's conjecture that the two new spheres were used by Callippus to move the sun on a hippopede which would periodically modify the mean progress of the sun.[5] Aristotle, however, seems not to have been convinced of the existence of a solar anomaly since he considered it possible that the two additional spheres for sun and moon could be omitted.[6]

Aristotle's attitude is surprising in view of the fact that Callippus did not accept the earlier recording of the inequality of the seasons without observing it himself. This is known through the so-called "Eudoxus Papyrus"[7]; near the end of this text[8] one finds a list of intervals between solstices and equinoxes according to Eudoxus, Democritus, Euctemon, and Callipppus. The length of the interval (s_4) from the vernal equinox to the summer solstice is always omitted but it can be readily determined under the certainly correct assumption that the "year" is schematically taken to be 365 days long. This gives for Callippus the following seasons:

$$s_1 = 92 \text{ days}, \quad s_2 = 89 \text{ days}, \quad s_3 = 90 \text{ days} \quad (s_4 = 94 \text{ days}). \tag{1}$$

These values agree so well with the facts[9] that their origin from observations can hardly be in doubt. Hence Callippus must have been convinced of the reality of an anomaly in the solar motion.

Damage to the papyrus creates some questions for the data concerning Eudoxus and Democritus. For s_1 we know only that both authors agreed on the

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[1] For the theory of solar latitude cf. below IV B 2, 2.

[2] Cf. Schiaparelli, Scritti II, p. 23–42; [1877], p. 120–136.

[3] Cf. above p. 625.

[4] Simplicius, Comm., ed. Heiberg, p. 497, 17–22; Schiaparelli, Scritti II, p. 100/101; [1877], p. 187.

[5] Assuming a maximum equation of about 2° one obtains according to p. 680 f. (2), (4), and (6) with $\gamma = 2°$ a negligible additional latitude ($\beta_{max} \approx 0;1°$ from $r \approx 0;0.1$) and $\approx 0;2^{\circ/d}$ as a maximum change of velocity, which is a reasonable amount.

[6] Simplicius, Comm., p. 503, 11; Schiaparelli, Scritti II, p. 107; [1877], p. 193.

[7] Written around 190 B.C. but based on an earlier version, possibly of the period around 300 B.C.; cf. below IV C 1, 3 A.

[8] Tannery, HAA, p. 294, No. 55; text: Blass, p. 25; Not. et Extr. 18, 2, p. 74 f.

[9] Cf. Schiaparelli, Scritti II, p. 83; [1877], p. 170. From the dates and intervals given in the "Geminus" parapegma (cf. above p. 581) one finds, however, for Callippus $s_1 = 92$, $s_2 = s_3 = 89$, $s_4 = 95$ (cf. below p. 1352, Fig. 4). The explicit statement in the papyrus seems to me the more reliable source.

same value. Beyond that we have only

for s_2 Eudoxus: 92 days, Democritus: 91 days
for s_3 both: 91 days.

Since s_4 is always omitted one can only guess the rest; it seems plausible to restore

$$\begin{array}{lllll} \text{Eudoxus:} & [s_1=91], & s_2=92, & s_3=91 & (s_4=91) \\ \text{Democritus:} & [s_1=91], & s_2=91, & s_3=91 & (s_4=92). \end{array} \tag{2}$$

This restoration implies equality of the seasons, the one exception 92 being arithmetically unavoidable. This would agree, at least for Eudoxus, with his above-mentioned restriction to only three spheres in the cinematic models for sun and moon.[10]

It is, of course, impossible to say whether Eudoxus purposely ignored the solar anomaly or not.[11] The central problem in his arrangement of homocentric spheres was the explanation of planetary retrogradations and he may well have disregarded the minute effects of the variability of the solar velocity. It is by no means necessary to apply even a well established insight to all possible cases for the mere sake of formal consistency.[12]

Also the parapegmata would better be considered under the viewpoint of arithmetical convenience than as witnesses for a theory of solar anomaly. If one looks, e.g., at the parapegma of Euctemon as reconstructed by Rehm[13] one finds nothing but a simple pattern of distributing 360+5 days over 12 zodiacal signs. The sun is conveniently assumed to traverse as many signs as possible in 30 days each; but 5 extra days must be stored away somewhere and this is done in a simple fashion symmetric to Aries: the sun is kept 31 days in each of the 5 signs from Aquarius to Gemini (cf. Fig. 3). Such an arrangement has nothing to do with the solar anomaly, as is evident, e.g., from the direction of the corresponding "apsidal line" (through Aries and Libra) which would place the extrema of the solar motion in the region of the mean motion.[14]

A different situation prevails with Rehm's reconstruction of the parapegma of Callippus.[15] He ended up with a neat scheme for the solar motion according to which the sun travels 32 days in Gemini, 29 in Sagittarius, and, symmetrically to this diameter, $2 \cdot 31 + 3 \cdot 30$ days in the intermediate signs. As far as I can see the meager evidence for this model is derived from the Roman "rustic calendar" of the time of Caesar which is based, supposedly, on Callippus' parapegma.[16]

[10] Tannery (Mém. Sci. 2, p. 236–247) detected in Eudoxus' parapegma a similar symmetry for the fixed star phases. Cf. also Boeckh, Sonnenkr., p. 110f. and Kl. Schr. 3, p. 343–345. In Ethiopic astronomy, which in many ways depends on hellenistic prototypes, we find a schematic year of 364 days, divided into four seasons of 91 days each.

[11] Rehm (RE Par. col. 1343, 48; also Parap., p. 39) calls it a "revolutionäre Tat" to ignore the earlier observations of Euctemon in order to obtain "harmony" as visualized by Plato.

[12] The Eudoxus Papyrus, e.g., accepts a strictly symmetric (linear) scheme for the length of daylight simultaneously with the recognition of seasons of unequal length (cf. below p. 706).

[13] Rehm [1913], p. 9. The Eudoxus Papyrus gives only $s_1 = s_2 = 90$ days, $s_3 = 92$ days.

[14] Surprisingly Pritchett-van der Waerden ([1961], p. 45) call this arrangement a "remarkable parallel" to the Babylonian solar model of System A (which, however, has an apsidal line through ♊20° and ♐20°; cf. Fig. 8, p. 1317).

[15] Rehm, RE Par. col. 1346f. and Fig. 2 there.

[16] Cf. above p. 595 and p. 596, note 27.

This procedure brushes aside the possibility that a Roman version of a Greek calendaric pattern may no longer, a century after Hipparchus, reflect a situation two centuries before Hipparchus.[17]

In fact we have direct evidence for the view that in this early period one had only a rather incomplete knowledge of the changes of the solar velocity with respect to the ecliptic. In the "Geminus" parapegma, e.g., about a century before Hipparchus,[18] the solar travel as shown in Fig. 4 cannot be considered to be representing a clearly understood model of the anomalistic solar motion. From the third century B.C. we have in the Almagest nine "Dionysian" dates[19] which, in principle, should tell us something about the underlying solar theory. The result, however, is purely negative since the given zodiacal data deviate in an irregular fashion both from true and from mean solar longitudes.[20] Again one is led to the conclusion that Greek astronomy before Hipparchus had nowhere reached the level of contemporary Babylonian astronomy.

2. Solar Latitude

Our sources leave no doubt that early Greek astronomers ascribed a motion in latitude not only to the moon but also to the sun. What is never explained in precise terms, however, is the definition of "latitude" when the ecliptic is not defined as the great circle in which the yearly solar motion takes place. Or formulated differently: why and how did one define a circle of reference which is not the path of the sun?

The zodiacal constellations which lie in the path of all planetary motions are much too irregular to lead directly to the definition of one specific great circle through their midst. We have, however, ample evidence of a much more direct but speculative approach to a definition of the ecliptic: it is the great circle which deviates from the equator by an angle which is exactly the 15th part of the circle's circumference.[1] Hence, as we find so often in early Greek astronomy some, perhaps "pythagorean," speculation was given precedence over accurate observation; and, as usual, such a speculative theory which imputes to the "cosmos" the orderliness and simplicity of the human mind creates problems which it takes generations (if ever) to overcome.

After having dogmatically fixed the position of the ecliptic as part of a numerological or ideal geometrical structure it is not too surprising that one found deviations of the sun from this abstract model as soon as one made observations in earnest. Of course it is not easy to determine the orbit of the sun with respect to the fixed stars or a circle of reference drawn ideally among them. What is relatively easy to determine, however, is the equator and its intersections with the horizon. Under favorable conditions it should also be possible to establish

[17] It also should be noted that the 32nd day in Taurus is actually attested in the text (Geminus, ed. Manitius, p. 232, 7).
[18] Cf. above p. 581.
[19] Cf. below p. 1066.
[20] Cf. Ideler, Astron. Beob., p. 264.
[1] For our evidence for this definition of the obliquity of the ecliptic cf. below p. 733 f.

points on the horizon that mark the rising- (and setting-)amplitude of the ecliptic which is considered to be defined by its inclination toward the equator. These points can be compared at the solstices with the points of actual rising and setting of the sun. It seems indeed that in this way deviations of the sun from the postulated ecliptic were discovered [2], which meant to ascribe to the sun a "latitude." All sources agree that these latitudes were not found to be large.[3] Unfortunately we do not know which maximum was assumed by Eudoxus; later sources, from the first to the fifth century A.D.,[4] agree on $\pm 1/2°$. Only Pliny [5] (perhaps mistakenly) mentions $\pm 1°$.

Chalcidius, in the fourth century A.D., assumes [6] in Libra a latitudinal variation of $\pm 1/2°$. The origin of this peculiar, and not very clearly formulated, statement probably lies in the previously mentioned observation that at the solstices the sun rises or sets at points which do not agree with the ortive amplitude of the ideal ecliptic. Hence, if the extremal latitudes occur at the solstices, the nodes will be in Libra and Aries. Chalcidius may still have meant to say that the inclination between solar orbit and ecliptic is about 1/2° in Libra. Martianus Capella in the fifth century, however, was already of the opinion [7] that the latitude of the sun varies only in Libra between $\pm 1/2°$, being zero on the rest of the ecliptic. Kepler tried to explain this "*absurditas et insolentia*" as caused by a misinterpretation of refraction at the observation of autumnal equinoxes [8], being much too kind to mediaeval astronomy. In fact not only Martianus but also the previously mentioned passage in Pliny NH II, 67 [9] has produced a long chain of more or less absurd pronouncements by mediaeval authors.[10]

Theon of Smyrna [11] gives some numerical information about the solar motion without, unfortunately, telling us who introduced these parameters. He only says that the majority of the mathematicians assumes that the sun's return to the same longitude, to the same latitude, and to the same anomaly takes practically the same time, i.e. 365 1/4 days. But nevertheless he gives the following specific values:

return to same anomaly:	365 1/2 days [12]	
return to same solstice or equinox:	365 1/4 days	(1)
return to same latitude:	365 1/8.	

[2] Simplicius, Comm., p. 493, 15–17 ed. Heiberg (Schiaparelli, Scritti II, p. 96; [1877], p. 182); Hipparchus Ar. Comm. I, IX (p. 88, 14–22 ed. Manitius) in criticizing the opinions of Eudoxus and Attalus. For the importance of the ortive amplitudes cf. above I A 4, 4 and below p. 977 f.

[3] Hipparchus l.c. p. 88, 21 f.; cf. also above p. 278 and below p. 807.

[4] Theon of Smyrna (about A.D. 130, mainly based on Adrastus, about a generation earlier) Expositio ..., Astron., Chap. 38 (ed. Hiller, p. 194, 4–8; Martin, p. 314/315; Dupuis, p. 212/213); also Chap. 12 (ed. Hiller, p. 135, 12–14; Martin, p. 174/175; Dupuis, p. 222, 10f./223, 8f.). For Adrastus cf. Theon, ed. Hiller, index p. 213.

[5] Pliny NH II 67 (Loeb I, p. 214/215; Budé II, p. 29).

[6] Chalcidius 88, ed. Wrobel, p. 159, 10–12; cf. also Chap. 70, ed. Wrobel, p. 137, 12f.

[7] Martianus Capella, De nuptiis VIII 867 (ed. Dick, p. 457, 2–5).

[8] Ad Vitellionem IV (Werke 2, p. 137, 3–17).

[9] Cf. above note 5.

[10] Cf., e.g., the collection given in Lattin [1947], p. 215, no. 87.

[11] In Chap. 27 (ed. Hiller, p. 172, 15–173, 16; Martin, p. 258/263; Dupuis, p. 278/281); also Schiaparelli, Scritti II, p. 30f.; [1877], p. 126f.

[12] Some manuscripts seem to give 365 1/6 but the emendation 365 1/2 is secured by the additional remark that the return occurs every two years at the same hour.

This looks like another example of numerology, particularly when one notices that in consequence of (1) the solar apogee returns to the same longitude in 1461 years, i.e. exactly in one Sothic period, while the return of the nodes takes two Sothic periods, i.e. 2922 years. In the present context another consequence of (1) is of interest: the motion of the apsidal line is direct, of the nodes retrograde. This contradicts the assumption of the Eudoxan model according to which the innermost sphere which carries the sun has the same sense of rotation as the second sphere which produces the motion of the nodal line along the ecliptic.[13] The combination of solar anomaly and latitude in an arithmetical pattern based on the Sothic period speaks more in favor of a hellenistic origin than of an early Greek theory.[14]

3. The Trepidation of the Equinoxes

Let the ecliptic now be defined as the great circle of the solar orbit and let there be measured on it "sidereal longitudes," i.e. ecliptic arcs with respect to some fixed star. Hipparchus discovered[1] that in such a coordinate system the vernal point (i.e. the intersection between ecliptic and equator) is not fixed but moves slowly in the direction from east to west. This motion is known as the "precession of the equinoxes." As far as one can conclude from the discussion in the Almagest it was Ptolemy who first insisted on the constancy of the rate of this motion (1° per century) whereas Hipparchus himself seems to have left this question in doubt.[2]

From a short section in Theon's "Small Commentary"[3] to the "Handy Tables" we learn about a theory which positively asserted the existence of a periodic variation in the sidereal position of the vernal point. Because of the limited amplitude of this motion one refers to it usually as the "trepidation of the equinoxes." All later authors who mention the early history of trepidation depend more or less explicitly on this short notice by Theon[4] as their only source of information; and this has remained so to the present day.

[13] Schiaparelli, Scritti II, p. 26; [1877], p. 122. Cf. above p. 625, note 8 for the erroneous interchange of the roles of the second and the innermost sphere.

[14] Schiaparelli, Scritti II, p. 33 ([1877], p. 128) suggested a connection between Theon's latitude theory and the octaeteris. But a luni-solar intercalation cycle has nothing to do with the solar latitude; furthermore one should not separate in (1) the motion of the nodes from the motion of the apogee which is part of the same speculative doctrine.

[1] Cf. above I E 2.

[2] Cf. p. 294 and I E 2, 2 C.

[3] This Commentary was written in the second half of the 4th century A.D.; cf. below p. 966 f. The historical interest of the section in question was first recognized by Delambre (1817; HAA II, p. 625–627); the text was edited by Halma (1822; HT I, p. 53) with a French translation. Nallino (1903; Batt. I, p. 298) gave a Latin translation; Dreyer (1906; Plan. Syst. p. 204) translated it into English. Duhem (1914; SM II, p. 194) again into French. For an Arabic version, found in the Picatrix (second half of the 11th century) cf. the German translation by Plessner-Ritter, p. 82 (1962).

[4] Battānī substituted the name of Ptolemy for Theon (Nallino, Batt. I, p. 126, Chap. 52). Also Bīrūnī in his Chronology (written A.D. 1000) refers to a work by Ptolemy "On the spherical art" (trsl. Sachau, p. 322). This is apparently the same work which Ṣā'id al-Andalusī in his Ṭabaqāt al-umam (written 1068) ascribes to Theon (trsl. Blachère, p. 86). Bīrūnī in his Astrology (written 1029) properly reverts to Theon's authorship (trsl. Wright, p. 101, No. 191).

Because of the profound effect on mediaeval astronomy of this early Greek theory I give a translation here of the whole relevant chapter in Theon's Commentary. The title is either simply "On the solstices" (Περὶ τροπῶν,[5] where τροπαί stands for all four cardinal points, as is common in texts of this period) or "On the motion of the solstitial points."[6] The text then gives the following description of an oscillatory motion of 8° amplitude:

"According to certain opinions the ancient astrologers[7] pretend that the solstitial points move 8° in the direct sense, from a certain initial moment on, then turn back again by the same amount; but this is not so in Ptolemy's opinion because without this additional term the said[8] computations, made with his tables, agree with the instrumental data. Nor do we recommend this correction; nevertheless we shall explain their computational procedure.

They assume that 128 years before the beginning of the reign of Augustus the greatest shift occurred in the direct sense of these 8°, and also the beginning of the return motion; then they add to it the 313 years from the beginning of the reign of Augustus to the beginning of Diocletian and to it the additional years[9] since Diocletian; and they take the position corresponding to this total, such that their motion amounts to one degree in 80 years, and they subtract from 8° the degrees which result from this division. They add the remainder as the shift of the solstices to the result obtained by the said computations of the positions of the sun and the moon and the five planets."

The principle underlying this rule is very simple: to the solstices and equinoxes is ascribed a motion following a linear zigzag function with a difference of 1° per 80 years and with the sidereal longitudes $\lambda^* = 0°$ and $\lambda^* = 8°$ as extrema (cf. Fig. 5). The vernal equinox, e.g., is at the point $\lambda^* = 0°$ in the years -797 and $+483$; in the year -157 (= Augustus -127) the vernal point is at $\lambda^* = 8°$ at which moment it changes its motion from direct to retrograde.[10] Consequently, from this moment on, to the time of Theon and beyond, the tropical longitudes of all

[5] The title shown in Halma's edition is Περὶ τροπῆς and is indeed found in Par. gr. 2399 fol. 12ᵛ. Par. gr. 2400 fol. 13ʳ, 2423 fol. 140ᵛ, and Vat. gr. 208 fol. 80ᵛ, however, all have the plural. There is no basis for Halma's new technical term "De la conversion" which has been repeated everywhere in the modern literature.

[6] Vat. gr. 1059 fol. 112ʳ II. All these texts are unpublished, excepting Halma's Par. gr. 2399.

[7] Παλαιοὶ τῶν ἀποτελεσματικῶν.

[8] All the above mentioned MSS have εἰρημένας. The senseless ἰσημερινὰς is Halma's mistake, religiously followed in all modern translations. I do not list additional minor variants.

[9] Delambre gives here the specific number of 77 years since Diocletian and Dreyer includes it in his translation. Neither Halma's printed text nor any of the unpublished MSS known to me has such a number.

[10] From VI A 2, 3 one derives the following chronological table:

λ^*	year of Augustus			Thoth 1
8°	−127	S.E. 154	−157 Oct. 2	Egypt.
6;24	1	S.E. 282	−29 Aug. 30	Alex.
2;29	314	Diocl. 1	284 Aug. 29	Alex.
0°	513	Diocl. 200	483 Aug. 30	Alex.

Delambre HAA II, p. 626 gives incorrectly 170 B.C. as the equivalent of Augustus −127.

fixed stars increase at a constant rate of 0;0,45° per year, thus faster than according to the Ptolemaic precession (0;0,36° per year). Obviously, according to this theory, the motion of trepidation is a substitute (in fact improvement) for Ptolemy's precession. Not until the Middle Ages was a (trigonometric) trepidation term superimposed over a monotonic precession.

It is impossible to say what argument had been used by the ancient astronomers to postulate an amplitude of 8° for a displacement of the equinoxes and solstices. It is tempting to assume a connection with the Babylonian norm ♈8° for the vernal point in System B in relation to the Greek norm ♈0°.[11] The use of a linear zigzag function certainly points in the same direction and suggests a relatively early period as time of origin of this weird theory. Perhaps we are dealing here with an early form of the theory of precession, if not by Hipparchus himself, at least near to his time and under the influence of his discovery. It may not be accidental that Hipparchus' earliest observations of equinoxes[12] are dated

$$-161 \text{ Sept. } 27, \quad -158 \text{ Sept. } 27, \quad -157 \text{ Sept. } 27.$$

The last mentioned year is exactly the year Augustus -127 when precession sets in at $\lambda^* = 8°$. In Babylonian terminology the solar longitude at these equinoxes would be ♎8°. This looks very much as if the λ^*-coordinates were simply the longitudes of Babylonian astronomy. We have no proof for this identification but it would explain why we are never told that a specific fixed star, like Regulus at a later period,[13] is the point of reference for the sidereal longitudes.

Without Theon the theory of trepidation would hardly have survived beyond antiquity. Proclus (around A.D. 450) alludes to it in the "Hypotyposis"[14] without giving any details or quoting his source. It is only in connection with Theon's Handy Tables that Islamic astronomers were introduced to the concept of periodic variations in the motion of precession.

Though the theory of trepidation was by no means accepted by all Islamic astronomers — Battānī, e.g., rejected it[15] — it found its widest following in the west, as late as Copernicus.[16] Mediaeval astronomers could indeed find good

[11] Cf. above IV A 4, 2 A and p. 600. Apparently an explanation of this kind was also in Bīrūnī's mind (Chronology, trsl. Sachau, p. 322, 15–25; cf. also his Astrology, trsl. Wright, p. 101, No. 191).

[12] Cf. above p. 276 and Table 28 there. Excepting the summer solstice of -431 June 27 which is the traditional epoch date for the Metonic cycle (cf. above p. 622) and Aristarchus' summer solstice of -279 reported by Hipparchus (cf. below p. 634), the above listed equinoxes are the earliest ones mentioned in the Almagest.

[13] Cf. below V C 4, 3 C.

[14] III 54 (Manitius, p. 66/69); expressly mentioned (with an amplitude of 8°) in scholion 316 (Manitius, p. 275). Manitius misinterpreted Proclus (p. 287, note 7) when he related this remark to the Eudoxan hypothesis of a solar latitude (for which see above IV B 2, 2), perhaps misled by Schiaparelli who also connected the theory of trepidation with the theory of solar latitude. Schiaparelli remarked correctly (Scritti II, p. 31/32; [1877]. p. 127) that a vibration of the equinoxes (i.e. of the intersections of the solar orbital plane with the equator) is a necessary consequence of the assumption of a rotation of the nodes of the solar orbit (i.e. of the intersections with the ecliptic). Assuming an inclination of 1/2° between orbital plane and ecliptic (cf. above IV B 2, 2) and $\varepsilon = 24°$ one finds that the true equinoxes deviate from the mean at most about 1;15° with a period of 2922 years (because of (1), p. 630). Obviously these parameters cannot explain Theon's data, even if it were permissible to combine the relations (1) which imply the existence of a solar anomaly with a homocentric model which excludes anomaly.

[15] Cf. Nallino, Batt. I, p. 127.

[16] De revol. III, Chap. 1 to 12.

arguments against a constant rate of precession in the contradictory results obtained by the measurements of their predecessors. It was again Tycho Brahe who reached the conviction that observational errors were the sole source for the phenomenon of trepidation.[17]

§ 3. Sizes and Distances of the Luminaries

Ptolemy, using a method developed by Hipparchus,[1] came to the conclusion that the sun is about 1200 earth radii distant from us, i.e. he assumed[2] 0;2,51° for the sun's horizontal parallax. It is essentially this estimate which was accepted during the whole Middle Ages, including Copernicus[3] and Brahe.[4] Kepler argued[5] for a distance about three times the traditional value but not before the end of the 17th century did one reach the correct order of magnitude of 0;0,10° for the solar parallax,[6] which showed that Ptolemy's estimate for the sun's distance was wrong by a factor of at least 17 and that the sun's volume was some 5000 times larger than any of the ancient evaluations had assumed.

It is not surprising that the early attempts at determining the size and distance of sun and moon in relation to the earth ended with wrong results. The ancient methods are of necessity based on trigonometric arguments in combination with visual estimates of very small angles and one naturally had the tendency to falsify such estimates in the wrong direction.

The first mathematically controllable method known to us is due to Aristarchus (first half of the third century B.C.), to be followed some decades later by Archimedes' discussion of the observational difficulties. These two sources give us the best understanding of the problems the early astronomers had to face when they tried to determine the size of our planetary system.

Beyond these two treatises from the third century B.C. we have a great number of references to evaluations for the apparent and for the absolute sizes of sun and moon, usually without letting us know how and when such estimates had been reached. Material of this type is collected in the last section of this chapter (IV B 3, 4).

1. Aristarchus

Aristarchus' lifetime is known from an observation of the summer solstice of 280 B.C., credited to him in the Almagest,[1] and through Archimedes' references

[17] Brahe, Opera II, p. 255f. and Dreyer's remarks in Opera I, p. XLVIf.
[1] Cf. above I B 5, 4 A.
[2] Cf. above p. 112.
[3] Who found it convenient to reduce the distance of the sun to 1179 earth radii (cf. Neugebauer [1968, 2], p. 101).
[4] His table of parallaxes, Progymn. I 80 (Opera II, p. 65) gives 0;3° for the horizontal parallax of the sun.
[5] Epitome Astron. Cop. IV, 1 (Werke 7, p. 279).
[6] Flamsteed and Cassini (cf. Houzeau, Vadem., p. 405).
[1] Alm. III, 1 (Manitius I, p. 144, 5–145, 3).

in the "Sandreckoner," written before 216 B.C.[2] It is from this latter source that we know about the suggestion of a heliocentric hypothesis which causes so much delight to modern historians, in particular because Copernicus knew of it but crossed out a reference to it in his final manuscript.[3]

In the following we shall discuss Aristarchus' only preserved treatise "On the sizes and distances of the sun and moon" of which Heath has given a modern edition and English translation in his great monograph on Aristarchus and early Greek astronomy.[4] In late antiquity Pappus gave extensive references to this treatise and added some useless commentary to one of its propositions.[5] Vitruvius (time of Augustus) recites the explanation given by Aristarchus for the phases of the moon,[6] based on assumptions which are also fundamental in the above mentioned treatise: sphericity and illumination by the sun.

A. Aristarchus' Assumptions

At the beginning Aristarchus, in the tradition of Greek mathematics, formulates the assumptions on which the subsequent propositions rest. He asserts that the moon is illuminated by the sun, that the (circular) lunar orbit has a smaller radius than the solar orbit (i.e. $R_m < R_s$[1]), and that parallax may be ignored, allowing us to identify our eye with the center E of the earth which is also the center of the lunar and solar orbit.

For Aristarchus' method the following assumptions are of basic importance:

(A) The elongation η of the moon from the sun at half-moon is $<90°$ (cf. Fig. 6), specifically[2]

$$\eta = 87°. \tag{1}$$

(B) Moon and sun are of equal apparent diameter (cf. Fig. 7)

$$r_☾ = r_\odot \tag{2}$$

specifically[3]

$$r_☾ = 1^\circ. \tag{3}$$

The equality of the apparent diameters of sun and moon is expressly stated as "Proposition 8," supposedly based on the observations of solar eclipses. In fact, however, the validity of (2) is assumed from the very beginning.

(C) The radius u of the shadow at a lunar eclipse (cf. Fig. 8) is a known multiple of the lunar radius

$$u = k \cdot r_☾ \tag{4}$$

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[2] Cf., e.g., Heath, Arist., p. 299, p. 302.

[3] E.g. Heath, Arist., p. 301.

[4] Heath, Arist., p. 352–411. Tannery, Mém. Sci. I, p. 371–396 reached conclusions in many respects similar to the following arguments.

[5] Cf., e.g., Heath, Arist., p. 412–414 and below p. 640.

[6] Vitruvius, Archit. IX, 2, ed. Krohn, p. 208, 1–25; Loeb II, p. 228/231; Budé, p. 17f. and notes p. 124–130.

[1] Notation: the actual radii of earth, moon, sun, and of the shadow at the moon's distance are denoted by r_e, r_m, r_s, r_u, respectively; the apparent radii are $r_☾$, $r_\odot$, u, respectively. The geocentric distances of sun and moon are R_s and R_m, respectively.

[2] For the actual formulation of (1) cf. above p. 590.

[3] In the text (3) is expressed in the form that the apparent diameter of the moon is 1/15 of a zodiacal sign.

specifically
$$k=2. \tag{5}$$

From these assumptions Aristarchus derives upper and lower bounds for the ratios of the actual diameters d_s/d_m (Propos. 9), d_s/d_e (Propos. 15), and d_e/d_m (Propos. 17). In short, the diameters of sun and moon in terms of the earth's radius r_e are determined within narrow limits. The distances, however, are not computed by Aristarchus, in spite of the title of his treatise; we shall return to this point presently.[4]

Before discussing the procedure adopted by Aristarchus in his treatise we shall first solve the problem at hand in full generality (Section B). Then we shall substitute in the formulae thus obtained the numerical data accepted by Aristarchus and compare them with his own numerical results (Section C). In this way one obtains a better insight into the interaction between purely mathematical conclusions and empirical data in Aristarchus' treatise.

B. Mathematical Consequences

From (A) it follows (cf. Fig. 6) that
$$R_m=c R_s, \qquad c=\cos\eta. \tag{6}$$
Aristarchus, who has no trigonometric functions, gives boundaries
$$c_1<c<c_2 \tag{7}$$
which, substituted in our general formulae, will give inequalities instead of equations. For the underlying arguments this is of no significance.

From (B) one concludes (cf. Fig. 7) that also
$$r_m=c r_s. \tag{8}$$
It follows from Fig. 8 that
$$(r_e-r_u)/R_m=(r_s-r_e)/R_s \tag{9}$$
where the diameter r_u of the shadow is a known quantity because of (4).

Consequently: from (9), (4), (6), and (8):
$$\frac{r_e-k r_m}{c R_s}=\frac{r_e-k c r_s}{c R_s}=\frac{r_s-r_e}{R_s},$$
hence
$$\frac{r_e}{c}-k r_s=r_s-r_e$$
or
$$r_s=\frac{1+\frac{1}{c}}{1+k}\,r_e \tag{10}$$
and with (8)
$$r_m=\frac{1+c}{1+k}\,r_e. \tag{11}$$

[4] Cf. below p. 643. Surprisingly the enumeration of results at the beginning mentions only the boundaries for the ratios R_s/R_m and d_s/d_m (Propos. 7 and 9) and for d_s/d_e (Propos. 15) but not for d_e/d_m (Propos. 17).

Hence we know now the size of the sun and of the moon in comparison with the earth.

In order to obtain also the distances we must introduce the apparent size of the moon (or sun, according to (2)):

$$r_{☾} = \alpha. \tag{12}$$

Then (cf. Fig. 9) with (11)

$$R_m = \frac{r_m}{\sin\alpha} = \frac{1+c}{(1+k)\sin\alpha}\, r_e \tag{13}$$

and with (6)

$$R_s = \frac{1+\dfrac{1}{c}}{(1+k)\sin\alpha}\, r_e\,, \tag{14}$$

the distances expressed in earth radii.

C. Numerical Consequences

Accepting Aristarchus' numerical assumptions

$$\eta = 87°, \quad r_{☾} = r_{\odot} = 1°, \quad k = 2 \tag{15}$$

we can now determine the numerical results which he could have obtained had he followed the direct mathematical procedure outlined so far. Only for the trigonometric functions must we make convenient additional assumptions which, as we shall see, come in part quite close to the estimates reached by Aristarchus.

Since $\cos 87° \approx .052$ we assume conveniently

$$c = 1/20. \tag{16}$$

Thus from (10)

$$r_s = \frac{1+20}{1+2}\, r_e = 7 r_e \tag{17}$$

and from (11)

$$r_m = \frac{21}{60}\, r_e = \frac{7}{20}\, r_e = 0;21\, r_e \quad \text{or} \quad r_e \approx 2;51\, r_m. \tag{18}$$

Since $\sin 1° \approx 0.0175 = 7/400$ we obtain from (13) and (18)

$$R_m = \frac{7}{20} \cdot \frac{400}{7}\, r_e = 20\, r_e \tag{19}$$

and from (6), (16), and (19)

$$R_s = 20\, R_m = 400\, r_e \tag{20}$$

for the distances.

Our results (10), (11) and (13). (14) were obtained through most elementary geometric considerations, fully at Aristarchus' disposal. For our

$$c = \cos\eta = \cos 87° \approx .052 \approx 1/20,$$

however, he had no tables at his disposal and determined boundaries (7) which we find in Proposition 7 in the form

$$18\,R_m < R_s < 20\,R_m \tag{21a}$$

and thus in Proposition 9

$$18\,d_m < d_s < 20\,d_m \tag{21b}$$

which is the equivalent of

$$c_1 = \frac{1}{20} < \frac{r_m}{r_s} < \frac{1}{18} = c_2 . \tag{21c}$$

Since we have assumed in (16) that $c \approx 1/20$ we have already at our disposal the values for one of the boundaries which also Aristarchus could have reached by following the simple way outlined above. For the other boundary we substitute in (10) and (11) for c the value $c_2 = 1/18$. Thus

$$r_s = \frac{1+18}{1+2}\,r_e = \frac{19}{3}\,r_e = 6;20\,r_e$$

$$r_m = \frac{1+\frac{1}{18}}{1+2}\,r_e = \frac{19}{54}\,r_e = 0;21,6,40\,r_e .$$

Hence Aristarchus might have said

$$6;20\,r_e < r_s < 7\,r_e \tag{22}$$

instead of Proposition 15

$$6;20\,r_e < r_s < 7;10\,r_e \tag{23}$$

and

$$\begin{aligned} & 0;21\,r_e < r_m < 0;21,6,40\,r_e \\ \text{or}\quad & 2;50,31,\ldots\,r_m < r_e < 2;51,25,\ldots\,r_m \\ (\text{or}\quad & 2;50\,r_m < r_e < 2;52\,r_m) \end{aligned} \tag{24}$$

instead of Proposition 17

$$\begin{aligned} & 0;19\,r_e < r_m < 0;23,53,20\,r_e \\ \text{or}\quad & 2;30,\ldots\,r_m < r_e < 3;9,\ldots\,r_m \\ (\text{or}\quad & 2;30\,r_m < r_e < 3;10\,r_m). \end{aligned} \tag{25}$$

In other words his own boundaries for cos 87° would have given him sharper limits for the size of sun and moon in relation to the earth.

The lack of trigonometry would also affect possible estimates for R_m and R_s. In Proposition 11 one has

$$1/60 < \sin 1° < 1/45$$

or

$$45 < 1/\sin 1° < 60. \tag{26}$$

Consequently from (13) and (14)

$$15;45\,r_e = 0;21 \cdot 45\,r_e < R_m < 0;21,6,40 \cdot 1,0\,r_e = 21;6,40\,r_e \tag{27}$$

or with Aristarchus' values (25)

$$14;15\, r_e = 0;19 \cdot 45\, r_e < R_m < 23;53,20\, r_e. \tag{28}$$

Similarly for the sun with (23):

$$6;20 \cdot 45\, r_e = 4,45\, r_e = 285\, r_e < R_s < 7,0\, r_e = 420\, r_e \tag{29}$$

against what follows from Aristarchus' parameters

$$285\, r_e < R_s < 430\, r_e. \tag{30}$$

But, as remarked before, Aristarchus abstained from giving estimates for the distances R_m and R_s.

D. Aristarchus' Procedure

We now can turn to the analysis of Aristarchus' treatise. Its 18 "Propositions" fall into three groups: first the justification of the basic construction of a right triangle EMS as represented in Fig. 6 (Propos. 1–5); secondly trigonometry concerned with $\eta = 87°$ (Propos. 6–10); finally the numerical consequences, involving also the assumption $r_☾ = r_⊙ = 1°$ and related trigonometric evaluations. We shall follow now this simple outline of topics, postponing to the next section (E) the discussion of the astronomical and mathematical background against which this treatise should be seen.

Propositions 1 to 5. Since the moon is seen to eclipse the sun and since the two luminaries appear under the same apparent diameter, the sun is larger than the moon and the common tangential surface is a cone (Propos. 1). Consequently the illuminated part of the moon is greater than the dark one (Propos. 2), i.e. the radius t of the circle which separates darkness from light (the "terminator") is smaller than the moon's radius r_m. Aristarchus' whole method depends on solving the right triangle EMS, assuming that the center E of the earth lies in the plane of the terminator. Hence Aristarchus attempts to show that the terminator is only imperceptibly smaller than a great circle. Unfortunately the proof of this statement (Propos. 4) is slightly garbled in the extant text. It is not difficult, however, to restore the correct argument and we shall do this now before describing the traditional version.

Obviously the radius t of the terminator has its smallest value t_0 when the moon is nearest to the sun (Propos. 3), i.e. at conjunction (cf. Fig. 10). It is this situation which is taken up in the first part of Propos. 4 (represented in the upper part of Fig. 11, to the right). Obviously

$$r_m = \mathrm{MT} = \mathrm{E_0M} \sin\alpha = R_m \sin\alpha.$$

Aristarchus assumes $\alpha = 1°$ (cf. (3), p. 635 and estimates correctly[1]) that

$$\sin 1° < 1/45. \tag{1}$$

Consequently, making also $\mathrm{NMH} = \alpha = 1°$

$$r_m < R_m/45 = (\mathrm{E_0N} + r_m)/45$$

[1] Indeed $\sin 1° = 0;1,2,50 < 0;1,20 = 1/45$.

or

$$r_m < 1/44\ E_0N < 1/44\ E_0H$$

and because

$$\frac{\beta}{\alpha} < \frac{r_m}{E_0H}$$

also

$$\beta < \frac{\alpha}{44} = \frac{1^\circ}{44}. \tag{2}$$

Aristarchus claims that an object seen under an angle that small (in his terminology "1/3960 of a right angle") is "imperceptible to our eye."

We now turn to the situation which corresponds to the arrangement shown in Fig. 6 in which EM is perpendicular to MS (cf. Fig. 11 lower part), such that we look at the moon in the direction of the terminator. Since GU = NH we see the smallest terminator t_0 under the angle β which is according to (2) smaller than $\alpha/44$. According to Propos. 3 the terminator KL is greater than TU, hence nearer to the diameter FG, and therefore seen from E under an angle $\delta < \beta$. Hence LG which represents the deviation of the actual terminator from the diameter FG is imperceptible for an eye in E, q.e.d. Hence we may say (Propos. 5) that E and M lie in the plane of the terminator, as assumed in Fig. 6.

As remarked before the extant proof of Propos. 4 is garbled. The figure does not distinguish between the two positions E_0 and E[2] and the arc NH is not placed to one side of E_0M but, for no good reason, halved by E_0M. At the end, instead of saying that LG is seen under an angle δ still smaller than GU the text implies that GU is seen from E_0 under an angle smaller than β. This is obvious nonsense since the arc GU lies beyond the tangent E_0U and is therefore invisible. At any rate, it is of no interest for our problem to determine how large GU or FT would appear from E_0, even if it could be seen. The error is caused by suppressing in the figure the shift of viewpoint from E_0 to E.

The corruption is an old one. Pappus, in the fourth century A.D., did not find a proof in his text for the smallness of the appearance of FT as seen from E_0 and felt obliged to provide one[3] instead of realizing that what he attempted to prove made no sense. The six centuries between Aristarchus and Pappus provide ample space for the adjustment of the text to an unclear figure; the symmetrisation of HN with respect to the axis E_0M is part of the same process.

Propositions 6 to 10. Propos. 6 states in fact that $R_m < R_s$ and that the elongation η of the half-moon (cf. Fig. 6, p. 1352) is less than 90°; thus assumption (A) (p. 635) is justified. Accepting $\eta = 87°$ Propos. 7 shows that

$$18\,R_m < R_s < 20\,R_m. \tag{3}$$

This is a purely mathematical affair, the equivalent of saying that

$$0;3 = 1/20 < \cos 87° < 1/18 = 0;3,20 \tag{4}$$

[2] It is not unusual in ancient drawings to find that different cases are superimposed in the same figure; cf., e.g., the diagrams for stereographic projection (below V B 3, 2).

[3] Pappus, Coll. VI, ed. Hultsch, p. 560, 11–568, 11; trsl. Ver Eecke II, p. 430–435. The earliest extant manuscript of Aristarchus' treatise is Vat. gr. 204 of the 10th cent.; cf. Heath, Arist., p. 325.

which is correct since $\cos 87° \approx 0;3,8,25$. The proof of (3) need not to be discussed in the present context.[4]

Propos. 8 justifies assumption (B) $r_{\text{\(}} = r_{\odot}$ through the experience at total solar eclipses, unfortunately without referring to a specific occasion. Hence (Propos. 9) one concludes from (3) that also

$$18\, d_m < d_s < 20\, d_m \tag{5}$$

and therefore (Prop. 10) for the volumes

$$5832\,(=18^3) \text{ lunar vol.} < \text{solar vol.} < 8000\,(=20^3) \text{ lunar vol.} \tag{6}$$

Propositions 11 to 18. Finally we come to the core of the problem: to find estimates for the size of the sun (Propos. 15 and 16) and of the moon (Propos. 17 and 18) in terms of the size of the earth. The discussion is greatly complicated by bringing in the assumption (3), p. 635 made about the apparent diameter of the moon. As we have seen (Sect. B, above p. 636f.) this is not only entirely unnecessary but also adversely influences the numerical accuracy of the boundaries (Sect. C, above p. 637).

The first two propositions (11 and 12) are purely trigonometric in character:

$$1/30\, R_m < d_m < 2/45\, R_m \tag{7}$$

which amounts to saying (Propos. 11)

$$0;1 = 1/60 < \sin 1° < 1/45 = 0;1,20$$

(indeed $\sin 1° \approx 0;1,2,50$) and (Propos. 12)

$$89/90\, r_m < r_0 < r_m \tag{8}$$

where r_0 is the radius of the terminator for $\alpha = 1°$ as before in Fig. 11 (p. 1354).

Propos. 13 introduces a very specific interpretation of the assumption (C), p. 635f. according to which the earth's shadow covers twice the apparent diameter of the moon: the shadow is supposed to cover exactly twice the visible section of the moon in the way shown in Fig. 12: if $\alpha(=1°)$ represents the angle under which the radius r_m of the moon is seen from E then ON (perpendicular to the axis of the shadow) is seen under the angle 4α.

The diameter ON of the shadow is now compared with the diameter d_m of the moon, d_s of the sun, and with the diameter RQ of the extended shadow cone (cf. Fig. 13) at S. Using the previously established inequalities one finds by simple geometrical considerations[5]

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[4] Cf., however, below p. 774.

[5] The lower bound in (9) is derived from (8) which leads to $\frac{ON}{d_m} > \frac{1}{2}\frac{89^2}{45^2} = \frac{7921}{4050}$. Heath, Arist., p. 397, n. 1 thinks that the ratio 88/45 in (9) is obtained by expanding the ratio 7921/4050 into a continued fraction. Much simpler, however, is the following procedure

$$\frac{1}{2}\frac{89^2}{45^2} = \frac{1}{45\cdot 90}\cdot 89^2 > \frac{1}{45\cdot 90}(89^2-1) = \frac{1}{45\cdot 90}(89+1)(89-1) = \frac{88}{45}.$$

$$\frac{88}{45} < \frac{ON}{d_m} < 2 \qquad \left(\text{i.e. } 1;57,20 < \frac{ON}{d_m} < 2\right) \tag{9}$$

$$\frac{22}{225} < \frac{ON}{d_s} < \frac{1}{9} \qquad \left(\text{i.e. } 0;5,52 < \frac{ON}{d_s} < 0;6,40\right) \tag{10}$$

$$\frac{979}{10125} < \frac{ON}{RQ} \qquad \left(\text{i.e. } 0;5,48,5,\ldots < \frac{ON}{RQ}\right). \tag{11}$$

Propos. 14 shows, using (7), that

$$R_m > 675 \cdot TM \tag{12}$$

where TM is the maximum distance of the center of the moon from the shadow diameter ON (cf. Fig. 13). Using (10), (11), and (12) Propos. 15 is derived which states that

$$\frac{19}{3} < \frac{d_s}{d_e} < \frac{43}{6} \qquad (\text{i.e. } 6;20\, d_e < d_s < 7;10\, d_e) \tag{13}$$

which is the main result of the whole treatise. It is a trivial consequence of (13) and (21) p. 638 that

$$\frac{108}{43} < \frac{d_e}{d_m} < \frac{60}{19} \qquad (\text{i.e. } 2;30,41,\ldots d_m < d_e < 3;9,28,\ldots d_m) \tag{14}$$

(Propos. 17). The two remaining propositions, 16 and 18, give the exact numerical equivalents of (13) and (14) for the volumes of sun, earth, and moon.

E. Summary

We now come to the astronomical analysis of Aristarchus' treatise. It is obvious that its fundamental idea, the use of the elongation η at the moment of dichotomy, is totally impracticable. The elongation of the moon changes 1° in about 2 hours; thus one should be able to establish the moment of dichotomy within at least one hour. In fact one would be lucky to determine the night into which dichotomy falls; hence $\eta = 87°$ is a purely fictitious number.[1]

Next the apparent diameter of 2° for the moon, is about four times the actual value. Again this cannot be the result of direct measurement, because it would be easy to obtain a much better estimate. Furthermore we have the assumption (C) (above p. 635 f.) which says that the earth's shadow at a lunar eclipse has twice the diameter of the moon.[2] Thus a total lunar eclipse from first to last contact could correspond to an increase of elongation of about 6°, resulting in a duration of about 12 hours. It is clear that $d_☾ = 2°$ cannot be the reflection of any empirical fact. And indeed we know explicitly from Archimedes that Aristarchus assumed 1/2° for the lunar diameter.[3]

---

[1] The actual value is about 89;51° and must therefore elude direct determination by methods available to ancient observers.

[2] In fact $u = \frac{5}{2} r_☾$ would have been a better estimate.

[3] Cf., e.g., Heath, Arist., p. 311.

The fact that Aristarchus does not make the obvious transition from the diameters of sun and moon to the distances, measured in earth radii, supports our conclusion that $d_{☾}=2°$ is not to be taken as a valid observational datum. As we have shown (p. 636) r_m and r_s can be found without any information about the apparent diameters (a fact obscured in Aristarchus' treatise by his involvement with the terminator) whereas the distances depend essentially on this parameter and are very sensitive to its value because of the $1/\sin\alpha$ in (13) and (14) p. 637.

Finally the major part of the discussion in the Propos. 13, 14, 15, and 17 (above p. 641 f.) is astronomically without interest. It is pure mathematical pedantry to operate with the moon's terminator and the chords ON and RQ (cf. Figs. 12 and 13) perpendicular to the axis of the shadow cone instead of the parallel radii—note above p. 642 (12) which says that T differs from M by less than 1/675 of R_m. At the same time parallax is expressly ignored although Aristarchus' own data could lead to a lunar parallax of almost 3°. And it is ridiculous to pretend that observations show that the moon fits the shadow cone exactly in the way depicted in Fig. 12 and to make this the basis for all estimates.

I think this analysis leads to the conclusion that Aristarchus' treatise on the sizes and distances is a purely mathematical exercise which has as little to do with practical astronomy as Archimedes' "Sandreckoner" in which he demonstrates the capability of mathematics of giving numerically definite estimates even for such questions as the ratio of the volume of the universe to the volume of a grain of sand. If one looks at Aristarchus' treatise in the same way it becomes strikingly similar in spirit to Archimedes' essay. On the basis of some more or less admissible assumptions a purely mathematical argument is shown to produce numerical estimates for the relative size of sun, moon, and earth. The numerical data $r_{☾}=1°$, $u=2°$, $\eta=90-3°$ are nothing but arithmetically convenient parameters, chosen without consideration for observational facts which would inevitably lead to unhandy numerical details. On the other hand mathematical decorum is strictly maintained by the pedantry of formulation, unrelated to the complexities of empirical data. But the power of a mathematical approach to astronomical problems has been drastically demonstrated, in the same sense as Eudoxus, a century earlier, constructed cinematic models which could be related to planetary motions without solving a single specific problem.

2. Archimedes

A. *The "Sand-Reckoner"*

Among the works of Archimedes the "Sand-Reckoner" probably takes first place in fame. No doubt the reason for this is its extraordinary topic: Archimedes shows that it is possible to construct a systematic notation for large numbers, so large that it is, e.g., possible to enumerate the grains of sand that would be needed to fill the whole universe up to the sphere of the fixed stars.

To use modern mathematical terms, Archimedes counts in units of 10^8 and uses the exponents for the ordering of his classes of magnitudes. Hence he can easily find a class (the equivalent of 10^{63}) which contains the number of grains

of sand, grains of a well-defined very small size, more than sufficient to fill the volume of the sphere of the fixed stars.

This is not the place to describe the Archimedian notation for numbers and exponents.[1] In the present context it is only the astronomical aspect that requires our attention. First Archimedes makes some historical remarks which concern estimates of the size of celestial bodies; then there follows a discussion of an observational problem and its connection with physiological optics: the measurement of the apparent diameter of the sun. Finally, occupying a major part of the treatise, there is the evaluation of the influence of what we now would call parallax on the apparent diameter of the sun. It is this discussion which we shall describe in detail; at the moment we shall only underline the strange similarity of this topic with Aristarchus' intricate investigation of the difference between a great circle on the moon and the "terminator" between its dark and illuminated part.[2] In both cases the actual dimensions, according to the explicitly stated assumptions, are such that the difference in question is clearly negligible and could easily be absorbed in the crude roundings accepted everywhere else. Archimedes, e.g., multiplies the commonly accepted circumference of the earth with a factor 10; he also more than doubles the diameter of the sun in relation to the diameter of the moon, or he replaces a regular polygon of 812 sides by a 1000-gon — all perfectly justified steps in view of the only goal of obtaining secure upper bounds for the volume in question. And yet he undertakes a rigorous geometric discussion about the change of an angle observed from the earth's surface when shifted to the center of the earth. As soon as pure geometry is involved both Aristarchus and Archimedes proceed without mercy and completely ignore the practical significance of the problem.

We now turn to the above mentioned problem of parallax from the Sand-Reckoner. Through observations Archimedes has established that the sun appears under an angle α which lies within the limits

$$\frac{(R)}{200} < \alpha < \frac{(R)}{164}, \quad (R) = \text{right angle} \tag{1}$$

(i.e. $0;27^\circ < \alpha < 0;32,55,36,\ldots^\circ$). He wants to show that the chord AB inscribed in the solar orbit of radius $R_s = \mathrm{ES}$ and center E of the earth (cf. Fig. 14) is greater than the side s_{1000} of a regular 1000-gon, A and B being points on the tangents drawn from E to the sun.

It is obvious that the length AB equals the diameter d_s of the sun since

$$r_s = \mathrm{PS} = \mathrm{A}\Phi = 1/2\,\mathrm{AB}$$

thus

$$\mathrm{AB} = d_s. \tag{2}$$

Since Archimedes obtained his estimates (1) of the apparent size α of the sun by observing the sun at sunrise we have to determine β from α in a situation as described in Fig. 15. Since $\mathrm{ES} = R_s > \mathrm{OS}$ we have

$$\beta < \alpha \tag{3}$$

[1] A clear analysis is given, e.g., in Dijksterhuis, Archimedes, p. 370–373.

[2] Cf. above p. 639 f.

hence with (1):

$$\beta<(R)/164=(C)/656, \quad (C)=4(R)$$

and therefore

$$AB<s_{656}.$$

In his "Measurement of a Circle" Archimedes had shown that for a regular polygon of n sides

$$\text{perimeter of } n\text{-gon}/R_s<44/7$$

thus for $n=656$

$$AB<\frac{1}{656}\cdot\frac{44}{7}\cdot R_s=\frac{11}{1148}R_s<\frac{1}{100}R_s$$

and with (2)

$$100\,d_s<R_s. \tag{4}$$

Let CD be the shortest distance between the surfaces of earth and sun (cf. Fig. 15), and accept, following the opinion of the majority of astronomers, that

$$r_m<r_e<r_s. \tag{5}$$

Then we can conclude with (4)

$$100\,CD=100\,(R_s-(r_e+r_s))>100\,R_s-100\,d_s>100\,R_s-R_s=99\,R_s.$$

Since $EP<ES=R_s$ and $OQ>CD$ we also know that

$$100\,OQ>99\,EP \tag{6}$$

and, since $ES>OS$, for the respective tangents

$$EP>OQ. \tag{7}$$

We have now two right triangles, SPE and SQO, with

$$SP=SQ=r_s \quad \text{but} \quad EP>OQ \tag{8a}$$

such that the following trigonometric inequality holds

$$\frac{R_s}{OS}<\frac{\alpha}{\beta}<\frac{EP}{OQ} \tag{8b}$$

which Archimedes considers to be well known.[3] Consequently, using (6), one has

$$\frac{\alpha}{\beta}<\frac{EP}{OQ}<\frac{100}{99}$$

and thus with (1):

$$\beta>\frac{99}{100}\alpha>\frac{99}{100}\cdot\frac{(R)}{200}>\frac{(R)}{203}=\frac{(C)}{812} \tag{9}$$

(i.e., in modern notation, $\beta>0;24,13°$). Since AB is the chord subtended by the angle β (cf. Fig. 14, p. 1355) we see that

$$d_s=AB>s_{812}>s_{1000} \tag{10}$$

q.e.d.

[3] Cf. below p. 773 (2).

After this tour de force which astronomically tells us hardly more than what was known from observation[4] Archimedes now turns to his real topic, the determination of the volume of the sphere of the fixed stars. It is in this context that we hear that astronomers commonly consider the radius R_s of the solar orbit as the radius of the "cosmos" but that Aristarchus had proposed a vastly larger world (κόσμος) in which the sun is the center about which the earth rotates. In such a universe the sphere of the fixed stars is so large that compared with its radius R_f the diameter $2R_s$ of the earth's orbit is negligible. Archimedes criticises the loose form in which Aristarchus expresses this consequence of a heliocentric hypothesis and he gives it a definite mathematical meaning by postulating that

$$R_s : R_f = r_e : R_s. \tag{11}$$

Archimedes assumes that it is a sphere of this radius R_f, the largest "universe" ever proposed, that should be filled with grains of sand. What follows is only the substitution of some numerical parameters and the application of his notation for large numbers to the resulting huge power of 10.

The only astronomical parameter in this chain of dimensions is the assumption that the actual solar diameter d_s is at most 30 lunar diameters d_m, the factor 30 being chosen as a convenient exaggeration of earlier estimates.[5] Hence, because of (5), it is assumed that

$$d_s < 30\, d_e. \tag{12}$$

The perimeter of the 1000-gon inscribed in the circle of diameter $D_s = 2\, R_s$ is on the one hand greater than $3D_s$, on the other hand, because of (10), less than $1000\, d_s$ and because of (12) still less than $30000\, d_e$. Hence

$$D_s < 10000\, d_e. \tag{13}$$

For the size of the earth Archimedes increases an accepted value for the perimeter $c_e = 300000$ stades[6] by a factor 10, thus assuming

$$d_e < 1000000 \text{ stades} \tag{14}$$

and hence with (13)

$$D_s < 10^{10} \text{ stades} \tag{15}$$

and with (11)

$$D_f = \frac{D_s}{d_e} \cdot D_s < 10000\, D_s. \tag{16}$$

Finally it is assumed that one poppy-seed contains less than 10000 grains of sand and that 40 poppy-seeds lined up in a row exceed one finger breadth, 10000 of which make one stade. Then (15) and (16) lead to the result that a sphere of diameter D_s (i.e. the conventional "universe") contains less than 10^{51} grains of sand while the Aristarchian universe of diameter D_f has room only for less than 10^{63} grains.

There remains to be made a remark about the section in Archimedes' treatise where he describes the observational techniques that provided him with the

[4] Note that $360°/1000 = 0;21,36°$ as compared with $\alpha > 0;27$ from (1).

[5] Cf. for details below IV B 3, 3 C, but also below p. 664 (11) and (12).

[6] Thus $1° \approx 833$ stades. Aristotle, De caelo II, 14 (Loeb, p. 254/255) mentions an estimate of $c_e = 400000$ stades (thus $1° \approx 1100$ st.).

estimate (1) of the apparent solar diameter (above p. 644). In this context he describes not only a diopter that operates with a small vertical cylinder which can be moved on a horizontal ruler into a position which covers exactly the solar disk at sunrise but he also discusses an independent experiment with two very small cylinders in order to determine the width of the observer's pupil.[7] The apparent diameter of the sun is then measured as the angle between two tangents to the first mentioned cylinder and the little space which corresponds to the width of the pupil determined in the second experiment. Here we have an interesting combination of a problem in physiological optics and the traditional geometrical optics. Lejeune [1947] has given a careful analysis of all aspects of these experiments and it is regrettable that much must remain unexplained because of the sketchiness of Archimedes' description.

B. Cosmic Dimensions

Size and distance of sun and moon, expressed in relation to terrestrial distances, were the only cosmic dimensions which could be determined by some rational method available to antiquity. For the planets, however, not much beside speculation was possible about their position with respect to moon and sun, based on the plausible hypothesis that planets with a sidereal period greater than one year should be farther away than the sun – in proportion to the slowness of their sidereal motion. This provided at least relative positions for Saturn, Jupiter, and Mars. For Mercury and Venus neither their relation to one another nor to the sun could be convincingly established. Ptolemy still mentions[1] "older" astronomers who placed Venus and Mercury below the sun while some later ones assumed a position beyond the sun. We shall find presently both assumptions ascribed to Archimedes, with Venus in both cases nearer to the earth than Mercury.

The source which provides us with this information is a treatise known as "Refutation of all Heresies" by the Bishop of Rome Hippolytus (who died c. 236), well known as the first schismatic pope, Callistus being his detested but more successful competitor.[2] Hippolytus was still relatively well educated in Greek philosophy and science[3] which he considered, however, as the fountainhead of all heresies, following the well established principle to call "heresy" any opinion not held, or not understood, by the author. Nevertheless St. Hippolytus likes to show his learning and thus preserved for us[4] a long list of planetary distances proposed by Archimedes.[5]

[7] Lejeune [1947] is undoubtedly right when he renders ὄψις by the technical term "pupil" and not by the indefinite "eye" (cf. in particular p. 37, n. 3). The same conclusion had been reached by F. Schmidt, Instrum., p. 330 (1935).

[1] Alm. IX, 1.

[2] Hippolytus, Heres. IX 12 gives a vivid picture of the fierce strife in the christian community of Rome at the end of the second century A.D.

[3] He was, after all, a contemporary of Clement of Alexandria, Tertullian, Origenes, and other educated theologians (and therefore prone to fall into heresies).

[4] Heresies IV, 8-11, ed. Wendland, p. 41-44; trsl. Preysing, p. 51-54.

[5] Ridiculed by Hippolytus as being useless for the true faith. It is illuminating to compare this position with the attitude toward astronomy expressed in the contemporary epigram conventionally ascribed to Ptolemy (cf. below p. 835).

Before turning to the astronomical meaning of these data we must secure the transmitted numbers. Since all distances are expressed in stadia one is dealing with 8- or 9-digit numbers which are easily subject to errors of transmission. Nevertheless it was observed long ago that arithmetical relations exist between these data[6] and this will help us to make some (relatively few and simple) emendations[7] and thus to obtain a consistent set of numbers which are absolutely secure.

We distinguish two sets of data, called henceforth (A) and (B), respectively,[8] (A) giving the intervals between the celestial bodies, (B) their geocentric distances. As can be seen at a glance these two sets are grossly contradictory; nevertheless we shall establish precise relations between (A) and (B) which show an intimate connection. All numbers are counted in stades.

In order to justify our emendations and to make the existing arithmetical relations evident we introduce the following abbreviations:

$$a=5\cdot 10^6, \quad b=27\cdot 10^4, \quad c=2065, \quad d=b+c=272065. \tag{1}$$

Then we are given in (A) the following intervals:

from the surface of the earth to the moon:	$5544130 = 1a+2d$		(2)
from the moon to the sun:	50272065[9] $=10a+1d$		(3)
from the sun to Venus:	$20272065 = 4a+1d$		(4)
from Venus to Mercury:	50816195[10] $=10a+3d$	(A)	(5)
from Mercury to Mars:	40544130[11] $= 8a+2d$		(6)
from Mars to Jupiter:	20272065[12] $= 4a+1d$		(7)
from Jupiter to Saturn:	40272065[13] $= 8a+1d$		(8)
from Saturn to the zodiac:	20272065[14] $= 4a+1d.$		(9)

From these intervals one derives the following geocentric distances (or rather distances from the earth's surface):

to the moon:	$5544130= 1a+ 2d$		(10)
to the sun:	$55816195=11a+ 3d$		(11)
to Venus:	$76088260=15a+ 4d$		(12)
to Mercury:	$126904455=25a+ 7d$	(A′)	(13)
to Mars:	$167448585=33a+ 9d$		(14)
to Jupiter:	$187720650=37a+10d$		(15)
to Saturn:	$227992715=45a+11d$		(16)
to the zodiac:	$248264780=49a+12d.$		(17)

[6] Tannery, Mém. Sci. I, p. 393, but without following up the consequences of his observation.

[7] I am using Wendland's edition which is preferable to Heiberg's excerpts in Archimedes, Opera II, p. 552–554.

[8] (A): Wendland, p. 41, 13–22; (B): p. 42, 8–16.

[9] Text: 5026 ... (ϛ instead of ζ).

[10] Text: ... 7165 (ζ for ϛ and 60 copied from the preceding number).

[11] Text: 4554 4154; later repeated as 40541108 (Wendland, p. 43, 23).

[12] Text: ... 5065; repeated p. 43, 25.

[13] Text: 4037 ...; repeated p. 43, 26.

[14] Text: 20080045.

The numbers of (A′) are not found in our text. Instead we have the following list of geocentric (or geo-surface?) distances:

$$
\left.\begin{array}{ll}
\text{of Saturn:} & 22269\,2711 \\
\text{of Jupiter:} & 20277\,0646^{15} \\
\text{of Mars:} & 13241\,8581 \\
\text{of the sun:} & 12160\,4451^{16} \\
\text{of Mercury:} & 5268\,8256^{17} \\
\text{of Venus:} & 5081\,5160.
\end{array}\right\}\ (\mathrm{B})
\qquad
\begin{array}{r}(18)\\(19)\\(20)\\(21)\\(22)\\(23)\end{array}
$$

From this table one derives the following intervals:

$$
\left.\begin{array}{ll}
\text{from earth to Venus:} & 5081\,5160 \\
\text{from Venus to Mercury:} & 187\,3096 \\
\text{from Mercury to the sun:} & 6891\,6195 \\
\text{from the sun to Mars:} & 1081\,4130 = 1081\cdot 10^4 + 2c \\
\text{from Mars to Jupiter:} & 7035\,2065 = 7035\cdot 10^4 + 1c \\
\text{from Jupiter to Saturn:} & 1992\,2065 = 1992\cdot 10^4 + 1c,
\end{array}\right\}\ (\mathrm{B}')
\qquad
\begin{array}{r}(24)\\(25)\\(26)\\(27)\\(28)\\(29)\end{array}
$$

numbers which are not found in our text and contradict (A).

Nevertheless there exists a close relationship between (A) and (B′). If we add in (A) the intervals (6), (7), and (8) and in (B′) (27), (28), and (29) we obtain in both cases exactly the same number $20a+4d$:

$$
\begin{array}{l}
(6)+\ (7)+\ (8) = \text{Mercury to Saturn} = 10108\,8260 \\
(27)+(28)+(29) = \quad \text{sun to Saturn} \quad = 10108\,8260.
\end{array}
\qquad (30)
$$

In (A) we notice a similar relation:

$$
\begin{array}{l}
(3)+(4)+(5) = \text{moon to Mercury} = 12136\,0325 \\
(6)+(7)+(8)+(9) = \text{Mercury to zodiac} = 12136\,0325
\end{array}
\qquad (31)
$$

i.e. $24a+5d$ for both distances.

Hippolytus does not seem to know how Archimedes arrived at these numbers. He criticizes the intervals given in (A) as not producing harmonious ratios and he therefore suggests[18] a scheme of intervals based on Platonic numerology. Accepting Archimedes' value for the lunar distance

$$\alpha = 554\,4130 \text{ stades} \qquad (2)$$

he postulates the following intervals between the orbits:

$$
\begin{array}{llll}
\text{moon to sun:} & 2\alpha & \text{sun to Venus:} & 3\alpha \\
\text{Venus to Mercury:} & 4\alpha & \text{Mercury to Mars:} & 9\alpha \\
\text{Mars to Jupiter:} & 8\alpha & \text{Jupiter to Saturn:} & 27\alpha
\end{array}
\qquad (32)
$$

---

[15] Text: 20272 ... (cf. apparatus to Wendland, p. 42, 11).
[16] Text: ... 54 (δ for α).
[17] Text: ... 59.
[18] Heresies IV, 11.

giving in each case the numerical value.[19] The only basis for this pattern is, of course, the alternating sequence of powers of 2 and 3.

Exactly the same argument against Archimedes is also mentioned (around 400 A.D.) by Macrobius[19a]. He knows about Archimedes' attempt to determine in stades the intervals between the planets, beginning at the surface of the earth and ending at the sphere of the fixed stars. All this agrees with version A in Hippolytus' summary (unfortunately without numbers), excepting the order in which Macrobius enumerates the intervals, assuming the sequence ☾→☿→♀→☉ customary at his time.

How Archimedes obtained his figures I cannot say. It is clear that in (A) he assigned to Mercury a central position which in (B) is occupied by the sun (cf. Fig. 16). It remains a mystery why he spaced the planets so differently in (A) and (B); to us the distribution in (A) seems to be more plausible than in (B), e.g. as far as Venus and Mercury are concerned.

Hippolytus' text contains a few more numerical data which, however, are difficult to relate to the parameters in (A) or (B), in part because of lacunae [ ] in the text, in part because of corruptions in the numerals.

In IV, 8[20] we are told that the

$$\text{[diameter of the earth} = 80]\ 108 \text{ stades} \tag{33}$$

and that the

$$\text{perimeter of the earth} = [240]\ 543 \text{ stades;} \tag{34}$$

furthermore that the distance from the surface of the earth to the moon's orbit was assumed

$$\begin{aligned} &\text{by Aristarchus: } [\quad]^{21} \\ &\text{by Apollonius: } 5000000 \text{ stades.} \end{aligned} \tag{35}$$

Hence Apollonius had placed the moon exactly at the distance a, while Archimedes had assumed $a+2d$ (cf. (2), p. 648). One could expect that d is somehow related to an accurately computed value for the radius r of the earth but I see no connection between $2d \approx 54 \cdot 10^4$ and a plausible value of r in the order of magnitude of $4 \cdot 10^4$ to $5 \cdot 10^4$ (cf. below (37)).

Archimedes is again explicitly mentioned in IV, 9[22] as having assumed that the

$$\text{perimeter of the zodiac} = 447310000 \text{ stades,} \tag{36}$$

a sixth of which should be the distance of the sphere of the zodiac from the center of the earth, a number which must be reduced by the radius of the earth

$$r = 40000 \text{ stades} \tag{37}$$

if one wishes to measure from the surface of the earth. As we have seen before[23] Archimedes considered 50000 stades as an acceptable value of r.

---

[19] The distance from the earth to Saturn thus becomes $54\alpha = 2993830\,20$ stades; cf. above (16) and (18).

[19a] Macrobius, Comm. II, 3 (ed. Willis II, p. 106; trsl. Stahl, p. 196).

[20] Wendland, p. 41, 10–13.

[21] Perhaps 1680000 stades; cf. Tannery, Mém. Sci. I, p. 394, note * and Wendland's apparatus to p. 41, 13.

[22] Wendland, p. 42, 1–8.

[23] Above p. 646.

The number in (36) is obviously corrupt since the corresponding radius would be

$$R \approx 7455\,1667, \tag{38}$$

i.e. about the distance of Venus in (A) (12). Also (A') (17) does not lead to a plausible emendation of (36) or (38).

The value (37) for the terrestrial radius agrees fairly well with the restored value (33). On the other hand 543 in (34) contains the factor 3 which might suggest the emendation

$$d = [80]\,181 \tag{39}$$

in (33). Ironically one obtains with this value of d

$$22/7 \cdot d = 25\,1997 \approx 25\,2000 \text{ stades} \tag{40}$$

which is the traditional value for the circumference of the earth according to Hipparchus.[24] It is impossible to determine how far one can trust Hippolytus to have correctly reproduced his sources.

Equally obscure is the origin of another value, ascribed to "Eratosthenes and Archimedes" by Martianus Capella,[25] according to which the circumference of the earth contains

$$c = 40\,6010 \text{ stades.} \tag{41}$$

Since this number is divisible by 22 it is the exact equivalent to assuming for the diameter

$$d = 12\,9185 \text{ stades.} \tag{42}$$

For the length of one equatorial degree one finds

$$1^\circ = 1127\ 29/36 \text{ stades}$$

which is an implausibly high value, in particular as related to Eratosthenes.

3. Posidonius

In the time of Pompey and Cicero, Posidonius was considered one of the greatest philosophers. He was born around 140 B.C. in Apamea on the Orontes and died at Rhodes about 50 B.C.[1] Because not a single one of his works has survived a huge modern literature has come into existence, attempting to identify ideas and "fragments" of Posidonius' works in later writings and to reconstruct his doctrines.[2]

Practically the only source which gives us any information about his astronomy is a little treatise by Cleomedes "On the circular motions for the celestial

[24] Cf. below p. 734.

[25] Martianus Capella VIII, p. 451, 12, ed. Dick.

[1] For a detailed discussion of these dates cf. Marie Laffranque, Posidonios d'Apamée (Paris 1964), p. 99–108.

[2] Of greatest influence were the books by K. Reinhardt, Poseidonios (1921) and Kosmos and Sympathie (1926), summarized in his article in RE 22.1 col. 558–826 (1953). A useful discussion of the sources is found in Gronau, Poseid. (1914). For an excellent study of Posidonius' personality and influence see A. D. Nock, Posidonius, J. Roman Studies 49 (1959), p. 1–15.

bodies," written about five centuries after Posidonius[3] and probably based only on excerpts from secondary sources.[4] From it one gets the impression that Posidonius' astronomy was of an extremely elementary nature, far below, e.g., the level of Geminus' "Isagoge."[5]

All ancient sources seem to agree on the high literary quality of Posidonius' writings. In contrast Cleomedes' treatise is a thoroughly mediocre compilation, repetitious, badly organized, and heavily padded with polemics against statements ascribed to Epicurus (e.g.: the actual size of the sun is one foot), much too absurd to deserve such lengthy discussions. Yet, the references to Posidonius are so specific that some definite relation to ultimately Posidonian writings seems certain. In other words, what Cleomedes describes, e.g., as Posidonius' method to determine sizes and distances of the luminaries must reflect in essence the teaching of Posidonius. The little one can recover of Posidonius' astronomy in this way does not leave us with the impression that he had profited from Hipparchus' achievements.

Cicero tells us[6] that Posidonius had constructed (*effecit*) a planetarium that showed the motion of sun, moon, and planets day by day. However difficult it is to picture such an apparatus it can be taken at best only as evidence for mechanical skill but not for any planetary theory reaching beyond the basic facts.[7] The little that Cleomedes has to say about planets[8] apparently comes from some elementary treatises unrelated to Posidonius.

A. Measurement of the Earth

Cleomedes (I, 10) describes methods of Posidonius and of Eratosthenes designed to measure the length c_e of the earth's circumference.[1] Posidonius is said to have argued as follows:

It is assumed that Rhodes and Alexandria are located on the same meridian. As an "observation" is cited that the star Canopus[2] just touches the horizon at Rhodes while it culminates at Alexandria at an altitude of one part, a "part" ($\mu\acute{\epsilon}\rho o\varsigma$) being the 48th part of the circumference of a circle (thus $1^p = 7;30^\circ = 1/2$ "step" of 15° [3]). If furthermore the distance Rhodes-Alexandria amounts to 5000

[3] Cf. for Cleomedes below V C 2, 5.

[4] Gronau, Poseid., passim, in particular Chap. V and Reinhardt, Pos. p. 185. Cf. also the final (spurious) remarks in Cleomedes (Ziegler, p. 228, 1–5): "The preceding teachings are not the author's own opinion but collected from older or more recent summaries; much of it is taken from Posidonius."

[5] Cf. above IV A 3. 2. Some scholars tried to make Geminus a pupil of Posidonius. Even if it were not chronologically excluded (above p. 580) I see nothing that supports such a conjecture (cf. above p. 578, also Reinhardt, Pos. p. 178).

[6] Cicero, De natura deorum II 34/35, 88 (Loeb, p. 206–209).

[7] I do not see how the daily (mean) motions of sun and moon can be combined with the planetary retrogradations (even ignoring latitudes) in one spherical model. D. Price [1974]. p. 57f. tries to explain Archimedes' planetarium by means of gearings of a type he had discovered in the Antikythera mechanism that represents lunar motions. But even these intricate devices cannot produce more than the mean motions of the outer planets. Hence the most characteristic features of planetary motions, stations and retrogradations, are omitted and the inner planets must be ignored altogether.

[8] Cf. below V C 2, 5 D.

[1] Cleomedes, ed. Ziegler, p. 92, 3–100, 23.

[2] Cf. for Canopus (modern: α Carinae) above p. 576, n. 3.

[3] Cf. below p. 671. The same data in Pliny, NH II, 178 (Budé II, p. 78).

stades then one has

$$c_e = 48 \cdot 5000 = 240\,000 \text{ st.} \quad \text{thus } r_e = 40\,000 \text{ st.} \tag{1}$$

To Eratosthenes Cleomedes ascribes the method ever since associated with his name[4]: assume that Syene and Alexandria lie on the same meridian, again at a distance of 5000 stades and that the shadow of the gnomon at the summer solstice is zero at Syene, but covers 1/50 of the circle at Alexandria (i.e. 7;12°). Thus

$$c_e = 50 \cdot 5000 = 250\,000 \text{ st.} \tag{2}$$

It is clear that neither the "measurements" of distances[5] nor the astronomical "observations"[6] are more than crude estimates, expressed in convenient round numbers. It does not disturb Cleomedes that 5000 stades are in one case the equivalent of 7;30°, in the other of 7;12°, i.e. 660 2/3 stades or 694 4/9 stades per degree, respectively, both values being close enough to be rounded to

$$1° = 700 \text{ stades.} \tag{3}$$

The corresponding terrestrial dimensions are

$$c_e = 252\,000 \text{ st.,} \quad r_e = 42\,000 \text{ st.} \tag{4}$$

data accepted by Eratosthenes[7] as well as by Hipparchus.[8]

We know, however, from Strabo[9] that Eratosthenes did not trust the estimate of 5000 stades for the distance Alexandria-Rhodes. Some seamen took it to be only 4000 stades, an estimate which Eratosthenes conveniently modified to 3750 stades.[10] The reason for this adjustment is obvious: if, as before, the distance in question corresponds to a latitudinal difference of 360°/48 then one obtains a round number again for the meridian degree:

$$1° = 500 \text{ stades} \tag{5}$$

(and consequently

$$c_e = 180\,000 \text{ st.,} \quad r_e = 30\,000 \text{ st.).} \tag{6}$$

It is characteristic for this genesis of a new norm that estimates of sailing distances are taken into account while an easily checkable astronomical parameter ($\Delta\varphi = 7;30°$ instead of actually $\approx 5°$) remains unchanged.

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[4] Letronne (Oeuvres choisies, ser. 2, Vol. 1, p. 263) argues against the authenticity of the whole story but seems to have found no followers.

[5] Actually Rhodes is about 4;50° north, 1;50° west of Alexandria, Syene about 7;10° south, 3° east (cf. Fig. 17).

[6] Actually for Rhodes $90 - \varphi = 54°$ while the declination of Canopus is about $-52;30°$. Hence the star culminates at Rhodes at an altitude of about 1 1/2° which leads to a visibility of about 2 1/2^h (Drabkin [1943], p. 510, n. 5).

[7] Strabo, Geogr. II 5, 7 (Loeb I, p. 436/437); also Heron, Dioptra 35 (Opera III, p. 302, 12–17); Geminus (Manitius, p. 166, 2), etc.

[8] Cf. above p. 305, n. 27.

[9] Strabo, Geogr. II 5, 24 (Loeb I, p. 482/483). Cf. also Pliny NH V, 132 (Jan-Mayhoff I, p. 418, 2) where he ascribes to Eratosthenes the estimate of 469 miles, i.e. $\approx$ 3750 st.

[10] It is, of course, nonsense when Strabo says that this distance was determined by means of sun dials: these instruments can only furnish angles, never absolute distances.

Obviously not only Eratosthenes but, following him, Posidonius as well wavered between the estimates (3) and (5) since we know from Strabo[11] that Posidonius also assumed (6) whereas he still used (1) (p. 653) for his estimate of the moon's size.[12]

It should be noted that our simple explanation of the shift from the norm (3) to (5) avoids all discussions about changes in the norm of the "stades," a favored topic among modern historians of ancient geography. We do not know on what basis Marinus[13] adopted (5) as the definitive norm; being accepted by Ptolemy as well and hence by later geographers it obliterated the earlier assumption (3) almost completely.

It may be remarked that a variant in Pliny NH II, 245[14] gives 563 miles as distance from Alexandria to Rhodes. Ignoring a rounding of 4 stades = 1/2 mile this is the equivalent of 4500 stades from which one obtains exactly $1° = 600$ stades.

B. Size and Distance of the Moon

Aristarchus had assumed[1] that the apparent radius u of the earth's shadow at the distance of the moon was twice the moon's apparent radius:

$$u = 2 r_{☾}. \tag{7}$$

Cleomedes mentions[2] as the basis for this relation the experience that the totality of a lunar eclipse lasts at most the same time as the immersion or the exit. Fig. 18 shows[3] that this indeed implies (7).

If we believe Cleomedes it was this relation on which Posidonius built his estimate for the size of the moon. In order to obtain absolute dimensions, however, (7) had to be replaced by a relation for actual distances. At this point Posidonius seems to have assumed a cylindrical shadow, which provides at least an upper limit for the width of the shadow cone. In Hipparchian parlance[4] one can describe this assumption as the "case of no solar parallax."

Thus Posidonius assumed not only (7) but also for the actual dimensions

$$r_u = r_e = 2 r_m \tag{8}$$

which gives, combined with his estimate of the earth's radius

$$r_e \approx 40000 \text{ stades} \tag{9}$$

[11] Strabo, Geogr. II 2, 2 (Loeb I, p. 364/365). The estimate (6) is also mentioned by St. Basil (Hexaemeron IX, Sources Chrétiennes, p. 483) who correctly observes that Moses was not concerned with the shape or the size of the earth, a fact held against the sciences.

[12] Cf. below (9). Perhaps an obscure remark by Pliny (NH II 247, Budé II, p. 111, p. 266) can be taken as an indication that Hipparchus also did not accept (4) unreservedly: he is said to have added "a little less than 26000 stades" to the 252000. [Note: $c_e = 252000 + 25500 = 277500$ st. leads with the Babylonian approximation $\pi \approx 3;7,30$ to exactly $r_e = 44400$ st.].

[13] Cf. below p. 935.

[14] Jan-Mayhoff I, p. 227 ad line 3. The other variants (584, 583, 588, 573 miles) make no sense (NH V, 132 l.c. p. 418, 2 and var.).

[1] Cf. above p. 635 (4) and below p. 667.

[2] Cleomedes II, 1 (Ziegler, p. 146, 18–25); cf. also II, 3 (Ziegler, p. 178, 12).

[3] Cf. also Fig. 15 in the Eudoxus Papyrus (below p. 1453, Pl. VII col. XII).

[4] Cf. above p. 327 and below p. 963.

(cf. above p. 653 (1)), for the moon's actual diameter

$$d_m = 40000 \text{ stades.} \tag{10}$$

The Egyptians supposedly had found from the outflow of a water clock that the apparent diameter of the sun is the 750th part of its orbit:

$$d_{\odot} = \frac{c}{750} (= 0;28,48°). \tag{11}$$

If one furthermore assumes that sun and moon are of equal apparent size

$$d_{☾} = d_{\odot} \tag{12}$$

then one obtains from (11) and (10) for the length of the lunar orbit

$$C_m = 750 \cdot 40000 = 30000000 \text{ stades} \tag{13}$$

and for its radius

$$R_m = 5000000 \text{ stades.} \tag{14}$$

This seems to be the result reached by Posidonius. The same value (14) is ascribed to Apollonius by Hippolytus.[5]

On the basis of (9) one can express (14) in terms of terrestrial radii r_e:

$$R_m = \frac{5000000}{40000} r_e = 125 \, r_e. \tag{15}$$

This is about twice the correct distance, essentially already known to Hipparchus[6]; the philosopher seems to have ignored the results of his famous predecessor.

Cicero once remarks[7] that "the mathematicians proved that the moon is more than half the size of the earth". He gives no source for this statement but it seems possible that he had it from Posidonius, his teacher, since it follows from (8) that $r_m = 1/2 \, r_e$. This relation is based, however, on the assumption $r_u = r_e$ whereas in reality $r_e > r_u = 2 r_m$. Thus Cicero should have said $r_m \leq 1/2 r_e$ instead of $> 1/2 r_e$.

C. Size and Distance of the Sun

Posidonius' attempts (according to Cleomedes) to determine the size of the sun are rather naive[1] and make it difficult to understand that his astronomy was not ridiculed by authors like Cicero or Pliny who pretend to know the work of Hipparchus.

According to Cleomedes' narrative Posidonius suggested two possibilities which could lead, in his opinion, to an estimate of the size of the sun.

The first method[2] makes use of an "empirical" fact of extreme crudity: at the summer solstice at Syene a vertical gnomon casts no shadow over an area of

[5] Cf. above p. 650 (35). Note that Archimedes made $R_m = 5544130$ stades (above p. 649 (2)), using obviously the same type of stades as Apollonius and Posidonius.

[6] Cf. above p. 327 f.

[7] De natura deorum II, 103 (Loeb, p. 221).

[1] The famous paper by Hultsch [1897] on "Poseidonius über die Grösse und Entfernung der Sonne" is a collection of implausible hypotheses which are not worth discussing.

[2] Cleomedes II, 1 (Ziegler, p. 144, 22–146, 16).

300 stades in diameter.[3] If we furthermore assume (it seems only for the sake of argument) that the length C_s of the solar orbit is 10000 times the circumference of the earth

$$C_s = 10000\, c_e \tag{16}$$

then one finds for the actual diameter d_s of the sun from the verticality of the rays 300 stades apart (cf. Fig. 19)

$$\frac{300}{c_e} = \frac{d_s}{C_s} \tag{17}$$

thus with (16):

$$d_s = 3000000 \text{ stades.} \tag{18}$$

It should be noted that the verticality of the solar rays over an area of about 300 stadia in diameter is a consequence of the assumption (11), p. 655 combined with the arrangement shown in Fig. 19. Indeed it follows from Fig. 19 that the diameter x in question must satisfy

$$\frac{x}{c_e} = \frac{d_\odot}{360^\circ}$$

and hence with (11) and (1), p. 653

$$x = \frac{c_e}{360} \cdot \frac{360}{750} = \frac{240000}{750} = 320 \text{ st.} \approx 300 \text{ st.}$$

Thus we see that $x \approx 300$ st. and (11) are not independent "observations." And the relation (16) is, of course, entirely arbitrary.

Posidonius' second estimate is based on the "plausible" conjecture that sun and moon (generally: the planets) move in their respective orbits with the same linear velocity. Since

$$1 \text{ year} \approx 13 \text{ sid. months} \approx 13 \cdot 27\,1/2^d \; (= 357\,1/2^d)$$

we find with (14), p. 655 for the orbital radii

$$R_s = 13 R_m = 65000000 \text{ st.} \tag{19}$$

and with (12) and (10) for the actual diameters

$$d_s = 13 d_m = 520000 \text{ st.} = R_s/125 = 13 r_e. \tag{20}$$

Reckoned in earth radii (19) and (15) give

$$R_s = 1625 r_e. \tag{21}$$

Hence this second method reduces the distance of the sun to less than one sixth of the estimate (16)

$$R_s = 10000 r_e \tag{22}$$

of the first method.

If one expresses the two distances (21) and (22) by means of (1), p. 653 in absolute units

$$R_s = 65000000 \text{ st.} \quad \text{and} \quad R_s = 400000000 \text{ st.}$$

[3] Also Ziegler, p. 140, 7–9; cf. also below p. 726 n. 14. Cf. also Pliny, N.H. II, 182 (Budé II, p. 80).

respectively, one may compare them with the Archimedian alternatives (cf. above p. 648 (11) and p. 649 (21))

$$R_s \approx 56000000 \text{ st.} \quad \text{and} \quad R_s \approx 122000000 \text{ st.}$$

which testify to a similar lack of rigorous methods in the evaluation of the size of the solar system. Perhaps this duplication of estimates for the solar distance is based on considerations which also have guided Hipparchus.[4]

Posidonius is also mentioned by Pliny[5] as having reckoned 2000000 stades to the moon[6] and 500000000 stades (from the moon?) to the sun. Probably these numbers are garbled; at any event Pliny does not indicate the underlying argument. Instead he grumbled about the uselessness of attempts to measure the universe, a mood which should become prevalent soon enough among the Fathers of the Church.

4. Additional Material

A. Apparent Diameter of Sun and Moon

That sun and moon appear to be of the same size will be evident to the most casual observer. The almost exact equality of the two diameters can easily be established by simple instrúments of the diopter type. Hence it was perfectly reasonable that Aristarchus used exact equality of the apparent diameters in his mathematical deductions.[1] Only in the theory of solar eclipses did it become necessary to refine the whole discussion — we shall return to it in the next section.[2]

A diopter also makes it easy to obtain relatively accurate angular measurements of the lunar or solar diameter. If the movable cylinder or plate on the ruler is put into a position so that it exactly obscures the moon, an arc between stars can easily be found which is a small multiple of a lunar diameter.[3] Any device for independently measuring angular distances between stars will then produce a reasonably accurate estimate for the lunar (and hence solar) diameter.

The generally accepted result of such a procedure is again given as a round number: either $1/2°$ or as fraction of the circumference: 1/720[4] or a similar ratio. That here too refinements had been introduced is evident from Archimedes' discussion of the dioptra[5] whereas the simple procedure is described in the (badly damaged) first part of a papyrus of the first or second century A.D.[6] It became fashionable, however, to embellish the simple measurement by diopter

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[4] Cf. above I E 5, 4 B.
[5] Pliny, NH II, 21 (Budé II, p. 37 and p. 172–175).
[6] Actually only from the upper limit of the atmosphere (the region of winds and clouds) which adds 40 stades to the radius of the earth.
[1] Cf. above p. 635 (2).
[2] Cf. below p. 668.
[3] Distances expressed in lunar diameters are mentioned, e.g., in the Almagest (IX, 7 and IX, 10 in observations from the Era Dionysius (−264 and −261), in X, 1 from Ptolemy and Theon, A.D. 140 and 127). Also P. Lond. 130 for A.D. 81 (Neugebauer-Van Hoesen, Gr. Hor., p. 26).
[4] E.g. Aristarchus, according to Archimedes, Sandreckoner (Opera II, p. 222, 6–8); cf. also above IV B 3, I E and p. 592.
[5] Cf., e.g., above p. 647; also Schmidt, Instrum., p. 328–334.
[6] P. Oslo 73 (Pap. Oslo III, p. 30).

with an obviously fictitious story about the timing by means of a waterclock of the rising of the solar disk, supposedly resulting in a quantity of water 1/720 of the total daily outflow.[7] It is obvious that this story is only a literary cliché; in order to establish the pretended result it would be necessary, e.g., to guarantee an accuracy of 1/1000 in the measurement of the daily outflow. Ptolemy's criticism[8] should have been sufficient to discredit the whole story.

The popular appeal of such simpleminded explanations for basic parameters cannot be doubted, however. It is in keeping with this style of literature when Diogenes Laertius credits Thales with the discovery of the same ratio.[9] Also the previously mentioned papyrus tells the waterclock story,[10] pretending that the outflow was found to be 1/360 of the daytime quantum (forgetting about the variability of the length of daylight) while Macrobius credits the Egyptians with an equally fictitious procedure for measuring the time of sunrise at a sundial.[11] When arithmetically desirable the outflow from the waterclock can also be declared to be the 750th part, reported, e.g., by Cleomedes[12] as an Egyptian achievement.

Attention must be paid to the problem of units, in particular to the ambiguity in the term "digits" (*δάκτυλοι* = fingers). On the one hand "digits" are used to measure the apparent diameter of sun and moon, e.g. in connection with eclipses. On the other hand angular distances in the sky in general, e.g. distances between stars can be measured not only in degrees but also in "cubits" or "fingers" and the relations between these different units are by no means always easy to establish.[13]

In Plutarch[14] one finds the remark that the diameter of the moon at mean distance measures 12 digits. This makes only sense if taken as a definition which then must be implemented by the grading of instruments: one digit is the twelfth of the apparent diameter of the moon when at mean distance. This is in conformity with a statement by Sosigenes (≈A.D. 175) that on a diopter a disk sometimes of 11, sometimes of 12 digits in diameter is needed to cover the moon.[15] It is perhaps one of Ptolemy's clever innovations to define "digits" always as twelfths of the apparent diameter of sun or moon,[16] regardless of the eccentricity. Consequently an eclipse is always total according to Ptolemy's definition as soon as 12 digits are obscured, whereas an eclipse can be total with the previous definition, e.g., at 11 digits.

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[7] In the second part of P. Oslo 73 or Proclus, Hypotyp. IV, 73–75; etc.

[8] Alm. V, 14 (Manitius I, p. 305); Tetrabiblos II, 2 (Boer, p. 110/111; Loeb, p. 231); also Proclus, Hypotyposis IV, 80–86.

[9] Diels VS[5], p. 68, 6–8; cf. also Heath, Aristarchus, p. 21/22.

[10] P. Oslo 73, 21–23.

[11] Cf. below p. 661.

[12] Cf. above p. 655 (11).

[13] Cf. also above p. 591.

[14] De facie 935 D, Loeb, Moralia XII, p. 142/143.

[15] Simplicius (≈A.D. 530) in his Commentary to Aristotles' De caelo, ed. Heiberg, p. 504, 25–505, 19; French transl. Duhem SM I, p. 401. Cf. also Proclus, Hypot. IV 98f. (Manitius, p. 130/131). In the same context we read that Polemarchus (the contemporary of Eudoxus and Callippus, cf. below p. 676) considered the variation of the apparent lunar diameter negligible and ignored it on purpose because he preferred the theory of homocentric spheres (Heiberg l.c. p. 505, 21–23; Duhem l.c. p. 402).

[16] Cf. Alm. VI, 7 (Manitius I, p. 374, 28; 376, 31–377, 1); same in Cleomedes II, 3 (Ziegler, p. 172, 25).

In the popular literature it is of course impossible to determine the exact type of definition, e.g. when Cleomedes says that $d_{\odot}=d_{\rm ☾}=12''$ or that 1/5 of the solar diameter is "2 digits and a little."[17] Also when Hipparchus states[18] that predictions of lunar eclipses deviate at most 2 digits from the observed facts we do not know the definition of "digit" assumed in this context.

The convenient estimate of $1/2^{\circ}$ for the apparent diameters of sun and moon is surely not the outcome of very careful observations although it is fortunately very close to the correct value. Systematic observations were made, as far as we know, only by Archimedes and Hipparchus, both with carefully designed apparatus.[19] Archimedes found for the solar diameter[20]

$$0;27^{\circ}<d_{\odot}<0;32,55,\ldots^{\circ} \tag{1}$$

while Hipparchus obtained for the lunar diameter[21]

$$d_{\rm ☾}=\frac{c}{650}\,(\approx 0;33,14^{\circ}) \tag{2}$$

at mean distance. Martianus Capella's[22]

$$d_{\rm ☾}=\frac{c}{600}\,(=0;36^{\circ}) \tag{3}$$

is perhaps only a corruption of the Hipparchian value (2).

B. Distances of Sun and Moon

It was 3000 stades, then, from the earth to the moon, my first stage; and from there up to the sun perhaps 500 parasangs ...

Lucian, Icaromenippus (Loeb II, p. 269)

As far as one can judge on the basis of our fragmentary sources it was Aristarchus who first demonstrated that a few observational data combined with purely mathematical arguments can give us some information about the sizes and distances of sun and moon.[1] While Aristarchus did nothing more than to establish a methodological principle he nevertheless was the first to abandon mere speculation in favor of rational and empirical arguments.

Apparently it was mere numerological speculation that preceded Aristarchus and it was by no means the end. Plutarch, more than three centuries later, still

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[17] Cleomedes II, 3 (ed. Ziegler, p. 172, 22–27).
[18] Comm. Ar., Manitius, p. 90, 10.
[19] Archimedes, Sandreckoner; cf. Lejeune [1947] (above p. 647).
[20] Cf. above p. 644 (1). I do not understand a sentence in Plutarch (Moralia, Loeb XIV, p. 64/65) in which he ascribes to Archimedes the discovery of a certain ratio of the solar diameter to the circumference.
[21] Alm. IV, 9 (Manitius I, p. 237, 8); also Pappus, Coll. VI, 37 (Hultsch, p. 556, 14ff., trsl. Heath, Aristarchus, p. 413).
[22] De nuptiis VIII, 860 (ed. Dick. p. 452, 13f.); cf. also below p. 664.
[1] Cf. above IV B 3, 1.

finds it worthwhile to discuss "pythagorean" speculations[2] according to which

fire counter-earth earth Moon Mercury Venus Sun

have the following distances from the center of the universe

1 3 9 27 81 243 729

which are simply the consecutive powers of 3. And Hippolytus, Bishop of Rome in the third century opposes to Archimedes' numerology[3] only the numerological fantasies from the Timaeus.[4]

It is obviously a reflection of Aristarchus' result[5]

$$18R_m < R_s < 20R_m \tag{1}$$

to assume

$$R_s = 19R_m \tag{2}$$

as we read in Pliny.[6] At the same time he tells us about a pythagorean theory[7] according to which

$$R_m = 126000 \text{ st.} = a, \quad R_s - R_m = 2a, \quad \text{zodiac} - R_s = 3a = R_s. \tag{3}$$

The value for R_m is also mentioned by Censorinus[8] who correctly observes that $2R_m$ is the earth's circumference as found by Eratosthenes.

According to a badly corrupted tradition[9] Eratosthenes seems to have estimated

$$R_m = 780000 \text{ st.}, \quad R_s = 4080000 \text{ st.} \tag{4}$$

which may mean $R_s \approx 6R_m$[10] but the numbers are much too insecure for any convincing interpretation. Even the order of magnitude of these distances does not agree with more or less contemporary estimates, e.g. Archimedes' $R_m = 5544130$ st.[11] or Apollonius' and Posidonius' 5000000 st.[12] Obviously it is only Hipparchus who brings some systematic reasoning[13] into this chaos.

As usual, Pliny and Plutarch, both great admirers of Hipparchus, did not refrain from reporting more or less arbitrary estimates. Pliny,[14] on the authority of Nechepso-Petosiris, mentions for 1° at the lunar orbit a little more than 33

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[2] De animae procr., Moralia 1028 B (ed. Hubert, p. 183, 17–24).

[3] Cf. above p. 649, (32).

[4] The same intellectual level is still present in Hegel's "Dissertatio philosophica de Orbitis Planetarum," accepted (1801) "pro licentia docendi" at the University of Jena (Hegel, Sämtliche Werke I, Stuttgart 1927, p. 1–29; German transl.: Philos. Bibl., Leipzig 1928, p. 347–402).

[5] Above IV B 3, 1.

[6] Pliny, NH II 83 (Budé II, p. 36, p. 172–175); also Plutarch, De facie, Loeb XII, p. 75.

[7] He has it from Sulpicius Gallus ($\approx$ 160 B.C. – cf. below p. 666, n. 8). For the subsequent speculation about harmonies cf., e.g., van der Waerden, RE Suppl. 10 col. 857–859.

[8] Censorinus XIII 3 (ed. Hultsch, p. 23, 12f.).

[9] Cf. the variants in Lydus, De mensibus (ed. Wuensch, p. 54, 7–10) and Diels, Dox., p. 362, 25–363, 4. Tannery (Mém. Sci. I, p. 391 f.) suggested drastic changes of all numbers in order to obtain reasonable results.

[10] Exactly the ratio 6 would require $R_m = 680000$ st.

[11] Above p. 649 (2).

[12] Above p. 650 (35) and p. 655 (14); cf. also p. 657.

[13] Cf. above I E 5, 4 B.

[14] NH II 88 (Budé II, p. 33, p. 175).

stades (which would mean $C_m \approx 12000$ st. and $R_m \approx 2000$ st.), twice as much at the orbit of Saturn and for the sun the mean value $R_s = 3/2\, R_m$. The last remark is of interest in so far as one of the Archimedian patterns also places the sun about halfway between earth or moon and Saturn.[15] The plausible emendation of the given numbers by means of a common factor 1000 or 10000 does not improve things much.

Plutarch in De facie[16] seems to have some vague recollection of Hipparchus' work in stating that the lunar parallax is not negligible.[17] Later on he says that

$$R_m \approx 56\, r_e = 56 \cdot 40000 \text{ st.} = 2240000 \text{ st.} \tag{5}$$

was the highest estimate for the moon's distance, ignoring again Hipparchus,[18] whereas $r_e = 40000$ st. is a Posidonian value.[19] Falling back on (2) he then obtains[20]

$$R_s - R_m \approx 40300000 \text{ st.} \tag{6}$$

A concoction of most heterogeneous elements is found in Macrobius' Commentary to Cicero's Somnium Scipionis.[21] The Eratosthenian measures

$$c_e = 252000 \text{ st.} \quad \text{thus } d_e = 7/22\, c_e \approx 80000 \text{ st.} \tag{7}$$

are combined with the arbitrary (and in our sources unique) assumption, ascribed to the "Egyptians", that the length of the earth's shadow cone equals 60 diameters of the earth and reaches exactly to the solar orbit, thus

$$60\, d_e = R_s. \tag{8}$$

Instead of immediately concluding from $d_\odot = 2^\circ$ and (8) that

$$d_s = 2\, d_e = 160000 \text{ st.} \tag{9}$$

a pseudo-observation with a hemispherical sundial is dragged in for the determination of the apparent solar diameter $d_\odot$. This "observation" deals with the crossing of the horizon by the rising sun,[22] supposedly leading to the result

$$d_\odot = \frac{1}{9} \cdot \frac{360^\circ}{24} = \frac{360^\circ}{216} = 1;40^\circ. \tag{10}$$

It then follows from (7) and (8) that

$$R_s = 4800000 \text{ st.} \quad \text{thus } C_s = 22/7 \cdot 2\, R_s \approx 30170000 \text{ st.} \tag{11}$$

hence with (10)

$$d_s = 30170000/216 \approx 140000 \text{ st.} \tag{12}$$

[15] Cf. p. 1355, Fig. 16B.

[16] Moralia, Loeb XII, p. 44 and p. 75.

[17] This is the meaning of the first sentence on p. 44 (misinterpreted in note (a) as referring to the lunar eccentricity); cf. for the terminology, e.g., Archimedes Opera II, p. 218, 18.

[18] Cf. above I E 5, 4: even the mean distances are greater: 67;20 r_e or 77 r_e; the maximum is 83 r_e.

[19] Above p. 653 (1).

[20] Accurately 40320000 st.

[21] Book I, 20 (ed. Eyssenhardt, p. 564–570; trsl. Stahl, p. 168–174).

[22] Cf. the corresponding waterclock "observation" above p. 658. The coefficient $c/216$ is mentioned once before in Book I, 16 (Eyssenhardt, p. 550, 29–32; trsl. Stahl, p. 154).

"almost twice the diameter of the earth" says Macrobius, perhaps being aware of the direct result (9). His numerical procedure is, of course, without any interest but one would like to know what lies behind (8) or (10).

C. *Actual Sizes of Sun and Moon*

Aristotle proved the sphericity of all celestial bodies by adducing some pseudo-arguments about motion[1]; for the moon, however, he mentions our visual experience: the various shapes of the lunar phases and the crescent shaped part of the sun left visible at partial solar eclipses. Plutarch[2] seems to consider Philip (of Opus), Aristotle's contemporary, as the first person who demonstrated the sphericity of the moon.

Arguments in support of the sphericity of the luminaries are, of course, repeated over and over again, frequently combined,[3] for better effect, with a refutation of different most implausible shapes. Furthermore we know, e.g. through Cleomedes,[4] of a whole collection of absurd hypotheses concerning the physical constitution and the optical properties of the moon, hypotheses to which also Posidonius contributed some theory of semi-transparency of the moon.[5]

An early attempt to come to a numerical estimate of the relative sizes of sun and moon is reported by Archimedes[6] who ascribes to Eudoxus the ratio

$$d_s : d_m = 9:1. \tag{1}$$

No method is mentioned which may have been used to reach this result but the "Eudoxus Papyrus" (in a corrupt passage) seems to give the same ratio in the disguise of harmonies.[7] This suggests a purely speculative origin of "pythagorean" variety.

The result of Aristarchus' procedure[8]

$$18\,d_m < d_s < 20\,d_m \tag{2}$$

was apparently not accepted by Phidias, Archimedes' father, when he proposed[9]

$$d_s = 12\,d_m. \tag{3}$$

Archimedes thought himself to be on the safe side when he took

$$d_s = 30\,d_m \tag{4}$$

as an upper bound for the size of the sun.[10]

Perhaps (3) is the consequence of the hypothesis of equal linear velocity of sun and moon in their respective orbits. If one says that roughly 12 rotations of the

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[1] De caelo II, 11 (Loeb, p. 200/201).
[2] Moralia, Loeb XIV, p. 62/65.
[3] Cf. below IV C 1, 3 B the "Eudoxus Papyrus", Tannery HAA, p. 290, No. 33.
[4] Mainly in Book II, 4 and 5.
[5] Cleomedes II, 4 (Ziegler, p. 190, 4); a similar theory also in the Āryabhaṭīya IV, 47 (trsl. Clark, p. 81).
[6] Sandreckoner, Archimedes Opera II, p. 220, 20f. (trsl. Ver Eecke, p. 356); also Lasserre, p. 17, D 13.
[7] Tannery, HAA, p. 293 (No. 49) and note (1).
[8] Above p. 641 (5).
[9] Archimedes, Opera II, p. 220, 21 f.
[10] Opera II, p. 222, 1-3. Cf. also above p. 646.

moon correspond to one rotation of the sun one would have for the orbital radii

$$R_s = 12 R_m \tag{5}$$

which, with the equality of the apparent radii, would imply (3).[11]

For Eratosthenes we have only a late source, Macrobius (around A.D. 400)[12] according to whom the "measure of the sun" should be the 27-fold of the "measure of the earth". This probably should mean that Eratosthenes assumed for the radii

$$r_s = 3 r_e. \tag{6}$$

A very different statement comes from some (christian) anonymous[13] who ascribes to Eratosthenes the opinion that

$$r_s = 100 r_e \tag{7}$$

$$r_m = 39 r_e \tag{8}$$

whereas "the teachers of our Church" declare that the size of the sun is the same as the size of the earth.[14] It is unlikely that Eratosthenes held that the moon is considerably larger than the earth; in fact the coefficient 39 in (8) suggest a crude misunderstanding of a passage in Alm. V, 16 according to which the volume of the earth amounts to 39 1/4 times the volume of the moon. For the sun the same passage gives the factor 170.[15]

The same anonymous reports[16] that Serapion (a contemporary of Posidonius) assumed for the sun

$$r_s = 18 r_e \tag{9}$$

a ratio which is also mentioned in a scholion to Aratus[17] and in King Sisebut's letter to Isidore of Seville[18] ($\approx$ A.D. 615). The factor 18 is perhaps a residue of the Aristarchian result (2). The same is perhaps the case for Cicero's[18a]

$$r_s = 19 r_e. \tag{9a}$$

The list of contradictory estimates can still be enlarged from Plutarch. In De facie[19] he ascribes to the "Egyptians" the opinion that "the moon is 1/72 of the earth." Taking this as referring to the volumes one would have

$$r_m \approx 6/25\, r_e. \tag{10}$$

[11] The same argument, in a slightly improved form (sidereal rotations instead of synodic) is also found with Posidonius (cf. above p. 656). Cf. for this whole type of arguing Aristotle, De caelo II, 10 (Loeb, p. 196/199).

[12] Macrobius, Comm. XX, 9, Eyssenhardt, p. 565, 25f.; trsl. Stahl, p. 170.

[13] Cramer, Anecd. Gr. I, p. 373, 27–30; the author is not Dionysius as assumed by Cramer (cf. CCAG 8, 3, p. 10, F. 103; CCAG 8, 4, p. 5, F. 192'; CCAG 7, p. 45, F. 75').

[14] Also Joannes Damascenus ($\approx$ 700), Expositio fidei 21 (ed. Kotter, Patristische Texte u. Stud. 12, 1973, p. 60, 164 = Migne PG 94, 895C) quotes "the Holy Fathers" for the same opinion.

[15] The same numbers also in Proclus, Hypot. IV, 101 (Manitius, p. 132/133).

[16] Cramer, l.c. p. 373, 25f.

[17] Diels, Dox. p. 63n. 2 = Maass, Comm. Ar. rel., p. 445, 18–22.

[18] Isidore, Nat. rer., ed. Fontaine, p. 333, 31. Isidore himself talks only very cautiously about the sizes of the luminaries (Etym. III, 47, 48; Nat. rer. XVI).

[18a] Academica II, 82 (Loeb, p. 571).

[19] Loeb XII, p. 120/121.

In De animae procr.,[20] however, we find

$$r_m = 1/3\, r_e \quad \text{thus } \mathrm{vol}_m = 1/27\, \mathrm{vol}_e \tag{11}$$

and

$$r_s = 12 r_e. \tag{12}$$

Again in De facie[21] one of the interlocutors mentions the estimate of 30000 stades for the circumference of the moon, thus

$$r_m \approx 5000 \text{ st.} \tag{13}$$

and, using (10)

$$r_e \approx 20\,800 \text{ st.}$$

Since this badly contradicts such estimates as[22]

$$r_e \approx 40\,000 \text{ st.}$$

one probably must cast doubt on the authenticity of estimates like (10) and (13).

Martianus Capella concludes from alleged observations of solar eclipses[23] (without telling us how) that

$$d_m = 1/6\, c_e \quad (\text{thus } d_m \approx r_e) \tag{14}$$

and combining this with a "measurement" at the waterclock[24]

$$d_☾ = 360°/600 = 0;36° \tag{15}$$

he comes to the conclusion that the circumference C_m of the moon's orbit is

$$C_m = 100\, c_e. \tag{16}$$

§ 4. Eclipses

The successful prediction of an eclipse, one of the most striking natural phenomena, must have appeared to contemporaries as a great achievement of wisdom and insight. It is therefore not surprising when later generations ascribed such knowledge to the wise men of the remote past.

For our understanding of the development of astronomy these myths of origin, so dear to the Greeks, are of no help. Our task still consists in extracting from extremely fragmentary sources whatever factual knowledge existed concerning eclipses in the period preceding the Hipparchian lunar theory.

Thanks to a large body of information on eclipse theory from cuneiform sources we have a fairly accurate insight into the development and general methodology of Babylonian eclipse prediction. Its key is the construction of a common period (later known as the "Saros") of syzygies and latitudes that made it possible

[20] Moralia VI, 1, ed. Hubert, p. 184, 3–6.
[21] Loeb XII, p. 143/145.
[22] Cf., e.g., above p. 653 (1).
[23] De nuptiis VIII 859 (Dick, p. 452); cf. also below p. 668 and p. 964.
[24] Cf. above p. 659 (3).

to select those syzygies which could be accompanied by eclipses. At such times a careful watch was kept and the recording of observed details of eclipses which actually occurred quickly led to refinements in the lunar theory.[1] At the very foundation of this process lies the insight that eclipses are connected only with the nodes (i.e. zero latitude) of the lunar orbit, hence generally separated by six months, excepting straddling cases of five-month intervals. It should be noted that this genesis does not imply any geometric model for the sun, moon, earth, and shadow nor any empirical search for eclipse cycles; it is the periodicity of the components of the lunar motion from which one derives a common eclipse period, not vice versa.

The recognition of the role of six- and five-month intervals between eclipses is amply attested in Babylonian astronomy of the Persian and hellenistic period.[2] There we also find explicit rules for the relative frequency of these intervals, e.g. in a text which concerns eclipse magnitudes,[3] supposedly varying from eclipse to eclipse according to a linear saw function. Its period is

$$\frac{2,15}{23} = 5;52,10,26, \ldots \text{ syn.m.} \tag{1}$$

which means that 23 eclipses occur in 135 months, i.e. during a period which contains 20 six-month and 3 five-month intervals. A similar pattern is preserved in a demotic papyrus[4] with

$$\frac{2,9}{22} = 5;51,49,5, \ldots \text{ syn.m.} \tag{2}$$

as period, i.e. assuming 19 six-month and 3 five-month intervals in 129 months. If one computes from (1) and (2), respectively, the ratio of synodic to draconitic months one finds

$$\frac{4,30}{4,53} = 0;55,17,24, \ldots \tag{1a}$$

in the first case and

$$\frac{2,9}{2,20} = 0;55,17,8, \ldots \tag{2a}$$

in the second. The Babylonian ephemerides are based on a ratio near 0;55,17,20.

References to the spacing of eclipses by six- and five-month intervals are quite common in Greek sources, e.g. in Hipparchus,[5] Heron,[6] and Plutarch.[7] This

[1] Cf. above II B 4.

[2] Cf. above p. 549.

[3] ACT No. 93 (Vol. I, p. 123).

[4] P. Carlsberg 31 (probably second century A.D.); cf. Neugebauer-Parker, EAT III, p. 241–243, Pl. 79 A (not B). The character of this text was not understood in the edition because the scribe had consistently replaced "month" by "year". The proper explanation was found by A. Aaboe [1972], p. 111 f.

[5] Cf. above I E 5, 2 B.

[6] Heron, Dioptra 35; Opera III, p. 302/303, Sect. 22; cf. also Rome [1931, 1].

[7] Plutarch, De facie (Loeb, Moralia XII, p. 131) citing parameters also known from Babylonian astronomy; cf. above p. 321.

rule of thumb provides, with very little effort of observation, a fair chance of predicting lunar eclipses. For solar eclipses, however, it would be the sheerest luck if on such a primitive basis a solar eclipse had been correctly announced in advance. It makes little sense to speculate for some cases mentioned in the ancient literature whether we are dealing in fact with such a chance prediction or with another case of historical mythology.[8]

The first seemingly secure case of a prediction of a solar eclipse is recorded by Dio Cassius for the reign of Claudius.[9] According to Dio's story Claudius expected a solar eclipse to occur on his birthday (A.D. 45 August 1) and in order to forestall any public tumult in Rome he had in advance announced time and magnitude of the eclipse. Unfortunately Dio does not mention any numerical details but modern theory indicates for Rome a maximum obscuration of slightly less than 4 digits, hence an eclipse of only questionable visibility.[10] This, of course, is not an argument against the existence of an ancient prediction. It is clear that Dio correctly understood the basic facts involved – he explicitly mentions anomaly and latitude for the lunar motion and anomaly for the sun and he was also aware of the influence of the geographical element. We know from papyri of the Roman imperial period[11] that one had arithmetical procedures at one's disposal which were fully adequate to select those syzygies which could lead to eclipses. For the geographical component, however, we have no information in the extant texts. It seems unavoidable to assume the existence of an elaborate theory of lunar parallax if a prediction for a specific locality were to have a reasonable chance of success. In this connection it should be remembered that according to Cicero[12] solar as well as lunar eclipses could be predicted "for many years in advance" by those who "siderum cursus et motus numeris persequuntur" – one is tempted to translate "by those who use numerical methods."

Seneca tells us that Conon collected records of solar eclipses,[13] perhaps because Catullus mentioned solar eclipses in connection with Conon.[14] Pliny's story about 600 years of eclipse predictions by Hipparchus is obvious nonsense.[15] A remark by Achilles ($\approx$ A.D. 250)[16] according to which "many" had worked on solar eclipses in relation to the seven climata, naming specifically "Orio, Apollinarios, Ptolemy and Hipparchus," can be explained by the discussion in the Almagest on eclipse intervals which involve parallax, i.e. geographical elements.[17] Completely unknown, however, is "Orio" while an Apollinarios is also mentioned in other sources.[18]

[8] E.g. the "prediction" by Helicon of the (annular) eclipse of −360 May 12 (Ginzel, Kanon, p. 183, No. 15) or of the lunar eclipse of −167 June 21 by Sulpicius Gallus (Pliny, NH II 53; Budé II, p. 23f. and p. 142f.; Ginzel, Kanon, p. 190ff., No. 27).

[9] Dio Cassius, Roman History (completed about A.D. 230) LX, 26 (Loeb VII, p. 432–435).

[10] Ginzel, Kanon, p. 201.

[11] Cf. below V A 2, 1.

[12] Cicero, De divinatione II 6, 17 (Loeb, p. 388/389), written 44 B.C.

[13] Quaest. nat. VII, III, 3 (written between A.D. 62 and 65); cf. also above p. 572, n. 4. Tannery, Mém. Sci. III, p. 353 substituted "Chaldeans" for the implausible "Egyptians".

[14] Catullus, Carmina 66, 3 (around 50 B.C.).

[15] Cf. above p. 319ff.

[16] Maass, Comm. Ar. rel., p. 47, 13f.; cf. also above p. 321.

[17] Cf. above p. 131f. and p. 322.

[18] Cf. above p. 601, n. 2.

The correct explanation of the causes of eclipses seems to have been found at a relatively early time. Philip of Opus (≈ -340) is said to have argued against a hypothetical "Counter-Earth" as the cause of lunar eclipses[19] and Zeno, the founder of the Stoic school, (≈ -300) explained solar eclipses using a diagram, according to Diogenes Laertius' story.[20]

Of numerical parameters related to eclipses very little is preserved from the time before Hipparchus. When he remarks (in his polemics against the hypothesis of a latitudinal motion of the sun[21]) that the error in predicting the magnitude of lunar eclipses amounts to only 2 digits, and, with careful computation, considerably less,[22] one must conclude that the apparent diameters of moon and shadow were known with fairly high accuracy. What we actually know from still extant sources, however, are only round values. Aristarchus had assumed[23] for the radius u of the shadow

$$u=2r_{☾} \tag{1}$$

whereas Plutarch (on two occasions) mentions[24]

$$u=3r_{☾}. \tag{2}$$

In a (perhaps corrupt) passage in Geminus[25] the diameter of the shadow is assumed to be 2°.

Hipparchus himself considered

$$u=2;30\,r_{☾} \tag{3}$$

sufficiently accurate at the moon's mean distance.[26] Ptolemy's ratio for the mean distance is 2;36, at the moon's apogee 2;35, at the perigee 2;36,13, ...[27]

The usefulness of lunar eclipses for the determination of geographical longitudes on the basis of the difference in local time is a pretty obvious consequence of the sphericity of the earth. That Hipparchus was aware of this fact is explicitly stated by Strabo[28] but there is no compelling reason to assume that Hipparchus was the first to see this implication. At any rate, the total absence in antiquity of any scientific organization deprived the whole method of its practical importance. Heron, in the first century A.D., apparently had no reliable time interval at his disposal even for cities like Alexandria and Rome.[29] It is therefore not surprising that the data of the lunar eclipse of -330 Sept. 20 (famous by its connection with

[19] Stobaeus, ed. Wachsmuth, p. 221, 6; cf. also above p. 574.
[20] Diogenes Laertius VII 146 (Loeb II, p. 248/251).
[21] Cf. above IV B 2, 2.
[22] Hipparchus, Ar. Comm., p. 90, 10–12, ed. Manitius. Note that he says nothing about solar eclipses.
[23] Cf. above p. 635 f. and below p. 689; also Cleomedes (from Posidonius): above p. 654; cf. also below p. 963.
[24] Plutarch, De facie 923 B (Loeb, Moralia XII, p. 57, note d) and De anim. procr. 1028 D (ed. Hubert, p. 184, 12).
[25] Isagoge XI, 7 (Manitius, p. 134/135); cf. above p. 593.
[26] Alm. IV, 9; cf. above p. 313, n. 6.
[27] Cf. above p. 125 (1).
[28] Strabo, Geogr. I 1, 12 (Loeb I, p. 24/25).
[29] Cf. below p. 848.

the battle of Arbela) were also only approximately known, so that Pliny and Ptolemy give widely different data.[30] Interestingly enough Pliny's data are superior to Ptolemy's, both with respect to the time interval and to the phases of the eclipse (cf. Fig. 20[31]).

Solar eclipses are, of course, without value for longitudinal determinations (although included by Strabo in the above cited statement concerning Hipparchus). Nevertheless we know of one solar eclipse (A.D. 59 Apr. 30) for which the time difference between far distant places ("Armenia" and the area of Naples) was correctly noted (about 3^h).[32]

The classical example in ancient literature for the local difference in the appearance of solar eclipses is the eclipse of -189 March 14, described as total at the Hellespont, partial (4/5 eclipsed) in Alexandria. This eclipse obtained special significance through its use by Hipparchus in his attempts to determine the distances of the luminaries.[33] Undoubtedly fictitious but following the same pattern is a story in Martianus Capella[34] about eclipses which are total at the climate of Meroe (longest daylight $M = 13^h$), partial at Rhodes ($M = 14\ 1/2^h$), and zero at the climate of the Borysthenes ($M = 16^h$). This then is somehow used to deduce from the size of the moon's shadow on the earth the size of the moon itself in terms of the earth's circumference.[35]

Of crucial importance for the relative sizes and distances of sun and moon is the question of the degree to which the solar disk can be obscured by the moon. We know that Polemarchus, a younger contemporary of Eudoxus, did see an annular eclipse and this experience may have resulted in the statement found in the "Eudoxus Papyrus" that total solar eclipses are impossible.[36] Another annular eclipse (probably A.D. 164 Sept. 4) was observed by Sosigenes[37] and Proclus also says that "some astronomers" report this type of eclipses[38] but he notes that Ptolemy's parameters exclude annular eclipses.[39] Cleomedes simply denies the reality of annular eclipses in contrast to the opinion of "some older" astronomers.[40] Simplicius in the 6th century, however, remarks that due to the variation of the geocentric distances solar eclipses can be total as well as annular.[41]

The aspect of an eclipse, solar or lunar, which was considered indicative for wind and weather is known to us through Ptolemy's discussion of the "prosneusis"

[30] Pliny, NH II, 180 (Loeb I, p. 312/313; Budé II, p. 79 and p. 234, n. 5) gives the 2nd hour of night for Arbela, moon-rise (i.e. sun-set) in Sicily. Ptolemy, however, in Geogr. I, 4 (Nobbe, p. 11, 19f.; Mžik, p. 21) reports the 5th hour of night for Arbela, the 2nd for Carthage. Cf. also Ginzel, Kanon, p. 184f. (No. 18) and P.V. Neugebauer, Kanon d. Mondf., p. 42. Cleomedes I, 8 (Ziegler, p. 76, 8–16) gives 4^h as difference for eclipses seen in "Persia and Spain".

[31] All data are reduced to Arbela local time: for Syracuse $\Delta t = 1;55^h$, for Carthage $2;15^h$.

[32] Pliny, NH II, 180; cf. Ginzel, Kanon, p. 201f. (No. 39).

[33] Cf. above p. 327f. and for the identification of the eclipse p. 316, n. 9.

[34] Martianus Capella, De nuptiis VII, p. 859f. (Dick, p. 452f.).

[35] Cf. above p. 663f., and below p. 964.

[36] Cf. below p. 688 and p. 689, n. 21.

[37] Cf. above p. 104, n. 4.

[38] Proclus, Hypot. I, 19 (Manitius, p. 10/11).

[39] Proclus, Hypot. IV, 99 (Manitius, p. 130/131). For Ptolemy's parameters cf. above p. 106.

[40] Cleomedes II, 4 (Ziegler, p. 190, 17–24).

[41] Simplicius, Comm. Arist., Vol. III, ed. Heiberg, p. 505, 1–9; transl. Duhem, SM I, p. 401.

(in Alm. VI, 11). We described[42] how Ptolemy himself dealt with the corresponding problem of spherical trigonometry, i.e. with the determination of the points at which the great circle that connects the centers of eclipsed and eclipsing body meets the horizon. We do not know, however, how early such theories appeared in Greek astronomy and meteorology but a connection with Mesopotamian eclipse omina seems not to be excluded.[43]

§ 5. The "Steps" (βαθμοί)

One looks in vain for the concept "steps" in Ptolemy's works; neither the Almagest nor the Tetrabiblos nor the Phaseis mention this term which refers to a six-division of the quadrant, each "step" being the equivalent of 15°. But Ptolemy's disrespect for antiquated concepts and terminology by no means always succeeded in preventing their continued use, e.g. in the astrological literature, or even their final readmission into the works on mathematical astronomy of Almagest tradition — the theory of trepidation of the equinoxes[1] being the classical example.

From the viewpoint of theoretical importance the "steps" cannot be compared with the hypothesis of trepidation. For historical inquiry, however, the steps are much more than another example of the tenacious survival of obsolete concepts because, as it were, these units through their chronological and geographical spread provide us with some, however faint, reflection of historical conditions in pre-Ptolemaic astronomy.

The earliest evidence for the use of steps as independent variable in astronomical computations is found in the Anthology of Vettius Valens[2] (who is a younger contemporary of Ptolemy) and in a short treatise, preserved on papyrus (P. Ryl. 27[3]), written shortly after A.D. 250. In both cases we are dealing with the "argument of latitude" of the moon, counted in the traditional fashion from the northernmost point of the lunar orbit,[4] but reckoned in "steps" without using any specific terminology that could warn us that one is operating with units of 15°. Both texts are only concerned with the distance of the moon from the nodes and do not give the transformation to the lunar latitude itself. Unfortunately no tables for latitudes as function of the argument of latitude seem to have been preserved in Greek sources but the transition from steps to latitudes is still available in a Tamil table from the 13th century A.D.[5] In Byzantine treatises as well we find the computation of lunar latitudes with specific reference to the corresponding numbers and fractions of the steps.[6]

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[42] Above I B 6, 7; for later modifications cf. below p. 997.
[43] Cf., e.g., Schaumberger, Erg., p. 246-249 or Parker, Vienna Pap.
[1] Cf. above I E 2, 2 B and IV B 2, 3.
[2] Cf. below V A 2, 2 B.
[3] Below V A 2, 1.
[4] Cf. above p. 80 or the table in Alm. V, 8.
[5] Cf. below p. 819, Table 7.
[6] E.g. Marc. gr. 325 fol. 21ᵛ, 1-14 (unpublished). Here the argument $\omega' = 345;12°$ is described as "in the 6th step and about 1/60 [thus 0;15° for the 0;12°], northerly and ascending" (cf. Fig. 21).

The two above mentioned texts, Vettius Valens (I, 18) and P. Ryl. 27, show a peculiar norm in the counting of longitudes: the starting point is Leo 0°, not one of the equinoxes or solstices. Traces of this norm are found in earlier and later sources[7]; if Leo were chosen as the sign of the sun during the Egyptian month Thoth the period around A.D. 120 would correspond to it.

Theon, as in the case of the theory of trepidation, feels obliged to mention in his Commentary to the Handy Tables topics ignored by Ptolemy. Thus he tells us that solar declinations are reckoned beginning at the maximum at Cancer 0°, adding that the "astrologers" (*ἀποτελεσματικοί*) call the six 15°-sections of each quadrant "steps" (*βαθμοί*) or "sixths" (*ἑκτημόριοι*).[8] In some manuscripts of the Handy Tables we find indeed the numbers of the steps written at the margin of the tables for solar declinations and lunar latitudes.[9] Most likely through Theon and his redaction of the Handy Tables the concept of steps has also been transmitted to Islamic astronomy.[10] Indeed Theon's definition of the steps of solar declinations is repeated by al-Battānī (≈A.D. 900) and illustrated by a diagram[11] of a type also found in Byzantine manuscripts.[12]

Of some interest are two accretions to the simple trigonometric problem of tabulating lunar latitudes and solar declinations: (a) an extension to planetary latitudes or declinations, and (b) association of the quadrants with wind directions. Chapter headings in Vettius Valens (I, 18 and III, 4) speak about "steps and winds of the moon" or of "stars" (i.e. planets). These wind directions are "ascending" or "descending" and "north" or "south" in relation to the approach toward the extremal latitudes or nodes.

Looking for a motive to associate the lunar latitude or the solar declination with winds one could well imagine some similarity with the concepts underlying the parapegmata,[13] according to which the positions and phases of celestial objects are indicative for meteorological conditions. Unfortunately the extant sources do not support such a hypothesis: never is a wind predicted from a lunar or planetary latitude. In fact the term "wind" is only used in the function of a celestial coordinate, e.g. in the sentence "look also upon the lunar latitude, to what wind it belongs and to what zodiacal sign"[14] and similarly in many cases in which the astrologer is advised to take heed of latitude and wind of the moon or of a planet.[15] In all such cases "wind" simply means the direction of the motion in latitude and

[7] Cf., e.g., below p. 671, or P. Carlsberg 9 which starts its enumeration of zodiacal signs with Leo (Neugebauer-Parker, EAT III, p. 223).

[8] Halma HT I, p. 54.

[9] Halma HT I, p. 144/145; cf. below p. 979, n. 3. The steps are not indicated, however, in the corresponding tables of Vat. gr. 1291 (fol. 44ʳ, 45ᵛ, 46ʳ).

[10] Similar commentaries, e.g. by Stephanus of Alexandria (≈A.D. 620) demonstrate the continuity of the tradition (cf. Usener, Kl. Schr. III, p. 296, Chap. 22, unpublished); cf. below p. 1049).

[11] Nallino, Batt. I, p. 13, 22–26; II, p. 58, p. 221.

[12] E.g. Vat. gr. 1059 fol. 112ʳ (unpublished). A unique V-shaped diagram is found in Vat. gr. 1291 fol. 47ᵛ (originally the last page) but with incorrect legend (latitudes of Mercury instead of solar declinations; the MS was written about A.D. 820; cf. below p. 969 f.).

[13] Cf. above IV A 3, 3. Cf. also Ptolemy, Tetrab. II, 12 Loeb, p. 209 and p. 213; ed. Boll-Boer, p. 99, 3, p. 100, 8.

[14] CCAG 8, 1, p. 261, 24 f. (from "Palchus").

[15] CCAG 7 p. 128, 12–24 (from Antiochus, ≈A.D. 200); CCAG 5, 1, p. 198, 8 f. (Anonymus of 379); CCAG 8, 1, p. 243 a, 12 f. and p. 243 b, 23 f. (from Rhetorius); CCAG 9, 1, p. 180, 19 (cod. 16, note 12); etc.

not an atmospheric phenomenon; probably "vector" would better represent the actual meaning.

The connection of the steps with the planets is twofold: either the steps are counted from the point of northernmost latitude, or "latitude" is replaced by the purely astrological concept of "exaltation" (*ὕψωμα*), obviously the result of a cheap play on words with different shades of the meaning.

The association with latitudes provides us with a welcome possibility of checking some basic parameters.[16] In the astrological version[17] we simply have the known points in which, according to doctrine, the planet's power is greatest.[18] The steps are again 15° sections beginning at the exaltation and related to the four types of "winds."

Applications of the steps and their fractions are not infrequent. A very elaborate horoscope cast for A.D. 497 Oct. 28 (probably by Eutocius[19]) gives steps and winds for the latitudes of the planets and the moon and for the declination of the sun. Similar data are given for the moon in an early horoscope from Oxyrhynchus, cast for A.D. 46 May 13.[20]

Still earlier evidence is concealed in a remark by Diogenes Laertius.[21] After having correctly explained the cause of eclipses he goes on to say that "the school (*οἱ περὶ*) of Posidonius" assumed the moon's latitude "right at the ecliptic" (*κατὰ τὸν διὰ μέσων*) in Libra[22] and Scorpio and in Aries and Taurus. As it stands this statement would imply the existence of a stable nodal line for the moon — contradictory to obvious facts. But when steps are counted from ♌ 0° (cf. Fig. 22) then the diameter at a right angle is indeed in the position of a nodal line with respect to ♌ 0° as northernmost point.

This is not the only evidence which relates Posidonius to the concept of steps; we have seen[23] him using "parts" to measure meridian arcs, i.e. 48th of a circle or half-steps. Thus we have reached the generation after Hipparchus who himself also used arcs of 1/24 of a circle as unit of measurement[24] as well as "signs" (of 30°) for arcs outside the ecliptic.[25] Hence we know now of an early sequence of primitive angular measures: "signs", "steps", "parts", i.e. 1/12, 1/24, 1/48 of the circle — eventually expressed in Babylonian units as 30°, 15°, 7;30°. On the other hand it may be significant that neither Aristarchus nor Archimedes use

[16] Cf. for the planetary latitudes in the Handy Tables below p. 1016. The customary names for the four quadrants of planetary latitudes are also found in Cleomedes I, 4 (Ziegler, p. 34, 23–36, 1) but no steps or winds.
[17] Cf., e.g. Vettius Valens III, 4 (Kroll, p. 140, 4–28).
[18] Cf., e.g.. Bouché-Leclercq, AG, p. 193–199.
[19] Neugebauer-Van Hoesen, Gr. Hor., p. 154 (cf. also p. 188 f. concerning the question of authorship of Julianus of Laodicaea or Eutocius).
[20] P. Oxy. 2555 (Vol. 31, p. 83–86).
[21] Diogenes Laertius VII 146 (Loeb II, p. 250/251).
[22] Also the use of *χηλαί*, i.e. "the Claws" (of Scorpio), for Libra conforms to an early date of Diogenes' source. It is a funny coincidence that Yonge in 1895 (in Bohn's Classical Library) mistook *χηλαί* for "Cancer" and that the same translation appears in the German translations of 1921, 1955, 1967, in the English translation (Loeb) of 1925 and 1950, in the French translation of 1933, and in the Italian translation of 1962. The older Latin translations give simply *chelae*.
[23] Cf. above p. 652; also Geminus V, 25 (Manitius, p. 52, 4).
[24] Cf. above p. 302.
[25] Cf. above p. 278.

these units.[26] The treatises of Autolycus, Euclid, and Theodosius on spherical astronomy never use other angular measures than signs and half signs.[26a]

It seems to me obvious that we have here the source of the norm in Indian astronomy and trigonometry which tabulates the sines in multiples of 3;45° which is the equivalent of a smallest unit of 7;30° in a table of chords.[27] This has been fully confirmed by G. J. Toomer's investigation[28] of Hipparchus' determination of the size of the lunar epicycle (or of the eccentricity), where he showed that the radius in the Hipparchian table of chords was also the same as in the Indian table of sines[29].

The "steps" are not only a good example of the spread of early Greek astronomical concepts but also of the tenacity with which antiquated methods were carried deep into the latest period of Byzantine astronomy. In the famous illuminated manuscript of the Handy Tables, Vat. gr. 1291 of the ninth century, we find a "Table for the Latitude of Mercury"[30] which shows in a circular diagram a V-shaped figure with the numbers from 1 to 6, four times, representing the "steps" between "North" and "South".[31]

A similar V-shaped figure, now in a rectangle and referring to the *βαθμοί* of the moon, is found in a group of manuscripts also related to the Handy Tables.[32] The terminology for the ascending and descending branches is the same as in the Mercury diagram and one may well conjecture that similar diagrams existed for all planets. The lunar diagram, however, also gives significant numerical data[33]:

left:	13	26	39	52	65	78	90	102	115	128	141	154	167	180
center:	1	2	3	4	5	6	☾	1	2	3	4	5	6	
right:	347	334	321	308	295	282	270	258	245	232	219	206	193	

The numbers 90 and 270 are characterized as "place for eclipses", i.e. as the lunar nodes. Obviously 0 represents the northern limiting point for the argument of latitude, tabulated in 24 sections of 13° each and four sections of 12° straddling the nodes. These 12° must represent the eclipse limits.

This rough scheme is refined and thus clarified by a large numerical table, headed "eclipse magnitudes", found with the previously mentioned notes to the

[26] Both operate almost exclusively with fractions of whole quadrants, not of sections of a quadrant; cf. below p. 772f.
[26a] Cf. below, IV D 3, 4.
[27] Steps of 15° are commonly used in Indian astronomy, e.g., just as in Greek astronomy, for solar declinations (Kh.-Kh. I 29. Sengupta, p. 31 for $\varepsilon = 24°$); also for the equation of center of the sun (Kh.-Kh. I 16, Sengupta, p. 19 for 2;14° as maximum equation). For the tables of sines cf., e.g., Āryabhaṭīya I 10 (≈ A.D. 500) or the "modern" Sūrya-Siddhānta II 15–17 (12th cent.). In the Middle Ages in Europe these tables are known as "kardaga"; cf., e.g., Millás-Vallicrosa, Est. Azar., p. 44 (in six steps to the quadrant) and Goldstein, Ibn al-Muthannā, p. 196/197. The same in Kh.-Kh. I 30 or IX 8 (Sengupta, p. 32 and p. 142) for $R = 150$.
[28] Toomer [1973].
[29] Cf. above I E 3, 1. p. 299f.
[30] Fol. 47v; cf. below V C 4, 1 D 2, p. 978.
[31] Terminology: *βορρά/νοτία/ἀνάβασις/κατάβασις*. Cf. also Tihon [1973]. p. 103.
[32] In the part published by A. Tihon [1973]; I am using here Marc. gr. 314 fol. 222r (unpublished).
[33] I correct trivial scribal errors.

Handy Tables.[34] The center is again a V-shaped table, 14 lines long on each side, flanked by two vertical tables of 37 lines each.

The numbers in the central table vary, again between "North" and "South", with a constant difference

$$\delta = 0,52,5 \tag{1}$$

which is replaced by

$$\delta' = 0,47,30 \tag{2}$$

near the "nodes" at 6,0,0 and 18,0,0. The left column runs, beginning at the "North", from 0,52,5 to 12,0,0, the right column from S to N from 12,52,5 to 24,0,0. Clearly, the units here are $\beta\alpha\vartheta\mu o\acute{\iota}$ of 15° each, covering a whole period from 0^{β} at the northern limit to 12^{β} at the greatest negative latitude, back to $24^{\beta} = 0^{\beta}$ at N.

Consequently we can now express the differences δ and δ' in degrees:

$$\begin{aligned} \delta &= 0;52,5 \cdot 15 = 13;1,15^{\circ} \\ \delta' &= 0;47,30 \cdot 15 = 11;52,30^{\circ}. \end{aligned} \tag{3}$$

The daily increment of the argument of latitude is about 13;13,45°[35], thus somewhat more than δ. Furthermore it takes 28 intervals of the table to complete one rotation, again more than a draconitic month which should be about 27;12,40 days. These discrepancies disappear, however, when one counts time not in days but in tithis[36]. Then we can say that the period of our table contains

$$24 + 4 \cdot \frac{47,30}{52,5} = 27;38,52,48^{\tau}$$

or, with

$$1^{\tau} \approx 0;59,3,40^{d}$$

the time of

$$27;38,52,48 \cdot 0;59,3,40 \approx 27;12,55,18^{d}. \tag{4}$$

The same argument explains also the preceding crude scheme which represents a period of

$$24 + 4 \cdot \frac{12}{13} \approx 27;41,32^{\tau} \approx 27;40^{\tau}$$

i.e. approximately

$$27;40 \cdot 0;59 = 27;12,20^{d} \tag{5}$$

for the draconitic month.

The eclipse limits in the refined pattern are given by

$$\pm\delta' = \pm 0;47,30^{\beta} = \pm 11;52,30^{\circ}.$$

Now the outer tables come into play: they relate eclipse magnitudes to the nodal distances. These tables contain strictly linear sequences with the constant difference

$$d = 0;2,38,20^{\beta} = 0;39,35^{\circ}. \tag{6}$$

[34] Marc. gr. 314 fol. 218ᵛ, *et al.* (cf. Tihon [1973], p. 52).
[35] Cf. Alm. IV, 4.
[36] Cf. above p. 358 and below p. 1069.

Since

$$36d = 1;35^{\beta} = 23;45^{\circ} = 2 \cdot 0;47,30^{\beta} = 2 \cdot 11;52,30^{\circ} = 2\delta'$$

we see that these sequences are constructed to bridge exactly the nodal zone from $5;12,30^{\beta}$ to $6;47,30^{\beta}$ and from $17;12,30^{\beta}$ to $18;47,30^{\beta}$.

Associated with these arguments of latitude is in each table one more column, denoted as "digits". There the numbers increase in steps of 1 from 1 to 12 digits, then they remain constant at 12 for the whole interval

$$6 \pm 0;18,28,20^{\beta} = 90 \pm 4;37,4^{\circ} \tag{7}$$

(and similar near $18^{\beta} = 270^{\circ}$), and thereafter decrease again linearly to 1 at the other end of the table. Hence we see that we are dealing with lunar eclipses which are total at oppositions within $\pm 4;37,4^{\circ}$ from the nodes. Ptolemy's limits are $\pm 4;48^{\circ}$ at maximum distance, $\pm 5;24^{\circ}$ at minimum distance[37]. Our arithmetical scheme obviously represents a quite reasonable approximation to the actual phenomena.

Operating with *βαθμοί* and tithis is not the only archaic feature in this group of texts. In both lunar schemes described here we also find zodiacal symbols associated with the V-shaped arrangement, such that ♋ stands near "North", ♑ near "South". The copyist no longer understood the function of zodiacal signs in relation to lunar latitudes and placed them in an irregular fashion along the *βαθμοί*. Nevertheless it seems evident that we have here a simple counting of arcs such that each sign covers 2 *βαθμοί*. Since the argument of latitude does not increase in integer "steps" per tithi the placing of the signs could not follow a simple pattern and thus resulted in general disorder. The same counting by signs appears also in other diagrams of these notes[38], thus supporting our impression that we are dealing here with remnants of an early level of Greek arithmetical schemes.

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[37] Alm. VI, 8 for $m = 12$ digits.

[38] Marc. gr. 314 fol. 222ʳ, 222ᵛ, 223ᵛ; also Tihon [1973], p. 70, note 1 (counting from the solar apogee).

C. Early Planetary Theory

Even a very casual observation of astronomical phenomena cannot fail to lead to the discovery that the motion of the planets is subject to some regularity, both with respect to time and to position among the fixed stars. To establish definite numerical rules for these regularities was the great achievement of Babylonian astronomy. The attempt to "explain" the planetary motions by means of cinematic models represents another and extremely fruitful approach which initiated Greek mathematical astronomy.

We have no reason to assume that the early phase of Greek cinematic planetary theory was developed under Babylonian influence. Otherwise it would be very difficult to understand the almost complete absence of nontrivial numerical data within Greek planetary theory before Hipparchus. For us, the heirs of hellenistic science, it is difficult to imagine two or three centuries of discussion about cinematic planetary models without a manifold of numerical parameters, derived from observational data. Nevertheless not only do we not have evidence for numerical data in the construction of Eudoxus' homocentric spheres but it would also be difficult to understand how his theory could have survived a comparison with observational parameters. The same situation seems to be characteristic to some extent for the whole pre-Hipparchian period, even including Apollonius.[1] When numerical data appear in early Greek astronomy then they can usually be connected with arithmetical methods and thus reveal Babylonian influence, unrelated to geometrical concepts. The unique role of Hipparchus lies in the combination of both approaches, Babylonian as well as Greek, supplemented by systematic observations of his own and by the development of new mathematical tools for dealing with numerical problems.

§ 1. Eudoxus

1. General Data

In spite of some discrepancies in detail it is clear from our sources that Eudoxus' lifetime practically coincides with the first half of the fourth century B.C.[1] Unfortunately nothing has survived of his writings and our knowledge of his work is therefore based entirely on secondary references in the ancient literature. A collection of these "fragments" – or what is assumed to be one – has been

[1] Cf. above p. 271.

[1] For references cf. above p. 573. It also has been noted above (p. 599, n. 10) that Rehm tried to distinguish two astronomers named Eudoxus.

published by Lasserre.[2] The so-called "Eudoxus Papyrus"[3] can at best be considered as representing the general level of astronomy in the fourth century but tells us nothing about specific Eudoxan elements.

We know from several references in Hipparchus' Commentary on Aratus[4] that Eudoxus had written two very similar works on the risings and settings of constellations, one called "Mirror" (ἔνοπτρον) — a title difficult to explain — the other "Phaenomena." To the same area of interest belong weather prognostics which are quoted under his name in the parapegma literature from Pseudo-Geminus to Ptolemy[5] and beyond.

Eudoxus' dubious authorship of a work on the "Oktaeteris" has been mentioned before.[6] The "Dog-Dialogues"[7] may have gotten their name from the Dog-Star, i.e. Sirius, and its calendaric interest.

Another work, probably called "On Velocities" (περὶ ταχῶν),[8] is concerned with an entirely different subject. In it the attempt is made to explain the typical phenomena of apparent planetary motions as the result of the superimposed rotations of concentric spheres. We shall presently describe the details of this theory.[9]

Eudoxus seems to have had considerable influence on the following generations. After a first, and apparently not very successful, stay in Athens and some travel in Egypt[10] he founded a school in Cyzicus (on the south shore of the Sea of Marmara) which attracted so many pupils that he is said to have succeeded upon his return to Athens in embarrassing Plato.[11] Among his pupils[12] is Menaichmos, known as the inventor of the conic sections,[13] who also worked on the homocentric spheres[14] and Polemarchus whose pupil Callippus became well known through his modifications, accepted by Aristotle, of the Eudoxan theory of homocentric spheres[15] as well as through his calendaric cycle[16] and his contributions to the parapegma literature.[17]

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[2] Berlin 1966; cf. also the review Toomer [1968].

[3] Cf. below IV C 1, 3.

[4] E.g. Hipparchus, Ar. Comm., Manitius, p. 9.

[5] Collected in Lasserre as Fragm. No. 146–267. Cf. also above p. 588 and below p. 929.

[6] Cf. above p. 620f.

[7] Mentioned by Diogenes Laertius VIII 89 (Lasserre, Fragm. T 7, p. 5, 17–20; Loeb, Diog. L. II, p. 402/403). The title has inspired a long sequence of learned (often funny) conjectures.

[8] This title is taken from a sentence by Simplicius (Comm. in Arist. De caelo, ed. Heiberg, p. 494, 12; Lasserre Fragm., p. 69, 10).

[9] Below IV C 1, 2.

[10] The often repeated stories about Eudoxus learning astronomy from Egyptian priests, or about his observatories in Egypt, are not worth refuting. Among the "observatories", shown to Strabo by his guides three centuries later, is mentioned (Strabo, Geogr. XVII 1, 30; Leob VIII, p. 84/85) "Kerkesoura in Lybia" which is Kerkeosiris in the Faiyūm, near Tebtunis (cf. RE 11, 1 col. 291), some 120 km from Heliopolis.

[11] Diogenes Laertius VIII 87 (Lasserre, Fragm. T 7; Loeb II, p. 402/403). This does not agree too well with the familiar story that it was at Plato's suggestion that Eudoxus undertook to explain planetary motions by means of uniform rotations (Simplicius, Comm., ed. Heiberg, p. 488, 18–24; p. 492, 31–493, 5; also Schiaparelli, Scritti II, p. 95f.; [1877], p. 182).

[12] What we know about the school of Eudoxus can all be found in Böckh, Sonnenkr., p. 150–159.

[13] Cf., e.g., Heath, GM I, p. 251ff.; II, p. 110ff.

[14] Theon Smyrn., ed. Hiller, p. 201, 25; Martin, p. 332 and p. 58f.; Dupuis, p. 326/327.

[15] Cf. below p. 683.

[16] Cf. above p. 623; also Heath, Aristarchus, p. 212.

[17] Cf., e.g., above p. 581, n. 13.

The influence of Eudoxus on rigorous mathematics, the theory of irrationals and of integration, lies outside of our theme[18]; but it is clear that he must be counted among the great founders of Greek exact sciences.

2. The Homocentric Spheres

A. The Eudoxan Model

We know about Eudoxus' model of planetary motion through a short summary by Aristotle[1] and through Simplicius' commentary (written around A.D. 540) on Aristotle's De caelo.[2] These sources provide us with a general description of the arrangement of the four concentric rotating spheres which Eudoxus assumed for each planet.[3]

How a cinematic model of this kind could represent planetary motions was probably completely forgotten during the Middle Ages and was fully recovered only in 1874 by the work of Schiaparelli.[4] In particular it was Schiaparelli who first determined the mathematical character of the "hippopede" (i.e. horse-fetter) as the ancients called the curve on which the planet performs its synodic cycle.[5]

We shall first demonstrate the geometrical properties of the hippopede, independent of all astronomical applications. Our proof is such an immediate consequence of the cinematic construction that I do not doubt that we have correctly restored here the essential trend of Eudoxus' argument. For the convenience of the reader I am using modern terminology but I shall show in a note (p. 678, n. 7) that all steps are fully within the reach of early Greek geometry.

All sources agree that the hippopede is generated by the motion of two concentric spheres which rotate with constant but opposite angular velocity about two inclined axes (cf. Fig. 23). A point (the "planet") on the equator of the innermost sphere (axis $\Xi O \Xi'$) describes the curve in question. The poles (Ξ and Ξ') of this sphere are fixed on the second sphere and therefore participate in its rotation about the axis $X O X'$ which we call, for the moment, the "vertical" axis.

We investigate separately the effects of these two components of the motion. First we consider the motion of a point on the equator of the inclined sphere through an angle α[6] from A to P_1. Looking down perpendicularly onto the plane

[18] Cf., e.g., Heath, Euclid I, p. 137; II, p. 112; II, p. 365 (et passim).

[1] Greek text: Metaphysics Λ(=XI), 8 (Opera, ed. Bekker, II, p. 1073b, 17–1074a, 14). Italian trsl.: Schiaparelli, Scritti II, p. 94f. (German: [1877], p. 180f.); English trsl.: Heath, Aristarchus, p. 194f.; p. 212; p. 217.

[2] Greek text: Simplicius. Comm. in Arist. De caelo, ed. Heiberg, p. 493–507. Italian trsl.: Schiaparelli, Scritti II. p. 95–112 (German: [1877]. p. 182–198); English trsl. of the major passages: Heath, Aristarchus, p. 201f.; p. 213; p. 221–223. Greek text and German trsl. of what is considered to be an Eudoxan "fragment" proper: Lasserre, p. 67–74.

[3] For the corresponding theory for sun and moon cf. above p. 624f. and p. 627.

[4] Schiaparelli, Scritti II, p. 3–112; German translation (with some changes): Schiaparelli [1877]. For earlier work on the homocentric spheres cf. Heath, Aristarchus, p. 194.

[5] For the modern discussion of the mathematical properties of the hippopede cf. Schiaparelli, Scritti II, p. 53–55 ([1877], p. 145–147) notes. Also G. Loria, Curve sghembe speciali algebriche e transcendenti (Bologna 1925), Vol. I, p. 199–201.

[6] Because of obvious symmetries we may restrict α to the first quadrant.

of the horizontal equator AOB (cf. Fig. 24) the inclined equator appears projected into the ellipse $AP_1'C$ with OA and OC as half major diameters (cf. also Fig. 25). The point P_1' is the projection of P_1; as point of an ellipse P_1' is the vertex of a right triangle $P_0P_1'Q$ where $AOP_0 = \alpha$ and $OQ = OC$.[7]

We consider secondly the rotation about the vertical axis XOX' by the same angle α but in the opposite direction such that P_1 comes into its final position P (cf. Fig. 23). The projection P' of P is again the vertex of a right triangle AP'R which is congruent to $P_0P_1'Q$ (cf. Fig. 26). Therefore P' is a point of a circle of diameter $AR = P_0Q = BC = XY$ (cf. Fig. 25) and center H such that $AHP' = 2\alpha$.

Thus we see, first, that a pair of rotating spheres arranged as shown in Fig. 23 or 27 produces as path for a point P of the equator of the innermost sphere an 8-shaped curve (called "hippopede") which is the intersection of the second sphere by a cylinder of diameter XY; secondly that the projection of P onto the equator of the second sphere moves with constant angular velocity on the base circle of the cylinder but twice as fast as the spheres rotate. In other words the projection of P traverses its circle once while P is on the upper loop of the hippopede and a second time for the lower loop.

One may add that geometric constructions of this type are familiar to Greek mathematics of the same period. This is evident, e.g., in Archytas' solution for the duplication of the cube by intersecting three surfaces of revolution, a torus, a cone, and a cylinder.[8] It should be mentioned in this context that Archytas is named as a teacher of Eudoxus.[9] We may also note in passing that the hippopede is obviously a special case of the algebraic curves of the 4th order which determine the boundary of the earth's shadow on the moon. To an observer on a far distant point of the axis of the hippopede this curve would appear as a circle. The same holds for the shadow limit at a lunar eclipse; contrary to Aristotle's opinion this proves nothing for the shape of the earth.[10]

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[7] It is not necessary to know that the curve AC is an ellipse. In order to determine the position of P_1' we first consider in the horizontal plane a rotation by the angle α from A to P_0. Then we tilt this plane about OA until the axis OX reaches the position $O\Xi$ (cf. Fig. 25; inclinations in these figures are not drawn to scale) and B comes to B_1. As a result B_1 is projected into C and P_0 moves into P_1' such that $P_0P_1'G$ is perpendicular to OA.

In order to find the distance P_0P_1' we note in Figs. 24 and 25 that $DE = P_0P_1'$. Let Q be the point where $P_1'E$ intersects the radius OP_0. Then

$$\frac{OE}{OQ} = \frac{OD}{OP_0} \tag{1}$$

and

$$\frac{OE}{OD} = \frac{OE}{OF} = \frac{OC}{OB_1} = \frac{OC}{OP_0}$$

hence with (1)

$$\frac{OE}{OC} = \frac{OD}{OP_0} = \frac{OE}{OQ}$$

and thus $OQ = OC$. Hence the hypotenuse P_0Q of the right triangle $P_0P_1'Q$ is always of constant length $BC = XY$.

[8] Cf., e.g., Heath GM I, p. 246–249.

[9] According to Diogenes Laertius VIII 86 (Tannery HAA, p. 295; Loeb, Diog. L. II, p. 400/401) on the authority of Callimachus, librarian of the Museum in Alexandria in the third century B.C.

[10] Cf. below p. 1093.

We now turn to the astronomical applications, assuming that the two innermost spheres produce the motion of the planet on the hippopede. This motion with its oscillations between two extrema is reminiscent of the synodic motion of a planet between its stations. But the synodic cycles also participate in a continuous sidereal progress. Hence Eudoxus introduces a third sphere which moves the hippopede with constant velocity along the ecliptic which is made to coincide with the line of symmetry of the curve (SA in Fig. 28). Consequently B is one of the poles of the ecliptic and BO is the axis of the third sphere which carries the two previous ones from west to east with the speed of the sidereal mean motion of the planet (as in the later theory the epicycle moves along the deferent). Finally the sphere of the ecliptic is set into the sphere of the fixed stars which performs the daily rotation of all celestial bodies. This completes the Eudoxan model of the motion of a planet; no attempt is made to connect the spheres of different planets or to introduce assumptions about their geocentric distances.

Though it is quite obvious that the combination of the planet's motion on the hippopede with a properly chosen uniform translation along the ecliptic can produce paths reminiscent of the apparent motion of a planet it is equally obvious that serious discrepancies with easily observable facts must remain. It is clear, e.g., that the Eudoxan model will repeat in a strictly congruent fashion and at equidistant points of the ecliptic whatever path is the result of the combined motion of the three innermost spheres. This disagrees strikingly with the variety of shapes seen in the retrograde arcs of the planets.[11]

Simplicius says that the width of the hippopede represents the motion in latitude of the planet.[12] This again contradicts basic empirical facts. The latitudinal variation in the synodic loops is obviously different from the slow change of latitude, once in each sidereal period, along the mean orbit of the planet.[13] For this phenomenon there is no room in the Eudoxan model. As we hear from Simplicius[14] this shortcoming of the theory was noticed in antiquity.

It has been shown by Schiaparelli[15] that the numerical data for the synodic and for the sidereal periods are such that the Eudoxan model cannot lead to retrogradations for Mars and Venus. If, as it seems, the ancient astronomers overlooked this fact — at least our sources do not mention it — this would be a confirmation of our impression that empirical data were not used to test the numerical consequences of the general cinematic construction.

The argument against the incorrect representation of latitudes could have been easily countered on numerical grounds. The actual retrogradations of the planets are not very large; consequently the angles γ for the hippopedes must be small and therefore the diameters XY of the cylinders which carry the hippopedes are very small indeed.[16] Hence the latitudes produced by the motion on the hippopedes are almost negligible and one could simply have said that the Eudoxan model was designed only to explain the origin of the true longitudes from the

[11] Cf. the examples shown in Figs. 228, 233; in Fig. 35; in V C 4, 5 B 2, Figs. 115 and 116.
[12] Simplicius, Comm., ed. Heiberg, p. 497, 4; Schiaparelli, Scritti II, p. 100, [1877], p. 186.
[13] Cf., e.g., Figs. 155 and 156.
[14] Simplicius, Comm., ed. Heiberg, p. 497, 5; cf. above note 12. Also Heath, Aristarchus, p. 202.
[15] Schiaparelli, Scritti II, p. 70ff., p. 74ff.; [1877], p. 159ff.; cf. below p. 683.
[16] Cf. below p. 682.

mean ones. Our sources give no indication that this line of defense had ever been taken. Again one could take this in support of the assumption that Eudoxus did not associate numerical details with his model.

B. Numerical Data

The main cause for the difficulties which the Eudoxan model encounters in its attempt to represent the various phenomena of planetary motion lies in the fact that one has only one free parameter at one's disposal, e.g. the angle γ between the axes $\Xi O \Xi'$ and XOX', or the equivalent parameter (cf. Figs. 25 and 28)

$$XY = BC = AR = 2r \tag{1}$$

i.e. the diameter of the cylinder which carries the hippopede. Assuming always OA = 1 one has

$$r = 1/2(1-\cos\gamma). \tag{2}$$

Let P be a point of the curve (cf. Fig. 29). The latitude β of P is an arc PP_e of the great circle whose plane contains OP and which is perpendicular to the plane of the ecliptic AOX.[1] Consequently

$$\sin\beta = PP'' = P'U. \tag{3}$$

The latitude reaches its greatest value when $P'U = r$, i.e. for $\alpha = 90°$ (cf. Fig. 30); consequently

$$\tan\beta_{max} = \frac{r}{1-r} = \frac{1-\cos\gamma}{1+\cos\gamma}. \tag{4}$$

The longitudinal component of the motion of the planet P on the hippopede reaches its maximum velocity v_0 near A. We can obtain an estimate of its value in the following fashion (cf. Fig. 31). Let λ_0 be the increment of longitude near A during a small time interval t_0 during which the projection P' of P moves on the base of the cylinder through an angle α_0, starting from A. Since the angular velocity of P' is twice the angular velocity of each of the two spheres we have

$$t_0/\Delta t = \alpha_0/2 \cdot 360° \tag{5}$$

where Δt is the synodic time during which P traverses once the whole hippopede. If $\lambda_0 = AP_e$ is small we have (cf. Fig. 29)

$$AP_e \approx QP \quad \text{i.e. } \lambda_0 \approx h_0.$$

[1] The longitude λ of P, reckoned from A, can be found from the right spherical triangle PP_eX (Fig. 29). The altitude h of P as seen from O is given by

$$\cos h = OP' = \sqrt{OU^2 + P'U^2}$$

with

$$OU = 1 - r + r\cos\alpha, \quad P'U = r\sin\alpha.$$

Hence

$$\lambda = AP_e = 90 - \bar{\lambda}$$

with

$$\tan\bar{\lambda} = \cos\eta \tan\bar{h}, \quad \cos\eta = \frac{OU}{OP'}.$$

Thus with (5)

$$v_0 = \lambda_0/t_0 = h_0/t_0 = \frac{h_0}{\alpha_0} \cdot \frac{720}{\Delta t}.$$

But

$$\cos h_0 = 1 - r + r \cos\alpha_0 \quad \text{or} \quad 1 - 1/2\, h_0^2 = 1 - r + r(1 - 1/2\,\alpha_0^2)$$

hence

$$h_0 \approx \sqrt{r} \cdot \alpha_0$$

and thus

$$v_0 \approx \sqrt{r} \cdot \frac{720}{\Delta t} \tag{6}$$

for the velocity at which the planet traverses either in direct or in retrograde motion the double-point A of the hippopede.

The final motion of the planet is the sum of the motion on the hippopede and of the sidereal mean motion along the ecliptic. For obtaining retrogradations it is obviously necessary and sufficient that the sidereal mean motion is of a smaller amount than v_0 given in (6). We shall now investigate this relation which is crucial for the Eudoxan theory.

For the synodic times Δt Simplicius reports[2] the following values:

Saturn and Jupiter:	≈ 13 months	(7)
Mars:	8 months and 20 days	
Venus:	19 months	
Mercury:	110 days[3].	

The value for Mars is obviously wrong since it is only one third of the actual synodic period of about 780 days (i.e. 26 months). It makes no sense to retain such a parameter for our subsequent discussion because this would mean assuming that Eudoxus counted, e.g., three oppositions of Mars in an interval that actually contained only one. Hence we shall operate with the following estimates for Δt, reckoning as usual a month schematically as 30 days[4]:

Saturn and Jupiter:	390 days	(8)
Mars:	[780]	
Venus:	570	
Mercury:	110.	

For the sidereal mean motions we have no accurate data in Simplicius beyond the commonly used round values for the sidereal periods[5]:

Saturn: 30 years, Jupiter: 12 years, Mars: 2 years (9)

and, of course, one year for the inner planets. Since these data are basically correct estimates for the daily mean motions κ, deducible from (9), must also be nearly

[2] Simplicius, Comm., ed. Heiberg, p. 496, 6–9; Schiaparelli, Scritti II, p. 99; [1877], p. 185.

[3] Lasserre in his translation (p. 72) gives 103 days by mistake.

[4] Examples for this reckoning are collected in Heath, Aristarchus, p. 285f.

[5] Simplicius, p. 495, 26–28; Schiaparelli II, p. 98/99; [1877], p. 185. The same parameters are also found, e.g., in the "Eudoxus Papyrus" (cf. below p. 688).

correct. Hence we may assume the following velocities [6]:

$$\text{Saturn: } \kappa \approx 0;2^{\circ/d}, \quad \text{Jupiter: } \kappa \approx 0;5^{\circ/d}, \quad \text{Mars: } \kappa \approx 0;30^{\circ/d} \tag{10}$$

and $1^{\circ/d}$ for Venus and Mercury.

For the moment we restrict our investigation to the outer planets. We must first make some estimates for the inclinations γ between the axes such that one obtains a reasonable agreement with the observable arcs of retrogradation $\Delta\lambda_r$. Such data are not available in Simplicius or Aristotle but one may assume values about as follows [7]:

$$\text{Saturn: } 7^\circ, \quad \text{Jupiter: } 10^\circ, \quad \text{Mars: } 16^\circ. \tag{11}$$

The motion of the planet on the hippopede alone would produce a retrograde arc of 2γ during $1/2\,\Delta t$. From (9) it follows that during one synodic period Δt the sidereal progress of the planet will be given by the following arcs $\Delta\lambda$:

$$\text{Saturn: } 12^\circ, \quad \text{Jupiter: } 30^\circ, \quad \text{Mars: } 6{,}0^\circ.^{8} \tag{12}$$

Since the retrograde motion on the hippopede lasts during $1/2\,\Delta t$ the retrograde arcs $\Delta\lambda_r$ should satisfy

$$\Delta\lambda_r = 2\gamma - 1/2\,\Delta\lambda. \tag{13}$$

Consequently, from

$$\gamma = 1/2\,\Delta\lambda_r + 1/4\,\Delta\lambda \tag{14}$$

and using the empirical data (11) and (12) one finds

$$\begin{aligned} &\text{Saturn: } \gamma = 3;30 + 3 = 6;30^\circ \\ &\text{Jupiter: } \gamma = 5 + 7;30 = 12;30^\circ \\ &\text{Mars: } \quad \gamma = 8 + 1,30 = 1,38 > 90^\circ. \end{aligned} \tag{15}$$

Substituting these values in (2), p. 680 gives for the radius r of the cylinder of the hippopede (for $OA = 1$):

$$\begin{aligned} &\text{Saturn: } r = 0;0,12 \\ &\text{Jupiter: } r = 0;0,43 \end{aligned} \tag{16}$$

but for Mars the meaningless value $r = 0;34 > 1/2$.

Returning to (6), p. 681 we have according to (8) for Saturn and Jupiter

$$\frac{720}{\Delta t} = \frac{12,0}{6,30} = 1;50^{\circ/d}.$$

and with (16) and (10)

$$\begin{aligned} &\text{Saturn: } v_0 \approx 0;3 \cdot 1;50 = 0;5,30^{\circ/d} > 0;2^{\circ/d} = \kappa \\ &\text{Jupiter: } v_0 \approx 0;7 \cdot 1;50 = 0;12,50^{\circ/d} > 0;5^{\circ/d} = \kappa. \end{aligned} \tag{17}$$

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[6] Cf., e.g., the tables in Alm. IX, 4.

[7] Cf. for Ptolemy's estimates at mean distance above p. 193. These values also agree with averages obtainable from Babylonian schemes (cf. ACT II, p. 315, p. 312, p. 305, respectively).

[8] Even crude observations of Mars during one or two decades should have given a greater value, e.g. $\Delta\lambda \approx 6{,}50^\circ$, hence about $\gamma \approx 1{,}50^\circ$ in (15).

Hence it is certain that the Eudoxan model can be so normed that it produces a retrograde loop in each synodic period of Saturn and Jupiter. For Mars, however, the model fails to provide us with a possible value of r.[9]

For the greatest latitude one obtains from (16) and (4), p. 680

$$\text{Saturn: } \beta_{max}=0;11^\circ, \quad \text{Jupiter: } \beta_{max}=0;42^\circ \tag{18}$$

i.e. these two planets would only vibrate in a narrow zone along the ecliptic. If one would arbitrarily assign Mars a large but permissible value of γ, e.g. $\gamma=60^\circ$, one would find $\beta_{max}\approx 18;30^\circ$. Hence the planet would move in each synodic period on an S-shaped curve (but without retrogradation!) between latitudes $\pm 18;30^\circ$ in flagrant contradiction to the observable facts. For the faulty value $\Delta t=260$ days given in the text (cf. (7)) one would find $\gamma\approx 30^\circ$ hence $r\approx 0;4$ and then $v_0\approx 0;45^{\circ/d} > 0;30^{\circ/d}=\kappa$, hence the possibility for retrogradation (and $\beta_{max}\approx 4^\circ$). The price one would have to pay for this success is the assumption of three synodic cycles where there is actually only one. I do not think one should accuse Eudoxus of such total disrespect for obvious facts in order to uphold one number in a text composed centuries later. But we must conclude that he did not, or could not, design numerical parameters which should have made his model work.

For Venus the situation is as hopeless as for Mars. The synodic arc of Venus is about $360^\circ+220^\circ=9{,}40^\circ$, thus $1/4\,\Delta\lambda\approx 2{,}25^\circ$. The retrograde arc is about 16° thus $1/2\,\Delta\lambda_r=8^\circ$; hence (14) requires the inclination $\gamma\approx 2{,}33^\circ>90^\circ$ which is meaningless.

To determine suitable numerical parameters for Mercury is hardly worth the effort since one would have to make too many arbitrary assumptions. Schiaparelli, e.g., operated with the maximum elongations as a relatively easily observable parameter which could lead to an estimate for the amount of retrogradation. In this way he obtained a hippopede and a retrogradation on an S-shaped curve.[10] But even so the inner planets under no circumstances could have been of use to Eudoxus in providing specific data for his general model. Thus the whole theory of homocentric sphere rests for empirical confirmation at best on two planets alone, Saturn and Jupiter.

C. Later Modifications

We know about two major attempts to modify the Eudoxan model of homocentric spheres. One was made by Callippus (in Athens, around 330 B.C.) who increased the number of spheres, presumably in order to eliminate some of the shortcomings of the original arrangement. The second was made by Aristotle who accepted the Callippic increase in the number of spheres and combined them into one single structure of some 55 concentric spheres. These nested spheres give the impression of a unified cinematic model of the cosmos. In fact, however, there is no mechanism provided which could transmit the motion of one sphere to the next and one must assume that each single sphere is endowed with its own law of motion.

[9] This was stated, for Mars and Venus, from the very beginning by Schiaparelli (Scritti II, p. 70-72; [1877], p.159-161).

[10] Schiaparelli, Scritti II, p. 73f.; [1877], p. 162. Similar curves are also shown in Hargreave [1970], p. 342-345 but the numerical discussions are without interest for the historical problem.

It would be interesting to know the modifications suggested by Callippus. Unfortunately all that is preserved in the story told by Simplicius [1] is the increase in the number of spheres by two each for sun and moon [2] and by one each for the planets, excepting Jupiter and Saturn. This exception is annoying since it eliminates the simple suggestion that the motion in latitude of the two outer planets could have been improved by the addition of one sphere in order to make the motion along the ecliptic (produced by the new sphere) independent of the motion along the orbital plane, i.e. the line of symmetry of the hippopede. But apparently Callippus was of the opinion that producing retrogradations was all one had to ask for, regardless of the insufficient representation of the latitudes.

For Mars and Venus the Eudoxan model failed to produce retrogradation and it seems plausible to assume that the additional sphere had been introduced to repair this deficiency. Unfortunately we have no information about the details of Callippus' construction. Schiaparelli suggested an arrangement [3] which produces retrogradations at least for Mars; Venus remains incorrigible. The amplitude for the latitudes can be made reasonable in amount but both extrema are reached four times in each synodic period since the path of the planet has two triple points near the ends of an 8-shaped main branch. [4] All this does not sound very convincing and it seems to me it would be better to admit our total ignorance of the character of Callippus' modifications of the Eudoxan model.

So far the spheres of Eudoxus or of Callippus remained independent from planet to planet, exactly as in the Almagest where the cinematic model for each planet is only designed to produce correctly, as function of time, the coordinates λ and β of the specific planet. But just as Ptolemy had the unfortunate idea of constructing a system of contiguous spheres which enclosed the deferents and epicycles of each planet [5] so too did Aristotle when he designed a sequence of concentric spheres which connected the Callippic spheres of one planet with the spheres of the next one; each sphere provides the support for the next one below it. But while Ptolemy's cosmic machinery is based on geocentric distances the Aristotelian spheres follow one another without consideration of actual distances, hence implicitly contradicting another Aristotelian hypothesis according to which a planet's distance is proportional to its sidereal period. [6] There is no mechanism which would transmit the rotation of one sphere to the next below it;

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[1] Simplicius had it from Sosigenes (2nd cent. A.D., the teacher of Alexander of Aphrodisias, not Caesar's contemporary) who had it from Eudemus' "History of Astronomy" (4th cent. B.C.; cf. Simplicius, Comm. ed. Heiberg, p. 488, 18–21; Lasserre Fragm., p. 67 (F. 121); Schiaparelli, Scritti II, p. 95; [1877], p. 182). For the complicated relations between these sources (including Aristotle) cf. Schramm, Haitham. p. 32–63. The difficulties are increased by a gap in the text of Simplicius (cf. Schiaparelli, Scritti II, p. 101; [1877], p. 187).

[2] Cf. for sun and moon above p. 625 and p. 627, respectively.

[3] Schiaparelli, Scritti II, p. 79–82; [1877], p. 167–169. For his model he assumes the greatest permissible inclination between the two axes XX' and $\Xi\Xi'$, i.e. $\gamma=90°$. The planet P, however, is not located on the equator of the sphere with axis $\Xi\Xi'$ but on the equator of the new innermost sphere whose axis ZZ' is inclined to $\Xi\Xi'$ and rotates about it with twice the angular velocity of XX' with respect to $\Xi\Xi'$. The motion starts when $XZ\Xi P$ (in this order) are located on the same great circle (which then becomes the line of symmetry of the curve and represents the ecliptic).

[4] Schiaparelli, Fig. 19.

[5] Cf. below V B 7, 7.

[6] Cf. below p. 691.

each sphere must be assumed to know how it should rotate and the "Prime Mover" remains a useless figurehead, helpless and bewildered as all omnipotent creators.

Aristotle's idea is in principle very simple.[7] The motion generated by one set of spheres can be transformed to rest with respect to the sphere of the fixed stars by compensating each rotation by an exactly opposite rotation. Having achieved that much the next set of planetary spheres can again be put into motion according to its own rules, uninfluenced by the rotations of the preceding planet. As we shall see presently $n-1$ compensating spheres are necessary to eliminate the effect of n spheres. Hence one needs 3 compensating spheres for Saturn and Jupiter and 4 each for the remaining planets and the sun. Thus we have a total of $2\cdot 3+4\cdot 4=22$ compensating spheres until we reach the outermost sphere of the moon which remains without counteracting spheres. Since Callippus requires $2\cdot 4+5\cdot 5=33$ spheres Aristotle operates with $33+22=55$ spheres to describe the planetary motions, including sun and moon.[8]

To understand the working of the compensating spheres one can use Fig. 32 for the case of Saturn. The planet is mounted on the equator of the innermost sphere (D) which rotates about the axis ΞO. Combining this motion with the opposite rotation of the sphere C about XO produces the hippopede. The sphere B carries X with the sidereal mean motion along the ecliptic (about an axis which is perpendicular to the plane of our drawing) and sphere A provides the daily rotation of the sky. So far we have only the Eudoxan model for Saturn. Now we eliminate its effect by placing inside of D a sphere D′ which rotates about the axis ΞO with a velocity opposite to D. Hence D′ behaves as if it were in a fixed position with respect to C. Thus a sphere C′ rotating inside D′ around an axis in the direction XO, in a sense opposite to C with respect to B, brings C′ into a fixed position with respect to B. By attaching C′ to a last sphere B′ which rotates about the axis of the ecliptic one can eliminate the sidereal motion of the planet. As a result the last sphere (B′) represents again the motion of A, i.e. the daily rotation. Now one can put inside of B′ the Eudoxan spheres of Jupiter, and so forth. This leads to the above mentioned 55 spheres. Schiaparelli has rightly observed that B′ of Saturn could be used as outermost sphere (A) for Jupiter, etc., which brings down the total of spheres to $55-6=49$.

The system of homocentric spheres could, of course, never reach practical astronomical importance. As ideal prototype of strictest geocentricity, however, its influence was felt deep into the Renaissance. And as a piece of mathematical ingenuity Eudoxus' construction of the hippopede has rightly been admired ever since Schiaparelli recovered its meaning.

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[7] Arist., Metaphys. Λ (Opera II, Bekker, p. 1073^b, 38–1074^a, 14); also Simplicius, Comm. ed. Heiberg, p. 497ff.; Schiaparelli, Scritti II, p. 101–112; [1877], p. 187–198.

[8] For some ancient difficulties with the count of spheres cf. Simplicius, p. 503, 10–504, 3; Schiaparelli, Scritti II, p. 107f.; [1877], p. 193f.

3. The "Eudoxus Papyrus"

A. The Text

The first Greek papyrus which reached a European collection was found in 1778 near Gizeh[1] and was published in Rome ten years later. It took four more decades until larger collections of papyri were started, in part by private persons who had connections with Egypt, in part by the great Museums. The Paris collection was begun around 1826. Some 70 texts were published in 1865 in Vol. 18, 2 of the "Notices et Extraits"; among these, as No. 1 (hence now called P. Par. 1, at present in the Louvre), an astronomical text with an acrostic of 12 lines, spelling out *Εὐδόξου τέχνη* "Art of Eudoxus." Each of the first 11 lines contains 30 letters, the last 35, hence the total of 365, i.e. the number of days in the "*μέγας χρόνος*," the Egyptian year.

Beside, and written around the acrostic, the verso contains different administrative documents, written by later hands but as early as 165 B.C. Of the astronomical treatise on the recto 24 columns are preserved, including the end. The beginning is lost; how much is difficult to say. At present the length of the roll is almost 2 meters. The acrostic on the verso is located where columns VII/VIII are written on the recto. For the title of a roll one would expect a position much nearer to the beginning.

Bibliographical Data:

First published in Not. et Extr. 18, 2 (1865), p. 25–76; also published separately under the title: Les papyrus grecs du Musée du Louvre et de la Bibliothèque impériale, Paris 1866 (pages unchanged). Separate folio volume with facsimile: Planches, Paris 1865, Plates I–VI and IX.

Second, improved edition of the text, with Latin translation: F. Blass, Eudoxi ars astronomica..., Program Univ. Kiel, 1887, p. 1–25.

Col. XXI, 5–XXIII, 14: contained in Wachsmuth, Lydus, 1st ed. (1863), p. 273–275 and p. LVIII–LX, 2nd ed. (1897), p. 299–301 and p. LIX.

Acrostic, with English transl.: Page, Greek Literary Papyri I (Loeb Cl. L., 1952), p. 466–469.

French transl. of the whole text: Tannery, HAA, p. 283–294 (1892).

Discussion: Boeckh, Sonnenkr., p. 196–226 (1859–1863); Tannery, Mém. Sci. II, p. 407–417 (1889). Many passages quoted by Letronne, Journal des Savants 1840, p. 741–750; 1841, p. 65–78, p. 538–547.

Figures: shown in color in the facsimile[2]; in modernized form also with the printed text in Not. et Extr. 18, 2; not reproduced by Blass; selection in Weitzmann, Illustrations in Roll and Codex (Princeton 1947), Fig. 37 (cols. VI to XIV) and in Ancient Book Illumination (Harvard 1959), Fig. 2 (cols. VII to X).

Blass discovered that our treatise originally had the form of a didactic poem; the original metre is still preserved in several sections.[3] The name of the author

[1] With this find is connected the famous story of the Arabs burning papyri for the pleasant smell of the smoke (Not. et Extr. 18, 2, p. 6, note 1).

[2] The only color used is some reddish brown, indicated by shading in the edition, appearing dark on our photograph Pl. VII, p. 1453.

[3] Blass, p. 4f. (Nos. 27 and 50 in Tannery's count). Suidas (ed. Adler, II, p. 445 = Lasserre T 8) says that Eudoxus had composed a poem called "Astronomy".

(or final redactor) and an original title is perhaps contained in the last words of the text[4]: "To the kings (of Egypt), instruction on the heavens by Leptines...". We have no other information about this Leptines.

In col. XXII of the text[5] one finds the remark that according to Eudoxus and Democritus the winter solstice falls on the 20th or 19th of Athyr, a correspondence which is correct for some years around -190.[6] This would be the date the papyrus was written. The close parallels in our text with P. Hibeh 27 which was written about -300 suggests a similar date for the original version of the "Eudoxus Papyrus." This would be close to the time of Callippus who is mentioned near the end of the text for his parameters of the lengths of the seasons.[7]

The arrangement of the subject matter in the papyrus is not very orderly and several topics are mentioned twice in almost identical wording but columns apart.[8] Similarly some of the illustrations are also repeated and most of them are obviously out of place. The conclusion that the extant text is a rather careless compilation based on two similar prose renderings (or abridgements) of the original composition seems to me to be inescapable.

B. Summary of Contents

If there were any specific Eudoxan features in the "Eudoxus Papyrus" they should be recognizable in the planetary theory. In fact, however, there is no trace of the homocentric spheres visible in our text nor do the synodic periods for the inner planets agree with the values known from Simplicius. Without the acrostic nobody would have suggested any direct relation of the papyrus to Eudoxus or to his pupil Callippus.

All the text has to say about the motion of the planets is that their paths form spirals (ἕλικα περιφέρονται) because their distance from the fixed poles of the celestial sphere is variable; the same holds for sun and moon[1]; in contrast all fixed stars move on parallel circles. There is no word about retrogradations.

In col. V one finds for the synodic periods of the inner planets:

$$\begin{array}{ll} \text{Venus:} & \text{1 year 3 months 4 days} \\ \text{Mercury:} & \text{3 months [2]6 days.} \end{array} \qquad (1)$$

The number for Venus, $365+94=459$ days is certainly incorrect but no simple emendation seems possible.[2] At any rate neither number in (1) agrees with the

[4] A second hand wrote in the space between these lines "Work, you men, in order not to work" and "Oracles of Serapis" and "Oracles of Hermes". The words "Oracles of Serapis" are also found at the end of the preceding column (XXIII) and inside the zodiac in col. XXIV.
[5] No. 54 in Tannery's count (HAA, p. 294).
[6] Cf. above p. 600.
[7] Tannery No. 55 (HAA, p. 294); cf. also above p. 627.
[8] Adopting Tannery's sectioning of the text one has the following parallels: No. 1/No. 53; No. 7/Nos. 34, 35, 40; No. 9/No. 41; Nos. 26, 27/Nos. 42, 50; No. 39/No. 44.
[1] Tannery No. 27 and No. 50. We have seen that these sections belong to the earliest form of the treatise; cf. above p. 686, note 3.
[2] Tannery (p. 286) replaces 3 by 7 months but the text has words, not alphabetic numerals; nor is the result, 579 days, attested elsewhere.

parameters mentioned by Simplicius[3]

Venus: 19 months, Mercury: 110 days.

For the outer planets the papyrus and Simplicius agree on the sidereal periods of 30, 12, and 2 years, respectively, values which are much too common to establish any significant connection. No synodic periods are mentioned in the papyrus.

On the whole the "ars Eudoxi" is a very elementary treatise. It begins and ends with a few data from a parapegma,[4] including a list of the lengths of the seasons according to Democritus, Euctemon, Callippus, and Eudoxus.[5] A linear scheme for the variation of the length of daylight is based on the parameters for Lower Egypt[6] ($m=10^h$, $M=14^h$). After remarks about the lengths of the solar and lunar year (365 and 354 days, respectively) one finds a garbled description of the intercalation pattern of the octaeteris.[7] Both for the sun[8] and for the moon[9] mean values for traversing one zodiacal sign are given.

Finally some simple concepts of the celestial sphere are listed (horizon, equator, etc.) and some data about the visibility of fixed stars are mentioned, e.g. a delay of 1/2 sign after sunset.[10]

The section about eclipses is historically most interesting. Here we are told that solar eclipses can never be total but must remain at most annular.[11] This could be the result of the observation of annular eclipses by Polemarchus,[12] the younger contemporary of Eudoxus.[13]

Comparison of P. Par. 1 with P. Hibeh 27[14] seems to suggest a better ultimate source from which P. Par. 1 was derived. Two passages from the introduction of P. Hibeh 27 (lines 28–33 and 41–54) are found in almost exactly the same wording also in P. Par. 1 but far apart and in inverse order[15] (col. XXI No. 51 and col. III No. 8, respectively). Furthermore, P. Par. 1 begins with a parapegma, followed by a description of the variation in the rising amplitude of the sun during the year and the corresponding (linear) change in the length of daylight. This could be con-

[3] Cf. above p. 681 (7).
[4] This parapegma does not agree with what has been reconstructed as Eudoxus' parapegma; cf. Rehm, RE Par. col. 1322, 37–1323, 51.
[5] Cf. above p. 627.
[6] Cf. below p. 706 and p. 710.
[7] Cf. above p. 620.
[8] The sun is said to remain in each sign 30 days and 5 hours, supposedly as a result of the division by 12 of 365. This implies that one "hour" (ὥρα) is taken as 1/12 of one day. Similarly in No. 37 (Tannery HAA, p. 290; Lasserre F 128) 1/2 zodiacal sign is equated to 1/2 hour. Since Letronne (Journal des Savants 1839, p. 585–587; Blass, p. 8; Boll, Sphaera, p. 313, etc.) this has been considered as evidence for the use of the Babylonian "double-hours" (danna). To me this seems very implausible on general historical grounds. I would consider a simple arithmetical error of the redactor of our text much more likely; in particular, since a section from the original poetic version (No. 3, on the variation of the length of daylight) uses ὥρα for the equinoctial hours.
[9] For the moon 2 1/4 days per sign (Tannery No. 41 and No. 9).
[10] Tannery Nos. 37–39, No. 44.
[11] Tannery No. 49.
[12] Cf. Simplicius, Comm. p. 505, 3–23 ed. Heiberg; Schiaparelli, Scritti II, p. 109 f., [1877], p. 195 f.
[13] For the problem of relative size and distances of the luminaries cf. above IV B 3.
[14] Cf. above p. 687.
[15] This fits in with our observation of duplications in the contents (cf. above p. 687).

sidered an excerpt or summary of cols. IV to XII of the Hibeh papyrus which combines in a running consistent text a parapegma, a festival calendar and the same linear daylight scheme we have in P. Par. 1.[16] In the introduction to P. Hibeh 27 a "wise man from Sais" is named as the source of the subsequent teaching, a friend of the author who also lived in Sais for five years. All this sounds much more factual than the reference to Eudoxus in the acrostic of P. Par. 1.

A few remarks should be added about the illustrations in P. Par. 1. At first sight these 32 figures[17] seem totally unrelated to the text and inexplicable. Some have indeed no relation to the extant text, e.g. there is no mention of "Scorpio" to which Fig. 12 on Pl. VII could belong.[18] This again suggests the existence of a more complete archetype. Other figures, however, can be interpreted as belonging to our text, although usually somewhat out of place. Fig. 14 (col. X), e.g., shows a partial solar eclipse, thus belonging to the text of col. XVIII or XIX. The shading in this figure is perhaps opposite to what we would expect and also the words "sun" and "moon" would be better interchanged. Fig. 15 illustrates (sloppily) the assumption that the diameter of the earth's shadow covers two lunar diameters, a statement not found in the present text but well known from Aristarchus[19] and Posidonius.[20]

Particularly crude are the Figs. 11 and 19 which must be read as if consisting of ideograms: "earth-sun-moon in a straight line" (i.e. referring to a syzygy) – without giving the actual geometric order. In Fig. 11 the eclipsed body is the moon (again the visible part is shaded); in Fig. 19 the sun is shown at an annular eclipse – in agreement with the above mentioned text in col. XIX.[21] In both cases the earth is provided with a little shadow cone, pointing to the right. I do not know whether it is significant that the circle of the earth has sometimes an unshaded diametrical zone (Fig. 11), or is divided into six sectors (Figs. 16 and 19), two of which are usually shaded. This, and many other details, escape me but it seems likely that the figures, perhaps a little less crude, made sense in a larger original version.

To sum up: it seems to me very unlikely that P. Par. 1 has anything specific to do with Eudoxus, i.e. as little as P. Hibeh 27. Both texts illustrate the level of early hellenistic astronomy in Egypt; they were probably written for didactic purposes and therefore operate only with the simplest arithmetical and geometrical concepts,[22] far removed from observational accuracy or theoretical refinement. The Eudoxan astronomy was probably not much farther advanced in observations and numerical details. Its historical importance lies in the attempt to use cinematical models for the description of planetary motions, holding out some promise for an ultimate success.

[16] Cf. below p. 706.
[17] Cf. above p. 686. I have numbered all figures consecutively; some are still better preserved on the facsimile. Nos. 11 to 19 are shown on Pl. VII; this plate is based on photographs which could not be exactly matched to the text in col. XIII.
[18] As shown on p. 600 we have probably a reference here to the solar longitude in the month Thoth. using the Eudoxan norm for the zodiacal signs.
[19] Cf. p. 635 (4) and (5).
[20] Cf. p. 654 (7).
[21] No. 49. Another representation of an annular eclipse (why 3 concentric circles?) is found in Fig. 26, col. XVIII.
[22] Neither text nor figures refer to astrological doctrines.

§ 2. Other Planetary Hypotheses

1. Arrangement of the Planets

The order in which the planets are enumerated does not necessarily imply a corresponding arrangement according to distance. In particular neither the Babylonian nor the Egyptian sources suggest the existence of any theory of relative planetary ordering in space. The sequence commonly (but not always) used in Egyptian texts[1] is

$$♃ \quad ♄ \quad ♂ \quad ☿ \quad ♀. \tag{1}$$

It is quite different from the order occasionally called "Egyptian" by Greek or Roman authors which is only a variant of the Greek order.[2]

We know of two Babylonian sequences, an older one

$$♃ \quad ♀ \quad ♄ \quad ☿ \quad ♂ \tag{2}$$

and a younger one, the latter one in common use during the Persian-hellenistic period,[3]

$$♃ \quad ♀ \quad ☿ \quad ♄ \quad ♂ \tag{3}$$

in which, for unknown reasons, the positions of Saturn and Mercury are interchanged. Boll observed that (2) underlies the scheme of the astrological "terms" (ὅρια) that is called the "Chaldean" system[4] in Ptolemy's "Tetrabiblos." Fig. 33 describes this pattern: In ♈, ♌, and ♐ Jupiter rules the first 8°, Venus the next 7°, etc. For the next "triplicity" of signs, ♉, ♍, and ♑, Venus takes the lead with the first 8°, etc., always following the order (2). To give all five planets first place once, although there are only four triplicities available, the third one is divided between Saturn and Mercury in day- and night-rulership. The assignment of the degrees is made in good Babylonian spirit: strictly linear, with difference 1, thus for each sign

$$8+7+6+5+4=30°.$$

This does not mean, however, that the whole pattern is of Babylonian origin: on the contrary we have no reason to assume that the doctrine of "terms" is not an invention of hellenistic astrology,[5] only using a standard Babylonian sequence for the planets. Boll has also seen that in the "Egyptian" modification[6] of the pattern of terms the sequence (2) is used for the terms in Leo, the sequence (3) for Sagittarius. In Ptolemy's own system,[7] however, only (3) is attested (in ♏ and ♐); it is perhaps only accidental that Virgo follows the Egyptian sequence (1). The remaining signs, both of the Egyptian and of the Ptolemaic pattern, show all

[1] Cf. Neugebauer-Parker, EAT III, p. 175 ff. (cf. also the index).
[2] Cf. below p. 692.
[3] Discovered by Kugler; cf. for (2): SSB I, p. 9, p. 11; for (3): p. 40. Conveniently listed in Boll [1913], p. 342.
[4] Boll-Boer, p. 47, 17 ff.; Robbins I, 21 (Loeb, p. 98–101).
[5] We shall meet similar sequences, e.g., in the hellenistic shadow tables; cf. below p. 739 (5).
[6] Tetrabiblos, ed. Boll-Boer, p. 47 top; Loeb, p. 97.
[7] Boll-Boer, p. 51/52. The list of the Loeb edition (p. 107) has to be modified on the basis of the apparatus in Boll-Boer.

kinds of permutations in the order of the planets; also the degree intervals no longer follow a simple scheme. Obviously increasing speculation (or fraud) gained the upper hand over the transparent pattern shown in Fig. 33.

Eventually men "competent in heavenly phenomena"[8] accepted a sequence which meant real geocentric distances:

♄ ♃ ♂ ☉ ♀ ☿ ☾. (4)

It is the sequence of the deferents in Ptolemy's cosmic system[9] but it existed long before Ptolemy. Cicero,[10] Vitruvius,[11] Pliny,[12] and Plutarch[13] take it for granted.[14] From it is derived the sequence of the days of the week that in turn became the order of the planets in Indian astronomy[15]:

♂ ☿ ♃ ♀ ♄ ☉ ☾. (5)

The original order, indicating distances, thus relapsed again into a purely formal (astrologically motivated) pattern.

For the outer planets the arrangement in depth is suggested by the plausible (e.g. Aristotelian[16]) argument that the farther away a planet is the slower it seems to move. But the inner planets caused trouble. Ptolemy tells us[17] that "the older astronomers" placed Venus and Mercury inside the solar orbit, the more recent ones, however, outside. He mentions as an argument put forward against the inner position the absence of transits, an argument which he counters in the Almagest only superficially by referring to the latitudinal motion of the planets.[18] Only later, in the "Planetary Hypotheses," does he correctly consider the sun's brightness as a possible cause of the invisibility of transits.[19]

Real geocentric distances are obviously meant by Archimedes in his arithmetical construction described above[20]:

A: ♄ ♃ ♂ ☿ ♀ ☉ ☾ (6)

and

B: ♄ ♃ ♂ ☉ ☿ ♀ [☾]. (7)

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[8] *Οἱ περὶ μετέωρα δεινοί* (Achilles, Isag. 16; Maass, Comm. Ar. rel., p. 42, 25).

[9] "Planetary Hypotheses," below V B 7, 3.

[10] Cicero, De divin. II, 43 (Loeb, p. 474/5). In De nat. deorum II, 53 (Loeb, p. 174/5), however, he follows the order (6c), p. 692.

[11] Archit. IX 1, 5 (Budé, p. 11).

[12] NH II, 8 (Budé II, p. 36).

[13] Plutarch, De animae procr. 31 (Moralia VI, 1, ed. Hubert, p. 183, 20–25).

[14] For the astrological practice of enumerating the planets cf. Neugebauer-Van Hoesen, Greek Hor., p. 164. In a magical papyrus (a codex of the 4th cent. A.D.) the sequence (5), beginning with the sun, is called *ἑλληνικόν*, the sequence (4) *ἑπτάζωνος* (Preisendanz, PGM II, p. 120).

[15] Cf., e.g., Varāhamihira, Pc.-Sk., Neugebauer-Pingree, Vol. II, p. 13f., Vol. I, p. 121 (XIII, 39); cf., however, Vol. II, p. 109. Two different derivations of the sequence (5) were given by Cassius Dio (≈A.D. 200), one based on musical intervals, the second on the rulers of the hours (Roman History 37, 18 and 19; Loeb III, p. 128–131).

[16] Aristotle, De caelo II, 10 (Budé, p. 79; Loeb, p. 196–199).

[17] Alm. IX, 1 (Manitius II, p. 93).

[18] Cf. above I C 7, 3 D.

[19] Cf. Goldstein [1967], p. 6, 2.

[20] Above p. 648f.

The sequence (6) is frequently mentioned since it is ascribed to Plato,[21] e.g. in the Pseudo-Aristotelian treatise De mundo[22]; it is also the order adopted by the Stoic Chrysippus (second half of the 3rd cent. B.C.).[23] Philo (about A.D. 40) still assumed the order (7).[23a]

A variant of (6) that interchanges the positions of Mercury and Venus

♂] ♀ ☿ ☉ [☾ (6a)

was supposedly introduced by Eratosthenes, based on some mythological considerations.[24] Achilles, in the third century A.D., ascribes[25] (6a) to the opinion of "some", whereas "others" assumed the order

♂] ☿ ☉ ♀ [☾ (6b)

which might have some relation to a heliocentric hypothesis for the inner planets.

It should be noted that the two Archimedean arrangements (A) and (B) differ with respect to the sun in the way named by Ptolemy as "older" and "more recent". When Achilles says[26] that the "Egyptians" placed the sun at the 4th place, the "Greeks" at the 6th,[27] he considers (7) and (6) to be of different origin. In fact neither one corresponds to (1), known from the Egyptian monuments. The historicity of all these stories is extremely doubtful.

We know very little about the chronology of the Greek planetary sequences. The "Eudoxus Papyrus"[28] and the inscription of Keskinto[29] agree with (6) by following the order

♄ ♃ ♂ ☿ ♀. (6c)

It seems unlikely that Aristarchus developed any specific planetary theory because our sources are silent on this topic; Archimedes also would not have needed to introduce an artificial interpretation of Aristarchus' vague formulation concerning the radius of the sphere of the fixed stars.[30] And when Archimedes says that astronomers before Aristarchus considered the "cosmos" bounded by the solar orbit[31] one will find it difficult to associate such a concept with any specific theory of planetary distances. That in such a cosmos the planets are inside the lunar orbit

[21] The basis is a passage in the Timaeus (38 D) which seems to postulate the sequence ☾ ☉ ♀ ☿, though the relative order of Venus and Mercury is not clearly stated. It is therefore not surprising that Macrobius (Comm. I, 19 and 21; ed. Willis II, p. 73, 17–20 and p. 89, 31–34) assumes (6a) for Plato. In II, 3, however, he ascribes (6) to the "Platonists" (Willis II, p. 106, 20–107, 2).

[22] Loeb (Arist.), p. 350/353.

[23] Stobaeus, Ecl., ed. Wachsmuth I, p. 185, 14–19.

[23a] For references see W. Bousset, Jüdisch-Christlicher Schulbetrieb in Alexandria und Rom, p. 31 ff. (Göttingen 1915).

[24] Theon Smyrn. XV, ed. Hiller, p. 142, 7 ff.; Dupuis, p. 232/233. Similarly Chalcidius, Chap. 73, ed. Wrobel, p. 141, 2 ff.

[25] Achilles, Isag. 16 (Maass, Comm. Ar. rel., p. 42, 30–43, 2).

[26] Maass, Comm. Ar. rel., p. 43, 28.

[27] For the counting of positions in the sequence (4) beginning with Saturn, cf., e.g. Pseudo-Plutarch, De plac. II, 16 (Diels, Dox., p. 344, 17–345, 3); here (4) is ascribed to Plato (!).

[28] Tannery, HAA, p. 286 f.; above, p. 687 f.

[29] Cf. below, p. 699.

[30] Cf. above, p. 646.

[31] Cf. above, p. 646; this opinion is ascribed to Empedocles (5th cent.) by Pseudo-Plutarch, De Plac. II 1, 4 (Diels, Dox., p. 328, 1–3).

is expressly stated by Achilles when he says[32] that "some" assume the order

$$\odot \quad ☾ \quad ♄ \quad \text{etc.} \tag{8}$$

Hence it seems to me that all available evidence indicates that no systematic planetary theory existed before the time of Apollonius or Hipparchus. I exclude, of course, the Eudoxan model and its descendants because they are not more than clever philosophical toys; similarly I do not mean the rather crude, if practically useful, arithmetical procedures which we know from Greek and demotic papyri. Meaningful cinematic models and spatial structures could only come into existence with the development of epicyclic theories.

Purely numerological sequences for planetary distances remained in high favor long after the development of serious planetary models. We mentioned before the sequence that progresses with alternating powers of 2 and 3 (Hippolytus and Macrobius)[33] as well as Plutarch's pythagorean sequence of powers of 3.[34] Plutarch then proceeds[35] to mention round numbers for the relative sizes of the luminaries, of Venus, and of the smallest fixed stars, much in the same way as we know it from Hipparchus.[36] Plutarch assigns to sun, moon, and earth the ratios[37]

$$d_s = 12\, d_e, \qquad d_e = 3\, d_m \tag{9}$$

and gives for Venus and for the smallest fixed stars

$$d_{♀} = 2\, d_e, \qquad d_0 \geqq 1/3\, d_e, \tag{10}$$

respectively. This implies with (9) for the actual size of Venus

$$d_{♀} = 1/6\, d_s. \tag{11}$$

Cleomedes[38] states that for the apparent size of fixed stars

$$d_0 \geqq 1 \text{ finger} = 1/12\, d_{\odot}. \tag{12}$$

If we assume that the ratio mentioned by Plutarch concerns apparent diameters we would have

$$d_0 \geqq 1/36\, d_{\odot} = 1/3 \text{ finger} \tag{13}$$

which seems to be a more plausible estimate than (12).

2. Cinematic Hypotheses

What we know about early cinematic theories of planetary motion is very little. An insistence on the circularity of the orbits, appears relatively early, e.g. among

[32] Maass, Comm. Ar. rel., p. 43, 2–4.
[33] Above, p. 649 (32).
[34] Above, p. 660.
[35] De animae procr. 1028 C and D (Moralia VI, 1, ed. Hubert, p. 184, 2–17).
[36] Cf. above, p. 330.
[37] Mentioned before (p. 663); similarities with Hipparchian ratios (p. 326 (3) and (7)) are probably accidental.
[38] Cf. below p. 965.

the "Pythagoreans".[1] Their pronouncements on the structure of the cosmos are, of course, not meant to be subject to empirical scrutiny. Among speculations on cosmic harmonies and cosmogonic doctrines is also found a famous passage in Plato's "Timaeus" in which he speaks about the orbits of sun, moon, and planets.[2] These remarks are amplified by a description of the appearances typical for the inner planets: they are said to always run with the sun (ἰσόδρομοι) but endowed with the capacity (δύναμις) to overtake the sun and to be overtaken by it.

Two much discussed statements are attributed to Heraclides Ponticus,[3] a contemporary of Aristotle (who died in 322 B.C. and whom he survived). One is the remark that the celestial phenomena can also be explained by an axial rotation of the earth and a stationary sphere of the fixed stars.[4] The second passage (from Chalcidius' Commentary to the Timaeus) runs as follows[5]: "Heraclides Ponticus, when describing the circle *(circulum)* of Venus as well as that of the sun, and giving the two circles one centre *(unum punctum)* and one mean motion *(unam medietatem)*, showed how Venus is sometimes above *(superior)*, sometimes below *(inferior)* the sun." Although the text makes no distinction between the two concentric circles which carry Venus and the sun with equal mean motion, this passage has been interpreted by modern scholars as describing a heliocentric motion of Venus, assuming that "above" and "below" must refer to geocentric distances. But the terms *superior/inferior* correspond exactly to Greek ἀνώτερον/κατώτερον, attested in spherical astronomy as indicating variations in longitudinal positions.[6] The continuation of the text also refers only to the elongations of Venus from the sun, corresponding to the shift between morning star and evening star phases. In this whole discussion there is no reference to an epicyclic motion of Venus, nor is Mercury ever mentioned. It seems significant that the name of Heraclides never occurs in the ancient sources which describe a heliocentric motion of both inner planets. Obviously such theories have nothing to do with the astronomy of the fourth century B.C., neither with Heraclides nor with Plato.

Heliocentricity is perhaps alluded to by Vitruvius[7] but the passage in question is not only obscure but also apparently corrupt. Otherwise the first clear evidence for such a hypothesis in relation to the motion of the inner planets is found in the second century A.D. in Theon of Smyrna.[8] He mentions alternative possibilities: either one assumes for the sun and the inner planets three deferents and three epicycles whose centers always lie on the same line; or one may consider a helio-

[1] E.g. with Philolaus (second half of the 5th cent. B.C.); Diels, Dox., p. 378, 6–9.
[2] Translated, e.g., in Heath, Aristarchus, p. 165.
[3] From Heraclea in Pontus. He was nicknamed "Pompicus" by the Athenians; for some moderns he is "the Paracelsus of Antiquity" or "un des romanciers les plus lus" (Bidez-Cumont, Mages I, p. 14). At the death of Plato (347 B.C.) he was about 40 years of age.
[4] The main sources are two short passages, one in Pseudo-Plutarch and one in Proclus, and a more substantial discussion by Simplicius in his Commentary to Aristotle's De caelo (Ps.-Plut.: Diels, Dox., p. 378, 10–15; trsl. Heath, Aristarchus, p. 251. Proclus, Comm. Tim., ed. Diehl III, p. 137; trsl. Heath, p. 255, Festugière IV, p. 176. Simplicius, ed. Heiberg, p. 541, 29; trsl. Heath, p. 255).
[5] Chalcidius, Comm. Tim., Chap. 110 (ed. Wrobel, p. 176, 22–25; trsl. Heath, Aristarchus, p. 256).
[6] Theodosius, De diebus et noct. I 9, I 10, etc. (ed. Fecht); cf. below IV D 3, 3 B.
[7] Archit. IX 1, 6 (Budé, p. 11; p. 89–92).
[8] Theon Sm., Chap. 32 f. (Dupuis, p. 300–303). The epicycles are represented by "solid spheres" which carry the planet, rolling inside of "hollow" spheres that correspond to the deferents. The radii of the epicycles increase from the sun to Mercury and to Venus.

centric arrangement (with Mercury nearest to the sun, as required by its small maximum elongation[9]).

Macrobius, in the 4th century, discusses in his commentary on Cicero's "Somnium Scipionis" the positions of the inner planets with respect to the sun and says[10] that "it did not escape the skill of the Egyptians" how to reconcile the opposing hypotheses on the inner or outer location of these planets, simply by assuming that they rotate about the sun. The name "Egyptian System", found in modern literature, has no other basis than this passage. Martianus Capella, referring again to the heliocentric motion of the two inner planets does not mention the Egyptians.[11] There is no mention of outer planets.

This small group of sources has been obscured by a huge web of intricate hypotheses and arbitrary assumptions. A process of speculative interpretation begins as early as antiquity, following a well-known tendency to attribute later knowledge to early authorities. The modern mythology begins with the empty praise of Heraclides as "precursor of Copernicus",[12] though statements about obvious cinematic equivalences are a commonplace in ancient literature.[13] The next, and much more serious step, consists in the embellishment of the "system" of Heraclides. Van der Waerden, e.g., constructed a planetary model[14] according to which Heraclides not only assumed an axial rotation for the earth but also a motion in a circular orbit about an empty center, the earth's orbit being the outermost of three concentric circles on which sun, Venus, and earth are supposed to move. A careful refutation of these theories, point by point, by Pannekoek had no effect.[15]

Chalcidius terminates his discussion of the elongations of Venus by saying that all will be "still clearer" from a figure.[16] Of course, such a drawing is only Chalcidius' own illustration of the motion of the planet and proves nothing for a model assumed by Heraclides. Van der Waerden investigated the figures preserved in our manuscripts (none of which is nearer to Chalcidius than about 700 years) and he arrived at distinguishing three classes. Two types make no sense, showing concentric orbits for sun and Venus but drawing the tangents of maximum elongations either from a point of Venus' orbit, or even from the celestial sphere, to the circle representing the sun's orbit(!). The third group of figures (similar

[9] Since the sun retains its solid sphere one must assume that the orbits of Mercury and Venus have their center in the mean sun.
[10] Macrobius, Comm. I, 19, ed. Willis II, p. 73f.; trsl. Stahl, p. 162–164; Heath Aristarchus, p. 258f.; summary in Dreyer, Plan. Syst., p. 129.
[11] Martianus Capella VIII 857; ed. Dick, p. 450, trsl. Heath, Aristarchus, p. 256.
[12] Initiated by Schiaparelli (1873); Scritti I, p. 361ff.; II, p. 113ff. Actually this overlooks a fundamental aim in the work of Copernicus, that is to show that all technical details of the Ptolemaic numerical procedures can also be explained heliocentrically under the severe restriction to uniform circular motions.
[13] Cf., e.g., the simile of the ants on the potter's wheel or of the traveler on a boat: Vitruvius, Arch. IX 1, 15 (Budé, p. 15, p. 111); Geminus, Isag. XII, 18 (Manitius, p. 140, 23–142, 12); Achilles, Isag. 20 (Maass, Comm. Ar. rel., p. 48, 16–24); Anon. (Maass, p. 97, 33f.); Cleomedes I, 3 (Ziegler, p. 30, 8–15). Cf. also Āryabhaṭīya IV, 8 (Clark, p. 64); Bar Hebraeus (13th cent.), L'asc. I, 6 (trsl. Nau, p. 10).
[14] Van der Waerden [1944] and [1951, 2] (p. 69, Fig. 5).
[15] Pannekoek [1952]. Twenty years later van der Waerden ([1970, 1], p. 51) treats his theory of the earth's motion as an established fact.
[16] Chalcidius, Chap. 111 (ed. Wrobel, p. 178, 5–8).

to our Fig. 34), however, fits the discussion of Chalcidius by illustrating the maximum elongations of Venus (50 *momenta*[17]). Van der Waerden, however, considers the corrupt figures as remnants of Heracleides' theory of the earth's motion and by adding one more circle (of which there is no trace in any version) obtains a picture of the peculiar model he postulates for Heraclides.

Finally this hypothetical Heraclidean astronomy is freely used for the interpretation of the previously mentioned passage in the Timaeus. Van der Waerden sees in the latter a description of an epicyclic motion both for the sun and the two inner planets[18] although Plato says nothing that relates the limited oscillations of Venus and Mercury to a change in geocentric distance. This interpretation is still farther developed by Saltzer.[19] Although he does not accept van der Waerden's theory of a mobile earth he attributes instead to Heraclides a solar epicycle, traversed in the correct sense of rotation (i.e. clockwise), around which Mercury and Venus travel on their epicycles (counterclockwise) and in the correct order Sun-Mercury-Venus. All this is then applied again to the Timaeus passage. In support of the assumption of a solar epicycle the following arguments (beyond van der Waerden's) are offered: (a) Meton and Euctemon knew the inequality of the seasons, (b) a passage in Plato's "Symposium" on astronomy[20] could not concern "bloßes Erfahrungswissen" but must again allude to the inequality of the seasons (!), and (c) the two versions of 50 *momenta* and 46 *partes* for the elongation of Venus suggest a solar (!) eccentricity 8/5 times the Hipparchian value.

The argument (a) ignores the fact that the transition from the inequality of the seasons to the construction of an epicyclic model is by no means trivial and requires the complete Hipparchian apparatus of trigonometry and eccenter-epicycle equivalence. Fortunately, however, we know enough about the early parapegmata to see that Hipparchus also could not have used them to obtain reasonable results.[21] Point (b) is at best amusing: reading the passage in question in its context one will find that it deals with the influence of the seasons on the Eros in nature and with animals — so much for the solar epicycle. Finally (c) which makes no sense astronomically: the elongations of 50° and 46° are only parts of two parallel traditions[22] and their ratio has no meaning whatsoever.

The most serious objection against all theories, ancient or modern, that ascribe to the time of Plato and Heraclides a consistent epicyclic theory, lies of course, in the general historical situation. Should one really assume that a mathematician of the rank of Eudoxus, knowing about epicycles, would have tried to explain planetary motion with so inflexible a model as homocentric spheres? And should not at least Archimedes have seen the advantages of a cinematic model, already well known in the Academy? Finally is it not strange that Ptolemy never mentions that correctly arranged epicyclic devices had been suggested by Plato or his pupils and associates?

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[17] Cf. below p. 804 for the tradition of this parameter (Theon of Smyrna-Chalcidius-Cleomedes-Martianus Capella).
[18] Van der Waerden [1951, 2], p. 45f.
[19] Saltzer [1970].
[20] Symposium 188 B (Loeb V, p. 130/131).
[21] Cf. above p. 628.
[22] Cf. below p. 804.

The discussions about Aristarchus never ascribe to him any interest in an epicyclic planetary theory. All we hear about his general astronomical theories concerns the motion of the earth. The only passage which contains some specific information about Aristarchus' heliocentric hypothesis is the famous introduction by Archimedes to his "Sand-Reckoner" where he says that Aristarchus located the sun in the center of an unmovable sphere of fixed stars, while the earth travelled on a circular orbit about the sun and that the size of this orbit was negligible in relation to the size of the sphere of the fixed stars.[23]

That Aristarchus postulated a movable earth is repeatedly stated in the ancient literature, e.g. by Plutarch[24] or by Sextus Empiricus.[25] A necessary consequence of this assumption is the acceptance of an axial rotation of the earth in order to account for the change of day and night and Simplicius mentions explicitly this additional assumption.[26] But nothing is ever said about a corresponding planetary theory which would indeed amount to the insight that the epicycles are only a consequence of a heliocentric motion of the earth.

Much discussion has been caused by another remark of Plutarch[27] which Heath renders as follows[28]: "... the earth ... was not represented as being ... at rest, but turning and revolving, as Aristarchus and Seleucus[29] afterwards maintained (ἀπεδείκνυσαν) that it did, the former stating this as only a hypothesis, the latter as a definite opinion (ἀποφαινόμενος)." The last verb has a wide range of meanings, from "declare", "proclaim", to "prove". Van der Waerden insists that only the strictest meaning, i.e. "to prove", should be accepted[30] and he even suggested that this implies that Seleucus (or an otherwise unknown pupil of his) computed heliocentric planetary tables.[31]

But notwithstanding such fantasies, the passage in question clearly indicates that Seleucus went beyond Aristarchus in an attempt to demonstrate the motion of the earth. Thanks to the recently discovered[32] fragments of a work of Seleucus, preserved in Arabic in a work of Rāzī (around A.D. 900), we now have a fair example of what constituted a "demonstration" for Seleucus. In his discussion concerning the infinity of the world[33] there is not a trace of any mathematical argument and everything is treated in purely philosophical terms. There is not the

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[23] English translation by Heath, Aristarchus, p. 302. Cf. also above p. 646.
[24] Plutarch, De facie (Loeb XII, p. 54/55; Heath, Arist., p. 304); also Pseudo-Plutarch, De Plac. II, 24 (Diels, Dox.; Heath, Arist., p. 305).
[25] Sextus Emp., Adv. Math. X (= Against Phys. II) 174 (Loeb III, p. 298/9; Heath, Arist., p. 305).
[26] Simplicius, Comm. Arist. De caelo (ed. Heiberg, p. 444, 34; trsl. Heath, Arist., p. 254).
[27] Plutarch, Plat. Quaest. 1006 C (ed. Hubert, Moralia VI, 1, p. 129, 21–25).
[28] Heath, Aristarchus, p. 305.
[29] On Seleucus (ca. 170 B.C.) cf. above p. 610f.
[30] Van der Waerden [1970, 1], p. 6, p. 11. Ironically he mixed up the two verbs in question: he says that ἀποδείκνυμι must mean here "to prove" but he himself renders it by "sich vorstellen" whereas the crucial term is ἀποφαίνομαι about which he has nothing to say.
[31] Van der Waerden [1970, 1], p. 7, p. 51. Needless to say such tables would be useless for a terrestrial observer.
[32] Cf. above p. 611, n. 29.
[33] That Seleucus considered the cosmos to be infinite is also known from Pseudo-Plutarch (Diels, Dox., p. 328, 4–6); in Stobaeus' version not only Seleucus but also Heraclides Ponticus are credited with this opinion.

slightest reason to assume a different character for his support of Aristarchus' hypothesis.

Postulating an infinite size of the cosmos only created new difficulties; again in Plutarch one finds, e.g., discussions about the meaning of "midpoint" in an unlimited world.[34] The absence of an observable parallax, not only for the sun but also for the fixed stars, must have greatly strengthened the arguments against the hypothesis of a movable earth. Without the accumulation of a vast store of empirical data and without a serious methodology for their analysis the idea of heliocentricity was only a useless play on words.

Quite charming is a discussion in Pliny[35] of the possible causes for the increasing errors of the sun dial for which the obelisk on the Campus Martius in Rome served as gnomon.[36] Besides blaming it on changes in the foundations of the obelisk, Pliny mentions the possibility of irregularities in the motion of the sun, or perhaps a displacement of the earth from its central position. Pliny certainly did not dream that these remarks would ever qualify as "revolutionary" in the history of ideas.

§ 3. The Inscription of Keskinto

In 1893 a Greek from Lindos on the Island of Rhodes accidentally discovered in nearby Keskinto[1] a fragmentary astronomical inscription of which he made a squeeze that was sent to Athens and then to Hiller v. Gaertringen in Berlin. The stone itself was later transported to the Museum in Berlin.[2]

The text gives a list of planetary parameters (thus in principle reminiscent of the "Canobic Inscription"[3]) ending in a dedicatory line, written in large letters: "...] a thanksgiving to the gods." The date is extremely insecure. It is obvious[4] that the underlying planetary theory is more advanced than in the Eudoxan cinematic models but less developed than the Ptolemaic theory; epigraphic criteria[5] suggest a date around 100 B.C. Entirely without proof remained Hiller v. Gaertringen's conjecture[6] that it may have been Attalus of Rhodes who erected the stela, the same Attalus whose commentary to Aratus attracted the criticism of Hipparchus.[7]

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[34] Plutarch, De facie 11 and 14 (Loeb XII, p. 77–79; p. 89–91).

[35] NH 36, 72/73 (Loeb X, p. 56–59).

[36] Erected under Augustus; the hour lines on the pavement were designed by the "mathematician" Novius Facundus.

[1] Ancient Lartos, about 7 km west of Lindos (cf. the map on Pl. I in IG XII, 1). Note that the text does not come from a systematic excavation (as has been occasionally asserted).

[2] The text seems to have been lost in the Museum for many years (cf. Hiller v. Gaertringen [1942], p. 165), only a squeeze was reproduced in Herz [1894], p. 1144 (upside down) and in Tannery, Mém. Sci. 15, p. 119. After the second World War Prof. Derek Price of Yale University obtained a new squeeze from Berlin. The fragment measures about 76 by 28 cm.

[3] Cf. below V B 7.

[4] Tannery, Mém. Sci. 2, p. 500f.

[5] Tannery, Mém. Sci. 15, p. 175/176.

[6] Hiller v. Gaertringen [1942], p. 167.

[7] Hipparchus, Ar. Comm., Introduction et passim; cf. also above p. 278.

The last line of the text (preceding the dedication) contains the statement

> ...] the circle (contains) 360 degrees (μοιρῶν), 720 "points" (στιγμῶν) of the circle.[8] One degree (contains) [2] "points".

It is not clear why these units are mentioned here; no angular measurements occur in the preceding text as far as it is preserved. No other text seems to be known which uses units of $1/2^\circ$ called "points".[9]

The following publications contain the details of readings and interpretations:

Preliminary notice:	Archaeologischer Anzeiger 1894, No. 3. p. 125
Text:	IG XII, 1 (1895) No. 913 (p. 148f. and p. 207).
Discussion:	Herz [1894].
	Tannery, Mém. Sci. 2 (1895), p. 487–526;
	Mém. Sci. 15 (1939), p. 119–187: correspondence between Tannery, Herz, and Hiller v. Gaertringen.
	Hiller v. Gaertringen [1942].

The extant text is the lower part of a presumably much larger stela. Preserved are three groups, of four lines each, which concern the outer planets, preceded by one fragmentary last line from a section for Mercury. Thus we know at least the order of the planets: [Venus]-Mercury-Mars-Jupiter-Saturn, i.e. the same order which is found, e.g., with Archimedes.[10] Nothing can be said about sun and moon; there is no evidence for or against an inclusion of these two luminaries in our text.

For each one of the outer planets are listed four sets of integers, called, respectively,

κατὰ μῆκος ζωδιακοί	i.e.	zodiacal longitudes, here:	L	
κατὰ πλάτος τροπικοί		latitudinal variations	B	(1)
κατὰ βάθος περιδρομαί		rotations in depth	G	
κατὰ σχῆμα διέξοδοι[11]		returns in phase	A	

The numbers are given, for reasons unknown, always in two forms which differ by the constant factor 10. For the readings this is of great help; in our discussion, however, we usually operate with the smaller numbers.

For the inner planets we have only the entry 9[.] 8[...] for the phases of Mercury in the last line of the column which displays the numbers $10n$. I think one should not base conjectures concerning the inner planets on so minute a fragment.[12] The following discussion therefore concerns exclusively the outer planets.

[8] "Circle" is written herewith the symbol ⊙ which is the ordinary abbreviation for κύκλος in Byzantine manuscripts, a fact correctly emphasized by Tannery. At no time is ⊙ attested in Greek texts to mean "sun."

[9] Cf. however, the "stadium" of $1/2^\circ$ in Manilius III 282ff. (below p. 719) and the "solar-cubit" in P. Oslo 73 (above p. 592).

[10] Cf. above p. 691.

[11] Cf. for this terminology Tannery, Mém. Sci. 15, p. 182f. For the three preceding terms cf., e.g., below p. 933.

[12] Tannery. Mém. Sci. 2. p. 509 suggests restoring 918543 for the above mentioned number $10n$ for Mercury's phases.

In the following table we give what seems to be a reasonably secure set of readings for the parameters of the outer planets:

	Mars		Jupiter		Saturn	
	n	$10n$	n	$10n$	n	$10n$
L	[1]74[9]2	174920	[2157]	21570	[ ]	[ ]
B	[17]436	174360	[215]6	21560	[ ]	9810[14]
G	[ ]	491680[13]	24260	242600	[2]7176	[27]1760
A	[ ]	136480	26690	26[6900]	[2]8148	2[8]1480

(2)

The discussion of the astronomical significance of these numbers has led to the assumption of some scribal errors and corresponding emendations. We shall summarize at the end[15] what we may consider as the effective contents of the inscription of Keskinto.

1. In any planetary theory the revolutions of the planet in longitude (L) and the number of corresponding phases (A) should satisfy the following identity

$$\text{number of sidereal rotations} + \text{number of synodic periods} = \text{number of sidereal years.}^{16}$$

Consequently we should expect

$$L+A=N_0 \tag{3}$$

with probably the same value N_0 for all planets. This assumption is easily tested. We know, e.g., from Babylonian "Goal-year-texts"[17] how many synodic periods S correspond to N years:

$$\begin{array}{lll} \text{Mars: } N=79 & \text{Jupiter: } N=71 & \text{Saturn: } N=59 \\ \phantom{\text{Mars: }} S=37 & \phantom{\text{Jupiter: }} S=65 & \phantom{\text{Saturn: }} S=57. \end{array} \tag{4}$$

Hence we should expect that in our text

$$A\cdot\frac{N}{S}\approx N_0. \tag{5}$$

This is indeed nearly the case (cf. (2) and (4)):

$$\frac{13648\cdot 79}{37}\approx 29140.3, \quad \frac{26690\cdot 71}{65}\approx 29153.7, \quad \frac{28148\cdot 59}{57}\approx 29135.6 \tag{6}$$

hence

$$N_0=29100+\delta, \qquad \delta\approx 40 \tag{7}$$

years is a common period assumed for the outer planets. This also shows that the readings of the numbers A can be trusted.

[13] The initial 40 is doubtful.
[14] The final 10 is doubtful.
[15] Cf. below p. 704 (22).
[16] Cf. above p. 150 (1), p. 170 (1), and p. 389 (7) or p. 420 (2).
[17] Cf., e.g., above p. 151 (2).

Returning to the fundamental relation (3) and using the numbers in (2) we find

$$\begin{array}{llll} & \text{Mars} & \text{Jupiter} & \text{Saturn} \\ L: & 17492 & 2157 & [9..] \\ A: & \underline{13648} & \underline{26690} & \underline{28148} \\ N_0: & 31140 & 28847 & 29[...] \end{array} \tag{8a}$$

Since N_0 is known within narrow limits (cf. (6) and (7)) and since the A-s are practically secure one must emend the L-s. Tannery therefore suggested the following corrections:

$$\text{Mars: } 15492, \quad \text{Jupiter: } 2450^{18}, \quad \text{Saturn: } [992]. \tag{8b}$$

Then one obtains for $N_0 = L + A$ the value

$$N_0 = 29140 = 8,5,40 \tag{9}$$

in agreement with (7). In other words our inscription is based on the assumption that 29140 (sidereal) years contain an integer number of revolutions and phases for the outer planets.

2. The near agreement between the value (9) for the common time interval and the estimates (6) obtained from Babylonian period relations leads to the question of how close the agreement will be for other basic parameters which can be derived from A and L.

Let $\overline{\Delta\lambda}$ be the mean synodic arc for an outer planet and Π and Z the smallest integers such that

$$\Pi \cdot \overline{\Delta\lambda} = Z \cdot 6,0^\circ \tag{10a}$$

while

$$P = \Pi / Z \tag{10b}$$

indicates how many mean synodic arcs (integer plus fraction) can be fitted on 360° of longitude.[19]

For all three outer planets A gives the number Π of synodic periods during N_0 years. For Saturn and Jupiter the number L of sidereal rotations during N_0 years is the same as Z; for Mars, however, we have[20]

$$L = \Pi + Z. \tag{11}$$

Hence we have the following numerical data

	Mars	Jupiter	Saturn	
L = sider. rot. A = occur. = Π	$\Pi + Z = 15492 = 4,18,12$ $= 13648 = 3,47,28$ $Z = 1844 = 30,44$	$Z = 2450 = 40,50$ $= 26690 = 7,24,50$	$Z = 992 = 16,32$ $= 28148 = 7,49,8$	(12)

[18] From an epigraphic viewpoint the change from 57 to 50 (or rather from 570 to 500) is unpleasant but seems unavoidable.

[19] Cf. for these concepts above p. 377.

[20] Cf. above p. 388 f.

If we now compute P from (10b) we find indeed close agreement with standard Babylonian parameters[21]

P	Mars	Jupiter	Saturn
Keskinto	7;24,4, ...	10;53,37, ...	28;22,30
Babylonian	7;23,20	10;51,40	28;26,40

(13)

On the one hand this shows again that our numbers A and L in (12) are basically correct; on the other hand it is clear that the Keskinto data are not simply taken from Babylonian astronomy as is the case for certain Hipparchian parameters.

3. For the parameters B in (2) we have unfortunately no independent check nor is it clear that the emendations (8b) for the L-s should not also affect the B-s. The mere fact, however, that different sets of numbers are listed for the returns in longitude and in latitude shows that a motion of the nodal line is assumed. This motion is retrograde when $L<B$, direct when $L>B$. Now we have from (8b) and (2)[22]

	Mars	Jupiter	Saturn
L	15492	2450	992
B	17436	2156	981

(14)

According to these numbers the nodal line of Mars would recede, but progress for Jupiter and Saturn.

Obviously (14) cannot be based on real observations since the motion of the nodes is much too small to be determined with so high a numerical accuracy as, e.g., 11 rotations of Saturn's nodal line in 29140 years.

4. According to the terminology as we know it from the later sources a parameter called "depth" would represent the epicyclic anomaly. Here, however, this cannot be the correct meaning since we have for each planet two different parameters, one for the rotations in "depth" (G), another for the "phases" (A), while these periods would be identical for a classical epicyclic model.

N. Herz remarked[23] that a differentiation between G and A could be explained by a "sidereal" reckoning of the anomaly, i.e. by using on the epicycle at C the same coordinates as in O (cf. Fig. 35). Then the "sidereal anomaly" γ of the planet is given by

$$\gamma=s=\bar{\lambda}+\alpha \tag{15}$$

where s is the longitude of the sun, $\bar{\lambda}$ of the center C of the epicycle, and α the anomaly in the customary sense such that its revolutions count the number of phases.

This interpretation cannot be correct, however. Not only would (15) result in a relation for the periods

$$G=L+A \tag{16}$$

[21] Cf., e.g., ACT II, p. 283 or above p. 426.
[22] The latter without emendations; perhaps one should read 15436 for Mars and 2456 for Jupiter.
[23] Herz [1894], p. 1142.

which is obviously not satisfied in (2) but from (3) or from $\gamma = s$ one would have to conclude that $G = N_0 = 29\,140$ years, i.e. G should be the same for all planets, which is not the case.

Actually, however, one finds from (8b) and (2) that the periods for Jupiter and Saturn satisfy the relation

$$L + G = A + 20 \tag{17}$$

as is seen in the following table

	Jupiter	Saturn
L	2450	992
G	24260	27176
$L+G$	26710	28168
A	26690	28148
D	20	20

(18)

The relation (17) does not hold for Mars; we therefore postpone the discussion for this planet to a later section.[24]

The additional term $D = 20$ is very disturbing. Changing either A or L would upset (3) as well as (13).[25] Emending G by subtracting 20 would mean to change in the inscription (2) not only two numbers n by 20 but also two numbers $10n$ by 200, changes which cannot be justified epigraphically. Hence we must accept (18) as it stands and operate with an auxiliary period

$$L' = L - D \tag{19}$$

which leads us to

$$L' + G = A. \tag{20}$$

5. The terminology (1) makes it practically certain that we are dealing with an epicyclic model, i.e. not with a purely arithmetical pattern. The fact that $G \neq N_0$ suggests maintaining sidereal coordinates for the anomaly as in Fig. 35. But in order to replace (16) by (20) we must change the sense of rotation of the planet on the epicycle (cf. Fig. 36). The sense of rotation of the planet on the epicycle is irrelevant for the counting of periodic returns[26] as long as (3) is maintained. This requires only that the planet P on the epicycle makes the same angle α with the apsidal line OCA as the sun, be it in the same (Fig. 35) or opposite sense of rotation (Fig. 36). If in the latter case we count the "sidereal anomaly" γ in the sense of rotation now assumed we see from Fig. 36 that

$$\gamma = \alpha + 180 - \bar{\lambda}. \tag{21}$$

Additive constants play no role for period relations; complete revolutions of α again count the number A of phases whereas revolutions of $\bar{\lambda}$ must be given by L',

[24] Cf. below p. 704.

[25] Decreasing L by 20 would lead in (13) to $P \approx 10;59$ for Jupiter, to $\approx 28;57$ for Saturn, i.e. to values hardly permissible.

[26] Evidence for planetary models which assume an incorrect sense of rotation on the epicycle will be discussed in V A 1, 4.

returns of the sidereal anomaly by G. Consequently from (21):

$$G = A - L'$$

as required by (20).

6. We have been compelled to distinguish two periods of mean longitude, L' and L, which differ, according to (18), by $D=20$ revolutions in $N_0=29140$ years. With respect to sidereal coordinates a planet completes fewer revolutions in a given time than with respect to tropical coordinates. This could suggest the interpretation of L' as sidereal, L as tropical revolutions. But D is much too large for a correction of precession: 2 revolutions in about 3000 years would imply about 24° per century. Hence precession fails to explain the correction D in (19). I see no solution to this problem.

7. Equally inexplicable seems the case of Mars. The relation (20) cannot be satisfied since A is much smaller than L' and consequently G would become negative (-1824). The reason for this difference in behavior lies in the fact that for Saturn and Jupiter the number of revolutions in longitude is much smaller than the number of synodic periods whereas Mars returns more than once to the same mean longitude before it comes again in conjunction with the sun. The text seems to require for G a number ending in 168. I find no relation to the other parameters which could explain this figure. Hence the meaning of G for Mars completely escapes us and this must also cast doubt on our interpretations in the case of Jupiter and Saturn.

8. That so little can be securely established from our inscription is perhaps less surprising when one realizes that it is at best a set of 12 parameters which refers to a common interval of $N_0=29140$ years:

	Mars	Jupiter	Saturn
L	15492	2450	992
B	17436	2156	981
G	...168	24260	27176
A	13648	26690	28148

(22)

without any explanation concerning their mutual relations.

Nowhere else are these parameters attested. They show no relation to Babylonian data nor to the classical Greek tradition which begins for us with Apollonius and Hipparchus. Only in Indian astronomy do we find some parallels, e.g. in the fictitious motion of the nodal lines for the planets[27] and in the use of a huge common planetary period. More important may seem the reckoning in Indian astronomy of the epicyclic anomaly from a sidereally fixed direction.[28] The Indian theory represents eccentricity and epicyclic anomaly in a purely formal way by two concentric epicycles (cf. Fig. 37), the "manda-epicycle" of radius r_m and the "sīghra-epicycle" of radius r_s. The common center of these two epicycles is the mean planet $\overline{P}$.

[27] Cf., e.g., Sūrya-Siddhānta I, 41–44.

[28] Prof. Toomer reminded me of this parallelism; cf. also Toomer [1965], p. 62, Fig. V.

The manda-epicycle is of no interest to the period relations discussed here since the radius $r_m = \overline{\mathrm{P}}\mathrm{M}$ has a fixed sidereal position, parallel from O to the apogee A (hence $m = \lambda_A$). Its purpose is the representation of the eccentricity, a parameter which plays no role in the Keskinto inscription.

The sīghra-epicycle, however, corresponds to the motion in anomaly in the Apollonius-Ptolemaic theory. The point S on the sīghra-epicycle moves (in the correct sense of rotation) in such a fashion that the radius $\overline{\mathrm{P}}\mathrm{S}$ always remains parallel to the direction from O to the sun (thus the meaning "conjunction" of śīghra), while $\overline{\mathrm{P}}$ progresses with the mean velocity of the planet.[29] Since the position of the sun is represented by its sidereal longitude s the relation (15) is satisfied. Thus the Indian śīghra anomaly appears to have a predecessor in the "sidereal anomaly" of the Keskinto inscription. It is, of course, impossible to say whether this is purely accidental or not.

[29] For further details of the Indian procedure cf., e.g., Neugebauer-Pingree, Varah. Pc.-Sk. II, p. 101 and Fig. 59 there.

D. The Development of Spherical Astronomy

§ 1. Arithmetical Methods; Length of Daylight; Climata

We have described in Book II the high degree of perfection which purely arithmetical methods had reached in Babylonian astronomy. This does not imply, however, that simple approximations by linear sequences must be taken as evidence for Babylonian influences.

As an example for an independent application of a linear scheme can be mentioned a pattern from Egypt for the variation of the length of daylight. A Ramesside papyrus[1] (about 12th cent. B.C.) gives a list of lengths of daylight and night for the twelve months of the year based on the extrema $M = 18^h$ and $m = 6^h$, using a constant monthly increment of 2^h. Babylonian influence seems most unlikely at that period nor would the ratio 3:1 or the 24-division of the day lend support to such a hypothesis.[2]

Perhaps from the time of Necho (≈ 600 B.C.) comes another list of lengths of daylight and night, now probably based on the ratio $M:m = 14:10$. Unfortunately the text is not only badly preserved but also corrupt. Nevertheless a linear scheme operating with an increment of 24 minutes for each 15-day interval seems a possible reconstruction, assuming constant values $M = 14^h$ and $m = 10^h$, respectively, during one month each.[3]

We come nearer to the familiar scheme in two Greek papyri, P. Hibeh 27 (≈ 300 B.C.)[4] and the "Eudoxus Papyrus" P. Par. 1 (≈ 180 B.C.)[5]. Both texts assume a daily increase or decrease of $1/45^h$ during 180 days each, between $m = 10^h$ and $M = 14^h$, with no changes during 3 days and 2 days, respectively, for the extrema[6] in order to complete the 365 days of the Egyptian year (cf. Fig. 38). P. Hibeh 27, which gives all the details, expresses the resulting lengths of daylight and night in terms of Egyptian unit fractions[7] such that we can deduce from this text a table of multiplication for 1/45 (beginning: $\overline{45}$, $\overline{30}\,\overline{90}$, $\overline{15}$, $\overline{15}\,\overline{45}$, $\overline{9}$, $\overline{10}\,\overline{30}$, etc.). A Babylonian text for this parameter would simply give the multiples of 0;1,20.

The preceding examples illustrate what cannot be repeated often enough: simple arithmetical data should not be taken as accurate results of astronomical observations. This holds not only for Egypt but for Babylonian texts as well.

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[1] Cf. Bakir, Cairo Catal., p. 54.

[2] Cf. Neugebauer-Parker, EAT I, p. 119f. for the difficulties connected with the unexpected ratio $M:m = 3:1$ and for its possible connection with the origin of the 24-division of the day which appears for the first time in this period.

[3] Neugebauer-Parker, EAT III, p. 46, Fig. 10.

[4] Grenfell-Hunt, The Hibeh Papyri I, p. 152f.

[5] Tannery, AA, p. 284; also above IV C 1, 3.

[6] P. Hibeh 27 places the 3-day stretch at M, P. Paris 1 at m.

[7] I denote $1/n$ by $\bar{n}$.

One must not try to explain the ratio $M:m=3:2$ for Babylon by intricate astronomical or historical hypotheses instead of recognizing the arithmetical convenience as the prime motive for the choice of these numbers.

In the further development of arithmetical methods the step from linear variation of the length of daylight to a linear pattern for the rising times of the zodiacal signs constitutes a marked progress in the right direction.[8] The Greek arrangement of geographical zones according to a linear increase of M had far reaching consequences for ancient and medieval geography by providing a definite ordering principle,[9] though in fact it perpetuated an antiquated approach for many centuries.

That the spell of neat arithmetical patterns can also lead to completely meaningless constructions may be documented by a Syriac "astronomical letter", written in A.D. 716, concerning the heliacal rising of Sirius.[10] The writer, George "Bishop of the Arabians", most likely only reproduced what he had absorbed from some earlier astrological writings. An ultimate Alexandrian origin of his scheme seems likely, e.g. because of the use of morning epoch for the calendar dates.

The rising of Sirius is assumed to take place on Tammuz (=July) 19 at the 11th hour of night (i.e. one hour before sunrise). But the author knew that the length of a year is actually 365 1/4 days and that one day must be inserted every fourth year in order to keep the calendar in agreement with the solar year. Consequently he added each year 6 hours to the moment of rising of Sirius, thus obtaining the following rule:

July 19	11^h night
July 20	5^h day
July 20	11^h day
July 20	5^h night
July 19	11^h night

which restores the initial date July 19 every fourth year. That the three intermediate "risings" fall, e.g., into daylight does not disturb our Bishop. In fact he certainly did not invent this senseless pattern of Sirius risings; with some small modifications we find it six centuries earlier in the Anthology of Vettius Valens[11] which, in turn, cannot be taken as the original source.

The suggestive power of arithmetical patterns is not restricted to astronomical problems. It is also visible in countless instances in Greek astrological literature (cf., e.g., the play with "planetary periods"[12]) and is equally prevalent in the numerological speculations of Greek philosophy (e.g. what is called "Pythagorean").

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[8] Cf. below IV D 1, 2.
[9] Cf. below IV D 1, 3 A.
[10] Cf. Ryssel [1893], p. 50f.
[11] Vettius Valens, ed. Kroll, p. 22, 12-27. Here the four years of each julian cycle are associated with the risings of the signs ♋, ♎, ♑, ♈ at 11^h night, 6^h, 12^h day, 6^h night, respectively. Note the inconsistency in the spacing of the hours.
[12] Above p. 606 (3).

1. Length of Daylight

A ratio 2:1 of longest to shortest daylight would fit a geographical latitude of about 48;30°, i.e. a region in southern Russia.[1] When this ratio appears in ancient Mesopotamia then it is clear that time must have been measured by devices which do not show equinoctial hours or their equivalent. This conclusion is supported by the fact that these "times" are denoted by a term for weights (mana), thus suggesting the use of water clocks.[2]

Whatever pattern one intended to apply to the variation of the length of daylight, its association with the months of a real lunar calendar would be difficult, even assuming a definite intercalation rule, but impossible with the irregular intercalations in the period preceding the 19-year cycle.[3] Consequently in early Babylonian astronomy the equinoxes were associated with the dates I 15 and VII 15 in a schematic calendar, the solstices with IV 15 and X 15, a pattern which we find, e.g., in the series ^{mul}Apin.[4] With these dates as fixed-points the variation of the above mentioned time units was represented by a linear zigzag function. This is shown, e.g., by the following facts: at the equinoxes daylight and night amount to 3 mana each; at the solstices, however, the length of daylight corresponds to 4 mana and 2 mana, respectively. The daily increment is chosen as 0;0,40 mana, hence exactly 1 mana in 90 days, i.e. in each quarter of the schematic year.[5]

Under the assumption that the given weights represent the outflow of water from the bottom of a cylindrical container the ration 2:1 is not in flagrant contradiction to the Babylonian standard $M:m=3:2$ for the length of daylight since this latter ratio would correspond to about 9:4 for the outflowing quantity of water. Since also $M:m=3:2$ cannot be considered as the result of any accurate method of measuring time we may accept both ratios as fair arithmetical approximations of the same crude empirical data.

Many centuries later we find traces of the early Babylonian norm, but no longer related to water clocks, in Egypt and even in Ethiopia. In a parchment codex we have a Greek text[6] (written in the 7th or 8th century[7] over a badly erased Coptic text) which gives on one page for each of the twelve months of the Alexandrian calendar the length of daylight and night, varying with the constant difference of 1 1/3^h per month between $m=8^h$ and $M=16^h$.

Ethiopic civilization always had close connections with Egypt and this is true in particular of its Christian (monophysite) literature. It is probably by this road that the doctrine of a ratio 2:1 for the variation of daylight reached Ethiopia, rather than through direct Palestinian influences. What we find in Ethiopian sources[8] is nevertheless not identical with the above described Greek scheme.

[1] In ancient mathematical geography the name of this area is "(Mouth of the) Borysthenes" (i.e. Dniepr); cf., e.g., below p. 725 (1).
[2] We have textual evidence for the use of water clocks since Old-Babylonian times; cf. Neugebauer [1947, 2].
[3] For the 19-year cycle cf. above p. 541 f.
[4] Cf. above p. 598.
[5] Cf. for the texts Neugebauer [1947, 2], p. 41.
[6] PSI 1296 Fol. α, p. 1 (PSI, Vol. 13, 1, p. 2).
[7] Written by Christians since it contains, e.g., a discussion between Gregory of Nazianz and Basil.
[8] E.g. in the Book of Enoch, Chap. 72.

The monthly difference was not chosen to fit a 24-hour day but $d=1$, the arithmetically simplest parameter. This leads to the condition:

$$M=2m, \quad d=1/6(M-m)=1 \tag{1}$$

hence

$$m=6, \quad M=12, \quad 1^d=18 \quad \text{or} \quad 1=0;3,20^d(=1;20^h). \tag{2}$$

The peculiar time unit of 1/18 day in the Ethiopic pattern is thus explained as the consequence of a linear daylight scheme based on $M:m=2:1$. The result is

(3)

	month	dayl.	night	
	XII, VI	9	9	
	I, V	10	8	
	II, IV	11	7	
M	III	12	6	m

and symmetrically for the remaining months (to be understood in the Ethiopic calendar which is essentially the same as the Alexandrian-Coptic[9]).

In the final phase of Babylonian astronomy the variation of daylight is obtained by summation over the rising times of 180° of the ecliptic.[10] There exists, however, some indication that this level was preceded by a more primitive approach which assumed linear variation between the extrema. One lunar tablet, probably written around 400 B.C. (Artaxerxes II), contains data as function of the solar longitude that seem to satisfy the following linear scheme[11] with $M:m=3:2$ and $d=12°$ $(=0;48^h)$:

(4)

		Daylight	
♋		3,36°	M
♌	♊	3,24	
♍	♉	3,12	
♎	♈	3,0	
♏	♓	2,48	
♐	♒	2,36	
♑		2,24	m

Another piece of evidence that points in the same direction comes from arithmetical rules in an astrological treatise, designed to predict the month in which a person will die.[12] There occurs the following pattern:

(5)

	Day	Night
♈	90	90
♉	96	84
♊	102	78
♋	108	72

[9] Cf., e.g., Chaine, Chron., p. 73.
[10] Cf. above II Intr. 4, 2.
[11] Aaboe-Sachs [1969], p. 8f.
[12] Published CCAG 8, 4, p. 232. The treatise was perhaps compiled by Balbillus who had great influence on Nero and Vespasianus. (Serious objections against the commonly accepted conjecture that Balbillus was the son of Thrasyllus, the astrological advisor of Claudius, were raised by Gagé, Basilèia, p. 76ff.).

etc. which is based on $M:m=108:72=3:2$ and $d=6$. That the units in these numbers are astronomically meaningless is merely the result of the astrological game according to which the numbers are used as days of lifetime. For us the essential fact is only the combination of a strictly linear scheme for the duration of daylight and night, combined with the Babylonian parameter $M:m=3:2$.

Finally we find a linear daylight scheme with $M:m=3:2$ attested in India through a statement of Varāhamihira (6th cent. A.D.) who relates such a pattern to the Paitāmahasiddhānta (1st cent. A.D.?).[13] That this rule is of ultimate Babylonian origin and was transmitted to India through hellenistic astrology is an obvious conjecture.

In the introduction to the present section we mentioned[14] linear schemes for the length of daylight adjusted to the latitude of Alexandria, i.e. to $M:m=14:10$, and progressing with a constant difference of $1/45^h$ per day.

One would perhaps expect that the derivation in Babylonian fashion of the length of daylight from rising times ought to have eliminated the linear daylight schemes. This, however, was not the case since astrology, as usual, kept operating with long antiquated methods. Thus we find in P. Mich. 149 (2nd cent. A.D.) a section on "hearing, seeing, and perceiving signs"[15] which mentions for some zodiacal signs the length of daylight (D) and night (N). Although only five signs are expressly mentioned necessary symmetries ( ) and the relation $D+N=24^h$ secure the values for 10 signs, leaving us only with the trivial restoration [ ] of the two extrema $m=9^h$ and $M=15^h$:

(♑)		$D=[9]^h$	$N=[15]^h$
♒	(♐)	(10)	14
♓	(♏)	(11)	13
(♈)	(♎)	(12)	(12)
♉	(♍)	13	(11)
♊	♌	14	10
(♋)		[15]	[9]

(6)

The ratio $M:m=15^h:9^h$ is frequently represented in our sources in connection with linear schemes. For example an astrological fragment on the influences of the zodiacal signs (ascribed to Hipparchus)[16] gives for the 12 months of the julian calendar and in relation to the 12 zodiacal signs for each month the length of daylight and night, varying strictly linearly with $d=1^h$ between 9^h (in December) and 15^h (in June). We find the same sequence in a parapegma (of the first century A.D.?) for each of the 12 schematically equated julian, macedonian, and alexandrian months,[17] and in astrological treatises,[18] e.g. in brontologia.[19] Gregory of

[13] Pañcasiddhāntikā XII 5; cf. Neugebauer-Pingree, Varāham., Pc. Sk. I, p. 105, II, p. 83. For the Paitāmahasiddhānta cf. I, p. 10.
[14] Cf. above p. 706 and Fig. 38.
[15] Mich. Pap. III, p. 77f. (XII, 11–48), p. 104f., p. 114f.
[16] Published by E. Maass, Anal. Erat., p. 141–146 (from Par. gr. 2426; cf. CCAG 8, 3, p. 61, cod. 46, F. 9^v). Cf. also above p. 332, n. 15.
[17] E.g. March = Dystros = Phamenoth. The text was compiled for a city in Asia with a harbour; cf. CCAG 9, 1, p. 128–137.
[18] E.g. CCAG 11, 1, p. 33/34, F. 71 (also Revilla, Catal. I, p. 298, No. 7).
[19] E.g. CCAG 8, 3, p. 123–125, p. 168f. (ascribed to the Prophet David).

Tours gives the same scheme in his De cursu stellarum [19a], written about 580. Also menologia of the Eastern Churches [20] (e.g. Maronite [21] and Georgian [21a]) follow the same linear pattern with $M:m=15:9$ and $d=1$. The same scheme appears in the 7th century in Armenia with Ananias of Shirak [22] in astrological context. Back to the earliest period of Greek astronomy lead arithmetically constructed shadow tables [23] which refer to a monthly variation of the length of daylight between 9^h and 15^h.

Exactly the pattern (6) is also given by Porphyry [23a] (who died about A.D. 300) and he refers it to "clima V" which is indeed characterized, e.g. in the Almagest, [23b] by $M:m=15:9$, conventionally called "Hellespont". Yet the connection with the archaic method of linear variation of the length of daylight suggests a purely arithmetical foundation of this ratio, independent of the later theory of trigonometrically computed "climata". Indeed if we assume a linear variation of daylight during six months with $d=1^h$ as the simplest increment it leads with $m+M=24^h$ directly to $m=9^h$, $M=15^h$.

At an early period this norm has been considered representative of "Greece" in general. We know this from Hipparchus and Geminus with specific references to Eudoxus, Aratus, and Attalus, [24] i.e. for a period from the fourth to the second century B.C. Finally $M=15^h$ for the longest daylight is the only value in integer hours (excepting $M=14^h$ for Alexandria) which is connected with an arithmetical scheme of rising times ("System B" type), [25] obviously constructed in the spirit of early hellenistic imitation of Babylonian methods but applied to a traditional Greek parameter which is therefore expressed in equinoctial hours, not in Babylonian time degrees.

In relating $M=15^h$ to an early phase of Greek astronomy I do not wish to deny that $M=15^h$ represented the latitude of the Hellespont or the area of Rome [26] at a time when the relationship between M and φ was fully understood. But the fact that $M=15^h$ fitted the sequence of the trigonometrically computed rising times [27] should not obscure the role of this parameter in early Greek astronomy.

[19a] Monumenta Germ. hist., Scriptores rerum Merov. I, 2, p. 405. I owe this reference to D. Pingree.

[20] Acta Sanctorum, Propylaeum ad Acta Sanctorum Novembris. Hippolytus Delehaye, Synaxarium Ecclesiae Constantinopolitanae e Codice Sirmondiano, nunc Berolinensi. Bruxelles, Soc. Bolland., 1902 (Greek).

[21] Sauget [1967] (Syriac) and Patrol. Or. 10, p. 347-353 (Arabic).

[21a] Garitte [1964].

[22] Petri [1964] Table, p. 283 (without realizing its schematic character).

[23] Cf. below p. 739.

[23a] Porphyry, Introd. 193: cf. CCAG 5.4, p. 209, 1-7.

[23b] Cf. below p. 725 (1).

[24] Cf. above p. 581, notes 7 to 10.

[25] Cf. below p. 731.

[26] Strabo, Geogr. II 5, 40 (Loeb I, p. 513) says that $M=15^h$ belongs to an area south of Rome but north of Naples. Geminus, Isag. VI 8 (Manitius, p. 70, 16) says that $M=15^h$ means "around (περί) Rome." Also the "Calendarium Colotianum" (1st cent. A.D.) and "Vallense" (Degrassi, Inscr., p. 284-287, Pl. 81-86) assume $M=15^h$. Pliny, NH II 186 (Loeb I, p. 319) associates "Italy" with $M=15^h$. Hyginus, Astron. IV (ed. Bunte, p. 100f.) says that where he lives $M:m=5:3$ and he therefore divides the day in 8 parts. John of Damascus, in the 8th century, declares, without any geographical specification, that $M=15^h$, $m=9^h$ (Expositio fidei 21, ed. Kotter, Patristische Texte u. Stud. 12, 1973, p. 57, 69f. and 84f. = Migne PG 94 col. 889-892).

[27] Cf. below p. 725.

2. Oblique Ascensions

In I A 4, 6 we have remarked that the length of daylight is the rising time of a semicircle of the ecliptic. In II Intr. 4, 1 we have seen how Babylonian astronomy made use of this fact by assigning specific rising times to the single zodiacal signs (or rather to twelve consecutive 30-degree sections of the ecliptic, beginning at the vernal point). These rising times (or "oblique ascensions") were not obtained as the result of accurate observations, or of mathematical deductions, but they were chosen so as to fit arithmetical progressions under the condition that the extrema obtainable for a semicircle would agree with the adopted ratio $M:m=3:2$ of longest to shortest daylight.[1] We have also shown that the lengths of daylight resulting from these rising times of the individual signs agree well with the actual facts (cf. II Intr. 4, 2, Figs. 5 and 6), keeping in mind the very limited accuracy of the ancient methods of measuring time. At any rate the introduction of the oblique ascensions represents a great improvement over the description of the variations of the length of daylight by a linear zigzag function.

It will be useful for what follows to assemble here some simple relations for the length of daylight and the oblique ascensions $\rho_1, \rho_2, \ldots, \rho_{12}$ (ρ_1 concerning the first 30° after the vernal point, etc.). All quantities may be measured either in equinoctial hours or in equatorial degrees, one day being either 24^h or 6,0°. The geographical latitudes (on which rising times depend) will be restricted for the following discussion to the area of the traditional "climata," thus excluding the equator as well as high northern latitudes.

At the vernal equinox the ecliptic makes a much smaller angle with the eastern horizon than at the autumnal equinox (cf. Fig. 39). It is therefore natural to assume

$$\rho_1<\rho_2<\rho_3<\cdots<\rho_6. \tag{1}$$

It is equally evident that the following symmetries hold[2]

$$\rho_1=\rho_{12}, \quad \rho_2=\rho_{11}, \quad \rho_3=\rho_{10}, \quad \rho_4=\rho_9, \quad \rho_5=\rho_8, \quad \rho_6=\rho_7. \tag{2}$$

If C_i denotes the length of daylight when the sun is located at the beginning of the i-th sign then we know that

$$C_i=\rho_i+\rho_{i+1}+\cdots+\rho_{i+5}. \tag{3}$$

If m is the shortest, M the longest daylight at the given place another set of symmetries follows from (2):

$$\begin{aligned} m&=C_{10}, \\ C_{11}&=C_9, \\ C_{12}&=C_8, \\ C_1&=C_7=12^h=3{,}0°, \\ C_2&=C_6, \\ C_3&=C_5, \\ M&=C_4. \end{aligned} \tag{4}$$

[1] Cf. above p. 706f. or p. 366.
[2] Cf., e.g., above p. 35.

If the rising times are known all C_i can be computed by using (3). Thus one obtains

$$\begin{aligned} m = C_{10} &= \rho_{10}+\rho_{11}+\rho_{12}+\rho_1+\rho_2+\rho_3 = 2(\rho_1+\rho_2+\rho_3)\\ C_{11} &= \rho_{11}+\cdots+\rho_4 = C_{10}-\rho_{10}+\rho_4\\ C_{12} &= C_{11}-\rho_{11}+\rho_5\\ C_1 &= \rho_1+\cdots+\rho_6 = 1/2(m+M) = 12^h = 3{,}0^\circ\\ &\text{etc.}\\ M = C_4 &= 2(\rho_4+\rho_5+\rho_6). \end{aligned} \tag{5}$$

The relations (4) and (5) are valid for all rising times which satisfy the conditions (1) to (3). It is now possible to assume for the ρ-s approximate values which form simple arithmetical patterns beyond (1) to (3). By means of (5) one will then obtain a scheme for the variation of the length of daylight which will satisfy (4) and which will be approximately correct as long as the ρ-s deviate not too much from the actual values of the rising times.[3]

What one will require from any scheme will be the accurate representation of the most important data, i.e. the extrema m and M of the length of daylight for the given region. Any roughly sinusoidal variation between such given limits will be acceptable. Hence the real problem consists in choosing the pattern for the ρ-s in such a way that one obtains by using (5) the given values m and M. We know of two simple patterns which were constructed with this condition in mind, here denoted as "System A" and "System B", respectively. Both devices have found a wide variety of applications in ancient astronomy and mathematical geography as we shall see in the following sections.

System A is characterized by the assumption that the rising times increase strictly linearly (of course satisfying (2)):

$$\rho_{n+1}-\rho_n = d, \qquad n=1,2,\ldots,5. \tag{6}$$

Thus from (4) and (3)

$$\begin{aligned} m &= 6(\rho_1+d)\\ M &= 6(\rho_4+d) = 6(\rho_1+4d)\,. \end{aligned}$$

and therefore

$$d = 1/18(M-m) = 1/9(M-3{,}0^\circ) = 1/9(M-12^h) \tag{7}$$

and

$$\begin{aligned} \rho_1 &= 1/6m - d = 1{,}20^\circ - 5/18M = 5;20^h - 5/18M\\ &= 5/18m - 20^\circ = 5/18m - 1;20^h. \end{aligned} \tag{8}$$

Hence we can find to given longest daylight M the rising time ρ_1 and d and thus all ρ-s and finally all C_i.

System B. The difference between ρ_4 and ρ_3 is assumed to be $2d$, otherwise d:

$$\begin{aligned} \rho_{n+1}-\rho_n &= d, \qquad n=1,2,4,5\\ \rho_4-\rho_3 &= 2d. \end{aligned} \tag{9}$$

[3] These were correctly determined, e.g., in the Almagest; cf. above I A 4, 1.

Thus from (4) and (3)

$$m=6(\rho_1+d)$$

$$M=6(\rho_4+d)=6(\rho_1+5d)$$

and therefore

$$d=1/24(M-m)=1/12(M-3{,}0^\circ)=1/12(M-12^h) \tag{10}$$

and

$$\rho_1=1/6m-d=1{,}15^\circ-1/4M=5^h-1/4M. \tag{11}$$

Hence again all ρ-s and C-s can be found to given M.

For the length of daylight one finds from (5):

System A	System B
$C_{10}=m$	$C_{10}=m$
$C_{11}=C_9=m+d$	$C_{11}=C_9=m+2d$
$C_{12}=C_8=m+4d$	$C_{12}=C_8=m+6d$
$C_1=C_7=m+9d=3{,}0^\circ=12^h$	$C_1=C_7=m+12d=3{,}0^\circ=12^h$ (12)
$C_2=C_6=m+14d$	$C_2=C_6=m+18d$
$C_3=C_5=m+17d$	$C_3=C_5=m+22d$
$C_4=M\ =m+18d$	$C_4=M\ =m+24d.$

It is important to note that the variation of daylight during the year is completely determined by the single parameter M as soon as one has chosen the arithmetical pattern A or B.

With this fact is connected, no doubt, the use of M as characteristic parameter for geographical latitude in pre-Ptolemaic geography. The altitude of the pole is easy to observe but very difficult to use in the determination of the length of daylight. In contrast the above given relations show how simple it is to obtain reasonable results by combining M with a plausible arithmetical hypothesis concerning the increase of rising times.

Finally one should note that it was for a good reason that we excluded the equator from our considerations. Indeed, for $M=m$ both (7) and (10) would lead to $d=0$ and hence to a constant rising time of 30° for all signs, contrary to geometric evidence.[4] Geometrically this means we do not differentiate between ecliptic and equator. In practice early Greek astronomy was hardly concerned about this limiting case.

The above given rules show that both for System A and B the rising time ρ_1 as well as the difference d are linear functions of M. Since all rising times ρ_k are of the form $\rho_1+c_k d$ (c_k=const.) we see that every ρ_k is a linear function of M. Figs. 40 and 41 give a graphical representation of the rising times for the zodiacal signs for latitudes between the equator and $M=17^h$ ($\varphi\approx 54^\circ$).[5] Fig. 42 shows the trigonometrically computed rising times for the same region as obtainable from Alm. II, 8. The agreement with the arithmetical schemes (in particular with

[4] Cf. below p. 720.

[5] For $M=12^h$ we have, of course, $\rho=30^\circ$ for both systems. At $M=17^h$ one finds $\rho_1=9;10^\circ$, $\rho_6=50;50^\circ$ for System A, $\rho_1=11;15^\circ$, $\rho_6=48;45^\circ$ for System B.

System B) is remarkably good. The reason lies in the fact that the trigonometrically computed values for each ρ_k are also nearly linear functions of M. Hence the choice of the longest daylight M as characterization of geographical locations, instead of φ, produced accidentally an excellent agreement between accurate and arithmetical methods. [6]

A. *System A*

We know from a remark by Hipparchus[1] that even in his time astronomers found difficulties with the rising times, at least according to Hipparchus' interpretation of an explanatory passage of his contemporary Attalus[2] on some verses in Aratus. Even if Attalus did not think – as Hipparchus implies – that all signs always rise in the same time one must admit that he did not present a clear description of the problem of the rising times of the zodiacal constellations.

How Hipparchus himself would have determined the rising times we do not know. If he had – as is quite likely – stereographic projection at his disposal[3] he could have found the correct solution in the same way as we know it from Ptolemy's Planisphaerium.[4] It is equally possible, however, that he worked with the arithmetical schemes that were derived from Babylonian prototypes. In any case we have evidence from the time of Hipparchus for the existence of such patterns in Greek astronomy thanks to the short treatise by Hypsicles[5] on the rising times ("Anaphorikos").[6] If we are not deceived by the accidental state of preservation of our sources the time of closest contact with Babylonian astronomy seems to be the second century B.C. which would therefore be the period which determined the character of hellenistic astronomy for the next three centuries.

Among our sources the Anaphorikos is of unique interest since it expressly develops for the reader the mathematical relations upon which the computation of the rising times for Alexandria rests. As the only empirical datum appears $M:m = 7:5$ (or the equivalent $M = 14^h$). With it is then combined the mathematical assumption that the rising times form a linear sequence, increasing and decreasing with constant difference; in our terminology: the structure of "System A" is postulated. As we have shown above (p. 713) all rising times can then be found; we need not give any details here of Hypsicles' procedure since in essence it is the same as our modern one. It is only to be remarked that not only the astronomical methods but also the mathematical ideas are based on Babylonian prototypes (which are in the case of mathematics of much greater antiquity than the astronomical applications[7]).

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[6] For $M = 12^h$ one finds in Alm. II, 8 $\rho_1 = \rho_6 = 27;50°$, $\rho_2 = \rho_5 = 29;54°$, $\rho_3 = \rho_4 = 32;16°$ and $\rho_1 = 11;19°$, $\rho_6 = 44;21°$ for $M = 17^h$.

[1] Commentary on Aratus II, 1, 4–16 (Manitius, p. 124/125).

[2] Comm. Ar. I, III, ed. Manitius, p. 4, 3; cf. also Manitius, p. 25ff.

[3] Cf. below V B 3, 7 B.

[4] Cf. below V B 3, 4.

[5] The time of Hypsicles, about 150 to 120 B.C., is suggested by his preface to his treatise commonly known as "Book XIV" of the "Elements" of Euclid where he refers to Apollonius (cf., e.g., Heath, Euclid III, p. 512); cf. also Huxley [1963], p. 102f.

[6] Cf. ed. De Falco-Krause-Neugebauer, Hypsikles (1966).

[7] Cf., e.g., Neugebauer MKT III, p. 76–80; cf. also, p. 83 s.v. Reihen.

Hypsicles himself was a competent mathematician as is evident from his treatise on regular solids[8] and he was also interested in number theory as is shown by a theorem of his on polygonal numbers, mentioned by Diophantus.[9] This particular problem is related to the study of arithmetic progressions and it seems therefore likely that certain extensions of the formulae for rising times may be Hypsicles' own invention. At least nothing similar has been found thus far in Babylonian sources.

The problem in question concerns the rising times of single degrees, to be determined in such a fashion that their total correctly amounts to the rising time of the signs to which they belong. They should furthermore form an arithmetic progression of constant difference δ, in keeping with the character of System A assumed for the rising times of the signs.

We assume M, the longest daylight, to be known and hence also d, the difference for the rising times of the signs. We consider only the increasing sequence $\rho_1, \rho_2, \ldots, \rho_6$ since the decreasing sequence consists of the same numerical values. In order to find δ Hypsicles proves the following theorem: Let $a_1, a_2, \ldots, a_{2n}$ be a sequence of numbers such that

$$a_1 < a_2 < \cdots < a_{2n} \quad \text{and} \quad a_{i+1} - a_i = \delta; \tag{1}$$

then

$$\sum_{n+1}^{2n} a_i - \sum_{1}^{n} a_i = n^2 \delta. \tag{2}$$

The following outlines the idea of the proof in modern terms, Hypsicles himself uses the customary Greek geometric terminology and a specific number of terms, e.g. $2n=6$.

His argument amounts to the following procedure: because of (1) we have

$$\begin{array}{ll}
a_{2n} - a_{2n-1} = a_n - a_{n-1} & \text{or:} \quad a_{2n} - a_n = a_{2n-1} - a_{n-1} \\
a_{2n-1} - a_{2n-2} = a_{n-1} - a_{n-2} & \text{or:} \quad a_{2n-1} - a_{n-1} = a_{2n-2} - a_{n-2} \\
a_{2n-2} - a_{2n-3} = a_{n-2} - a_{n-3} & \text{or:} \quad a_{2n-2} - a_{n-2} = a_{2n-3} - a_{n-3} \\
\text{etc.} & \qquad \text{etc.} \\
a_{n+2} - a_{n+1} = a_2 - a_1 & \text{or:} \quad a_{n+2} - a_2 = a_{n+1} - a_1 \\
\text{additional identity:} & \quad a_{n+1} - a_1 = a_{n+1} - a_1.
\end{array}$$

In the right-hand set of n equations the right side of each line is the same as the left-hand side in the next line. Hence the sum of all right-hand sides is $n(a_{n+1} - a_1)$ and because of (1) we have $a_{n+1} - a_1 = n\delta$ thus $n(a_{n+1} - a_1) = n^2\delta$. The total of the left-hand sides is the difference shown in (2); q.e.d.

Let now a_i represent the rising time (measured in time degrees) of the i-th degree counted from the beginning of Aries. Applying (2) to the $2n=60$ rising times of the degrees within two consecutive signs one has

$$\sum_{30k+31}^{30k+60} a_i - \sum_{30k+1}^{30k+30} a_i = 900\,\delta = 15{,}0\,\delta \qquad k=0,1,\ldots,4. \tag{3}$$

[8] Book XIV of the Elements, mentioned above note 5; cf., e.g., Heath. GM I, p. 419–421.

[9] Opera I, p. 470, 17–472, 22 ed. Tannery; also Heath, Dioph., p. 125f.: p. 252f.

Each of the sums of the left-hand side must amount to the rising time of its sign:

$$\sum_{30k+31}^{30k+60} a_i = \rho_{k+2} \qquad \sum_{30k+1}^{30k+30} a_i = \rho_{k+1}.$$

Hence from (3)

$$15{,}0\,\delta = \rho_{k+2} - \rho_{k+1} = d$$

and therefore

$$\delta = \frac{d}{15{,}0}. \tag{4}$$

For Alexandria $M = 14^h = 3{,}30°$ Hypsicles had found (cf. (7), p. 713)

$$d = \frac{30°}{9} = 3;20° \tag{5}$$

hence he obtained

$$\delta = 0;0{,}13{,}20° \tag{6}$$

as increment for the rising times of the single degrees according to System A.

By a slight modification of the method used in the proof of (2) Hypsicles shows that

$$\sum_{k\cdot 30+1}^{k\cdot 30+30} a_i = 15(a_{k\cdot 30+1} + a_{k\cdot 30+30}), \qquad k = 0, 1, \ldots, 4 \tag{7}$$

hence, e.g., for Aries ($k = 0$)

$$\sum_{1}^{30} a_i = 15(a_1 + a_{30}) = \rho_1. \tag{8}$$

But for ρ_1 one finds from (8), p. 713 with $m = 2{,}30°$ $d = 3;20°$

$$\rho_1 = 1/6\ 2{,}30 - 3;20 = 21;40°$$

thus

$$a_1 + a_{30} = 2a_1 + 29\,\delta = \frac{21;40}{15} = 1;26{,}40$$

and with (6)

$$a_1 = 1/2\,(1;26{,}40 - 0;6{,}26{,}40) = 0;40{,}6{,}40° \tag{9}$$

for the rising time of the first degree of Aries. All subsequent rising times can then be found by always adding δ. The rising times for the 30° consecutive degrees will then give correctly the rising times for the individual signs:

$$\begin{aligned} \rho_1 &= 21;40° = \rho_{12}, & \rho_4 &= 31;40 = \rho_9 \\ \rho_2 &= 25 \quad\ = \rho_{11}, & \rho_5 &= 35 \quad = \rho_8 \\ \rho_3 &= 28;20 \ = \rho_{10}, & \rho_6 &= 38;20 = \rho_7 \end{aligned} \tag{10}$$

for the latitude of Alexandria.

Although Hypsicles always uses Alexandria for his numerical examples he formulates his method such that it is applicable for any given value of M and one may well conjecture that he initiated the extension of System A to a whole linear sequence of values of M, starting either from Alexandria ($M = 14^h = 3{,}30°$) or

from Babylon ($M = 14;24^h = 3,36°$). These patterns are widely used in later astrological literature, e.g. by Vettius Valens.[10]

Manilius, who wrote his astronomical poem in the second decade of our era,[11] claims the invention of the basic rules for the determination of the rising times of our "System A".[12] Even if we were not fortunate enough to have the treatise by Hypsicles as well as its Babylonian prototypes one would doubt the validity of Manilius' claim. Not only does he mix in his poem data from System A and System B[13] but his examples refer sometimes to Alexandria, sometimes (perhaps inadvertedly) to Babylon, and sometimes to Rome (or rather to a locality with $M = 15^h$), always without informing the reader. No author of a simple and consistent arithmetical system could have presented such a conglomerate of incompletely understood and thus contradictory rules.

Manilius gives[14] the numerical values for the rising times, supposedly for Alexandria and $M = 14\ 1/2^h (= 3,37;30°)$ as follows

$$\begin{aligned}
\rho_1 &= 1;20^h = 40 \text{ stades } (= 20°) = \rho_{12} \\
\rho_2 &= 1;36 \;\;= 48 \text{ stades } (= 24°) = \rho_{11} \\
\rho_3 &= 1;52 \;\;= 56 \text{ stades } (= 28°) = \rho_{10} \\
\rho_4 &= 2;\ 8 \;\;\;= 64 \text{ stades } (= 32°) = \rho_9 \\
\rho_5 &= 2;24 \;\;= 72 \text{ stades } (= 36°) = \rho_8 \\
\rho_6 &= 2;40 \;\;= 80 \text{ stades } (= 40°) = \rho_7 .
\end{aligned} \tag{11}$$

But $M = 14\ 1/2^h$ is the standard value for Rhodes, never for Alexandria, and the values of the ρ-s are the values from System A for Babylon ($M = 3,36°$).[15] Only the units

$$1 \text{ stade} = 1/2° \tag{12}$$

seem to be peculiar to Manilius; we shall return to this question presently.[16]

The rules which Manilius expressly claims to be his own amount to the following

$$\rho_2 = \frac{m}{6}, \quad \rho_5 = \frac{M}{6}, \quad d = 1/3\,(\rho_5 - \rho_2) \tag{13}$$

and are correct. They are identical with the rules given by Hypsicles, except for a trivial factor 2 because Hypsicles starts with[17] $\rho_4 + \rho_5 + \rho_6 = 1/2\,M$. The first two equations in (13) hold for both Systems, the third, however, is characteristic for System A because ρ_5 in System B would be equal to $\rho_2 + 4d$. The equivalent

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[10] Vettius Valens, Anthol. (p. 157, 14f., ed. Kroll) remarks that "the king" (i.e. Nechepso) — in contrast to Hypsicles? — gave the rising times only for the first clima (i.e. for Alexandria). What follows in this section are horoscopes which make use of System A for Alexandria and for Babylon (cf. Neugebauer-Van Hoesen, Greek Horosc. Nos. L 82 and L 102 IVa and IVb). The reference to Nechepso is, of course, historically valueless.

[11] Cf. RE 14, 1 col. 1116, 12–1117, 10.

[12] Astron. III, 394: "*mihi debeat artem.*"

[13] For System B cf. below p. 721.

[14] Astron. III, 247–293; ed. Housman III, p. 22–26 and commentary p. XIII–XX.

[15] Cf., e.g., above p. 368 (1).

[16] Cf. below p. 719.

[17] Cf. Hypsikles, ed. De Falco-Krause-Neugebauer, p. 16, p. 48f.

of (13) once more is found in Rhetorius[18] ($\approx$A.D. 500[19]), applied to Alexandria.

A trivial variant of the rule for d in (13) is found in Vettius Valens I, 7[20], again valid only for System A:

$$d = 1/5(\rho_6 - \rho_1) = 1/3(\rho_5 - \rho_2) \tag{14}$$

applied to the numerical parameters for Babylon. On many other occasions in the Anthology rising times based on System A are used, e.g. in Book VIII, Chap. 6 for Babylon,[21] with ♈0° as vernal point. As we have remarked before[22] the tables in the same book combine System A with the vernal point ♈8° of System B. One may doubt whether Vettius Valens was aware of such inconsistencies in different doctrines he had inherited from his predecessors.

There remains the enigma of the "stades" used by Manilius,[23] which represent half-degrees of time. Nowhere else in ancient astronomy is this unit attested; only in the inscription of Keskinto (of the second century B.C.) do we find a statement[24] that a circle has 360 degrees or 720 "points," a "point" (στιγμή) being 1/2°. The purpose of these units remains unknown in both cases. A Babylonian origin seems very unlikely.[25]

The arithmetical schemes for the rising times were also known to Ptolemy. In the Almagest he finds it unnecessary to mention such antiquated methods but in the Tetrabiblos[26] he criticizes data for Alexandria from System A ($\rho_5 = \rho_8 = 35°$, $\rho_6 = \rho_7 = 38;20°$).

Porphyry (about A.D. 300) in his "Isagoge"[27] says that "the old ones" gave for Alexandria a set of rising times – known to us as System A – measured in degrees, and he compares it with the trigonometrically computed values from the Almagest (II, 8). Toward the end of the fourth century Paulus Alexandrinus[28] once more gives the rising times for Alexandria, now reckoned in hours and fractions (e.g. $\rho_1 = 1\ 1/3\ 1/9^h$ for 21;40°).

Firmicus Maternus, in the middle of the fourth century, gives a list of rising times for six different climata "as it was handed down to us by the wisest Greek teachers."[29] Unfortunately only a rather garbled scheme has reached us in which, e.g., Alexandria and Babylon are given identical rising times, although listed as different climata.[30] We shall come back to the arrangement of the climata;

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18 CCAG 1, p. 163, 4–14.
19 For the date of Rhetorius cf. above p. 258, n. 14.
20 Ed. Kroll, p. 23 f.
21 Ed. Kroll, p. 304, 4, 9, 18.
22 Above p. 597, n. 31.
23 Above p. 718 (11). Manilius III 275, 279, 291, 418, 437; ed. Housman III, p. 24 ff., p. 41, p. 44; commentary p. XIV.
24 Cf. above p. .
25 Since 0;30° = 0;0,5^d and since 1 bēru = 0;5^d (the "double-hour" of the older literature, e.g., Bilfinger) one could say that the stadia are the "minutes" of the bēru. But in Babylonian astronomy the units below the bēru are the uš, i.e. the degrees (1/30 bēru) and their sexagesimal parts, and not units of 1/60 bēru.
26 Tetrab. I, 21 Boll-Boer, p. 46, 10–14 = I, 20 Robbins, p. 94/95.
27 CCAG V, 4, p. 211, § 41 (194 Wolf).
28 Ed. Boer, p. 3, 4–8, 2: p. 10, 17–11, 3; p. 81, 13–19.
29 Firmicus, Math. II, 11 (ed. Kroll-Skutsch I, p. 53–55).
30 In fact the above p. 718 (11) mentioned values for Babylon (in degrees).

at the moment it suffices to say that the numbers follow the pattern of System A or are derived from it by simple omission of fractions.[31]

The Greek wisdom of the rising times spread not only to Rome but also, presumably from Alexandria, to the East. Bardesanes of Edessa (around A.D. 200), famous as heresiarch (and duly excommunicated) was deeply influenced by dualistic and astrological doctrines. It is therefore not surprising that he was well acquainted with System A of rising times for Babylon. For these values he is still quoted by George "Bishop of the Arabians," five centuries later.[32] And exactly the same numbers are also given by Varāha Mihira (6th cent.) in his Brihat Jataka I, 19, although entirely unfit for the latitudes of India.

Applications. I. As we have seen linear rising times remained in use among astrologers for many centuries. They were applied even to the solution of problems in spherical astronomy long after the exact formulae could have been found in the Almagest or in the Handy Tables. As an example may serve the determination of the culminating degree of the ecliptic (M) from the given ascendant (H) in the "Isagogika" of Paulus Alexandrinus (Chap. 30[1]).

On p. 42 we mentioned, following the Almagest, a relation which permits us to find the right ascension $\alpha(\mathrm{M})$ from the oblique ascension $\rho(\mathrm{H})$ at the given geographical latitude:

$$\alpha(\mathrm{M})=\rho(\mathrm{H})-90^\circ. \qquad (1)$$

Paulus, however, gives the rule

$$\lambda(\mathrm{M})=\rho(\mathrm{H})+\text{♑}\,0^\circ. \qquad (2)$$

Since $\lambda(\text{♑}\,0^\circ)=270^\circ=-90^\circ$ the right-hand side of (2) is identical with the right-hand side of (1). The left-hand sides then show that Paulus identifies the longitude of M with its right ascension, which is the same as to say that he identifies the ecliptic with the equator. As we remarked on p. 714 this is indeed inherent in the linear schemes for the rising times because $M=m$ leads to $d=0$. For Alexandria, however, Paulus simply uses the rising times from System A, i.e. $\rho_1=21;40^\circ$, $d=3;20^\circ$.[2] For an example he assumes $\mathrm{H}=\text{♌}\,15^\circ$ and thus finds

$$\rho(\mathrm{H})=\rho_1+\rho_2+\rho_3+\rho_4+1/2\,\rho_5=124;10^\circ$$

hence

$$\lambda(\mathrm{M})=124;10^\circ+\text{♑}\,0^\circ=\text{♉}\,4;10^\circ$$

as the culminating point of the ecliptic. The tables in Alm. II, 8 would give ♉ 10°.[3] No wonder Ptolemy says that the rising times of System A are "not even close to the truth."[4]

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[31] Cf. below p. 729.

[32] Cf. Ryssel [1893], p. 47 f. Syriac text and Latin translation in Patrologia Syriaca, Vol. I, part 2 (Paris 1907), p. 513, No. 8.

[1] Cf. for Paulus ($\approx$A.D. 375) below V C 2, 4 B; for Chap. 30 (which is not found in all MSS) cf. ed. Boer, p. 81 f.

[2] Cf. above p. 717.

[3] Alm. II, 8 gives $\rho(\mathrm{H})=127;29^\circ$, thus $\alpha(\mathrm{M})=37;29^\circ$ while $\alpha(\text{♉}\,10^\circ)=37;30^\circ$. The Handy Tables give $\rho(\mathrm{H})=127;31^\circ$ and $\alpha'(\text{♉}\,10^\circ)=127;30^\circ$.

[4] Tetrab. I, 21 Boll-Boer, p. 46, 7-9; =I, 20 Robbins, p. 20.

II. A purely astrological application of the rising times is known to us through Pliny and Censorinus.[5] Pliny refers to "Petosiris and Nechepso" (i.e. to hellenistic Egypt) a doctrine according to which human life cannot exceed the greatest rising time obtainable for the given geographical latitude for a quadrant of the ecliptic, the degrees being counted as years. Thus the most favorable quadrant would give for the longest lifetime

$$\kappa = \rho_5 + \rho_6 + \rho_7 = \rho_6 + \rho_7 + \rho_8 = 3\rho_6 - d. \tag{3}$$

Since $M = 6(\rho_6 - d)$ in both Systems one can find κ from

$$\kappa = 1/2\,M + 2d. \tag{4}$$

These relations explain statements reported by Pliny as well as by Censorinus: Berosus ($\approx$300 B.C.) declared 116 years to be the maximum length of human life, whereas Epigenes ($\approx$250 B.C.?) said that 112 years cannot be reached. Indeed, computing with System A for Babylon and Alexandria, respectively, one obtains with (4)[6]

Babylon: $\kappa = 116$
Alexandria: $\kappa = 111;40$.

Pliny mentions, furthermore, 124 years for Italy, Censorinus 120 years according to "others." No check is possible for these numbers since we have no accurate geographical data. Assuming, however, with Geminus[7] for Rome $M = 15^h$ one finds $\kappa = 122;30$ for System A, $\kappa = 120$ for System B.

If Epigenes indeed belongs to the middle of the third century B.C. we would have here the earliest evidence for the use of linear rising times in Alexandria[8].

B. System B

It is always a precarious thing to use very fragmentary material as if it were statistically significant. Being fully aware of the possibility of a purely accidental distribution of the extant sources we may nevertheless call attention to the fact that we know of many fewer authors who use rising times of System B than we found before with System A. It may be equally accidental that System B prevails in the lunar theory[1] and that for the position of the vernal point the norm ♈8° of System B is the rule, System A the exception.[2] This is surprising since one would expect a fixed correlation between rising times and vernal point, as is always the case in the Babylonian ephemerides. But that much, at least, follows unequivocally from our material that hellenistic astronomy did not hesitate to combine norms from different systems.[3]

[5] Pliny NH VII 49, 160 (Jan-Mayhoff II, p. 55, 19–56, 6; Loeb II, p. 612–615 but unreliable in the translation); Censorinus, De die nat. 17, 4 (ed. Hultsch, p. 31, 15–24). The best presentation of these passages is given by Honigmann in Mich. Pap. III, p. 307–311.

[6] One has for Babylon $M = 3{,}36^\circ$, $d = 4^\circ$; for Alexandria $M = 3{,}30^\circ$, $d = 3;20^\circ$. For System B one would find $\kappa = 110$ for Babylon, $\kappa = 114$ for Alexandria.

[7] Geminus, Isag. VI, 8 (Manitius, p. 70, 16f.); or above p. 581.

[8] There is no proof, however, for Honigmann's conjecture (Mich. Pap. III, p. 316) to see in Epigenes the "inventor" of the "astrological climata".

[1] Cf. above IV A 4, 3 A.

[2] Cf. IV A 4, 2 A and 2 B.

[3] Cf., e.g., Neugebauer-Van Hoesen, Greek Horosc., p. 184.

Table 1. System B

Clima	1	2	3	4	5	6	7
$\rho_1=\rho_{12}$	22°	21°	20°	19°	18°	17°	16°
$\rho_2=\rho_{11}$	24;40	24	23;20	22;40	22	21;20	20;40
$\rho_3=\rho_{10}$	27;20	27	26;40	26;20	26	25;40	25;20
$\rho_4=\rho_9$	32;40	33	33;20	33;40	34	34;20	34;40
$\rho_5=\rho_8$	35;20	36	36;40	37;20	38	38;40	39;20
$\rho_6=\rho_7$	38	39	40	41	42	43	44
d	2;40°	3°	3;20°	3;40°	4°	4;20°	4;40°
m	2,28° = 9;52^h	2,24° = 9;36^h	2,20° = 9;20^h	2,16° = 9; 4^h	2,12° = 8;48^h	2, 8° = 8;32^h	2, 4° = 8;1[illegible]
M	3,32° = 14; 8^h	3,36° = 14;24^h	3,40° = 14;40^h	3,44° = 14;56^h	3,48° = 15;12^h	3,52° = 15;28^h	3,56° = 15;4[illegible]
$M:m$	53:37	3:2	11:7	28:17	19:11	29:16	59:31

Chronologically the first evidence we have for the use of System B (outside Mesopotamia) is the astronomical poem of Manilius, written at the beginning of our era; we have already mentioned that he had adopted System A for the rising times supposedly at Alexandria, actually at Babylon.[4] Now we find Manilius using System B for the increments of the length of daylight at a locality with $M=15^h$, probably Rome.[5] For it he gives[6] 1/2 hour, 1 hour, and 1 1/2 hours, respectively, as increase in the length of daylight, beginning with Capricorn. We shall find the motivation for this rule in Cleomedes.[7]

System B is next represented in a Greek papyrus, written in the second century A.D., P. Mich. 149.[8] For seven geographical zones, or "climata," are given the rising times for Aries and Libra, i.e., in our terminology, ρ_1 and ρ_6, and the corresponding differences d, all measured in time degrees ($\chi\rho\acute{o}\nu o\iota$). The problem of the climata we shall postpone to a later section.[9] Here Table 1 suffices to show how the given values of ρ_1, ρ_6, and d fit an arithmetical pattern based on System B for each clima and with linear increase of the longest daylight M by 16 minutes from clima to clima.[10] The second clima – called "Syria" in the text – shows a pattern identical with System B for Babylon.[11] A remark in the text saying that for all regions the rising times for Cancer and Capricorn are 30° is obviously wrong as it stands, unless one interprets it to mean that the 30° of the ecliptic that contains the solstices at midpoints rise with 30° of the equator. This could either be a reminiscence of the Eudoxan arrangement of the cardinal points or a loose description of the fact that for all climata the rising times which straddle the line of symmetry have 30° as their mean value, e.g. $\rho_3+\rho_4=60°$ or $\rho_1+\rho_6=60°$, etc.

[4] Cf. above p. 718.
[5] Cf., e.g., Geminus, Isag., ed. Manitius, p. 70, 16; cf. also above p. 581.
[6] In Book III, 458–462; cf. also Housman, Manilius III, p. XIX f.
[7] Cf. below p. 723.
[8] Mich. Pap. III; text: XI, 38–XII, 11 (p. 76/77); transl.: p. 114; commentary by Honigmann: p. 301–321 and Neugebauer [1942, 2], p. 255–257.
[9] Cf. below p. 730.
[10] It follows from (7) or (10), p. 713 f., respectively, that also d and each ρ_i belongs to an arithmetic progression if the M form such a sequence.
[11] Cf., e.g., above p. 368 (1).

It is obvious that the pattern displayed in Table 1 is not an invention of the second century A.D.; the mere presence of the Babylonian scheme (in column 2) is sufficient proof. Similarly the reference to a System B type of variation in the length of daylight in Cleomedes[12] tells us nothing about the time of origin; in particular there is no reason to associate it with Posidonius, although he is repeatedly (but not here) mentioned by Cleomedes.[13]

What Cleomedes tells us is that the length of daylight increases, beginning with the shortest daylight $m=C_{10}$ (cf. above p. 712 (3) and (4)) successively by $1/12\,(M-m)$, $1/6\,(M-m)$, twice $1/4\,(M-m)$, $1/6\,(M-m)$, $1/12\,(M-m)$. It is easy to see that System B leads to these increments which are, because of (10), p. 714, the same as $2d$, $4d$, twice $6d$, $4d$, $2d$, respectively, in agreement with (12) B, but not with A. As an example Cleomedes names $M-m=6^h$ which characterizes, as we know, either the Hellespont, or Greece in general.[14] It is useful to derive for this parameter the specific values for the C_i and the ρ_i; one finds:

$$\begin{array}{ll} C_{10}=m=9^h & \rho_1=18;45^\circ=\rho_{12} \\ C_{11}=\ 9;30=C_9 & \rho_2=22;30\ =\rho_{11} \\ C_{12}=10;30=C_8 & \rho_3=26;15\ =\rho_{10} \\ C_1\ =12=C_7 & \rho_4=33;45\ =\rho_9 \\ C_2\ =13;30=C_6 & \rho_5=37;30\ =\rho_8 \\ C_3\ =14;30=C_5 & \rho_6=41;15\ =\rho_7 \\ C_4\ =M=15 & d=\ 3;45. \end{array} \tag{1}$$

The values given in the left column of (1) are exactly those listed in Syriac menologia of the Jacobite church, extant in manuscripts of the late Middle Ages (12th to 15th cent.)[14a] but surely based on much older sources.

As we have seen the increments of these C_i follow the rule given by Manilius.[15] A rather garbled parallel to Cleomedes' statement about the increments in the length of daylight is found in Plutarch (about A.D. 100) who obviously did not understand what he was talking about.[16]

The rule about the increments of daylight in the form given by Cleomedes is also found in Martianus Capella[17] (first half of the 5th century A.D.) and quoted verbatim by Gerbert[18] (later pope Sylvester II) at the end of the 10th century. Neither Martianus nor Gerbert were able to apply correctly their own rules. Martianus gives the following example for the rising times[19]:

$$\begin{array}{lll} \rho_1=20^\circ & \rho_3=28;45 & \rho_5=35 \\ \rho_2=25 & \rho_4=31;15 & \rho_6=40. \end{array} \tag{2}$$

[12] Cleomedes I, 6, ed. Ziegler, p. 50f.
[13] Cf. above IV B 3, 3.
[14] Cf. above p. 711.
[14a] Patrol. Or. 10, p. 59–87 (menologium from Aleppo); slightly garbled versions: p. 93–97 (from Scete in the Wâdi Natrûn, Lower Egypt), p. 102–107 (Antioch), p. 127–151 (Aleppo).
[15] Cf. above p. 722.
[16] Cf. Neugebauer [1942, 3], p. 458f.
[17] De nuptiis VIII, 878 (ed. Dick, p. 463, 10–15).
[18] Gerbert, Opera, ed. Bubnov, p. 39.
[19] Martianus, De nupt. VIII, 844f. (ed. Dick, p. 444, 1–445, 13).

Obviously this is neither a sequence of System B nor of System A since the differences are 5°, 3;45°, 2;30°, 3;45°, 5° and hence cannot lead to a sequence C_i which satisfies the above stated rule. In fact the middle difference is smallest ($d=2;30°$), instead of largest ($2d$). If we nevertheless accept the numbers as they stand one would find for the longest daylight $M=3,32;30=14;10^h$ i.e. a little more than for Alexandria. The correct values for Alexandria, System B, would be

$$\begin{array}{lll} \rho_1=22;30^\circ & \rho_4=32;30 & M=3,30^\circ=14^h \\ \rho_2=25 & \rho_5=35 & d=2;30^\circ. \\ \rho_3=27;30 & \rho_6=37;30 & \end{array} \qquad (3)$$

The identity of the values of ρ_2 and ρ_5 and of d with the values in the abortive scheme (2) shows that the pattern for Alexandria was originally intended and then clumsily modified, probably in order to obtain $M=14;10^h$ as better fitted for Carthage, the home of Martianus. Curiously enough exactly the same rising times (though measured in hours and with some garbled numbers for rising and setting) are found in the "Liber de astronomiae disciplinae peritia" of Gergis "philosophus Antiochenus, incola Aegypti" of the 11th century, preserved in a Latin translation probably made by Gerard of Cremona.[20] No locality is mentioned in this text and it is probably a mere accident that $M=14;10^h$ would roughly fit, e.g., Cairo. Clearly Islamic medieval astrologers could be as incompetent as their western contemporaries.

Gerbert, after referring to Martianus for the increments of daylight gives as examples the data for the twelve months of the year at $M=18^h$ and at $M=15^h$, assuming the latter value for the latitude for the Hellespont. His numbers for $M=18^h$ are

I and XII:	$6^h=m$	IV and IX:	15^h
II and XI:	9^h	V and VIII:	17^h
III and X:	12^h	VI and VII:	$18^h=M$.

Had he applied the rule he had given a few lines earlier he would have found that the first increment $1/12(M-m)=1^h$ was missing. Similarly he gives for $M=15^h$ the list

I and XII:	$9^h=m$	IV and IX:	$13;30^h$
II and XI:	$10;30^h$	V and VIII:	$14;30^h$
III and X:	12^h	VI and VII:	$15^h=M$.

Comparison with (1), p. 723 shows again the omission of one value, here $9;30^h$. The error originated in both cases from arranging the 12 months in six pairs, instead of in five pairs flanked by a single month for m and M, respectively. We will meet exactly the same type of error in the arrangement of shadow tables.[21]

[20] Cf. CCAG 12, p. 216, p. 223-228. On Gergis cf. also Ruska, Turba, p. 26 (No. 27) and p. 56f.
[21] Cf. below p. 740.

3. Climata

The original meaning of the term κλίμα (also ἔγκλιμα,[1] Latin *clima*[2]) is geometric in character: "inclination."[3] In astronomical context it means the inclination of the earth's axis with respect to the plane of the local horizon, directly observable by the ἔξαρμα πόλου, the "elevation of the pole." Obviously this concept presupposes the discovery of the sphericity of the earth.[4]

In modern languages "climate" has lost all of its original geometric meaning, referring only to meteorological conditions, due to a shift of emphasis which begins already in antiquity. In order to avoid misunderstanding I shall use the Greek forms, clima and climata, whenever I mean the ancient technical term.[5] Then, however, "climata" means much more than simply "inclinations." Two concepts are essential for the definition: (a) the climata are associated with specific values for the corresponding longest daylight M, and (b) these values of M form an arithmetic progression.

The value of M is, of course, constant for any circle parallel to the equator. Hence one can also say that the "climata" are a set of parallels whose M-values form an arithmetic progression. Ptolemy in a chapter on shadow lengths[6] (Alm. II, 6) considers 33 "parallels" from $M=12^h$ ($\varphi=0°$) to $M=24^h$ ($\varphi=90-\varepsilon$), the first 25 with $\Delta M=0;15^h$, followed by four with $\Delta M=0;30^h$, finally four with $\Delta M=1^h$. In the Geography[7] $\Delta M=0;15^h$ is used only for the 14 parallels from $M=12;15^h$ to $M=15;30^h$, to be followed by $\Delta M=0;30^h$ until $M=18^h$ (No. 19), then $M=19^h$ and finally as the 21st parallel $M=20^h$ ("Thule"). In the Almagest II, 8 the rising times[8] are given for 11 parallels with $\Delta M=0;30^h$ from $M=12^h$ to $M=17^h$. In this set the seven parallels from $M=13^h$ to $M=16^h$ are commonly known as the "*seven climata*", conventionally associated with the following geographical locations which testify to the hellenistic-Alexandrian origin of the whole pattern:

I	13^h	Meroe	V	15^h	Hellespont	
II	13;30	Syene	VI	15;30	Mid-Pontus	(1)
III	14	Lower Egypt	VII	16	Borysthenes.	
IV	14;30	Rhodes				

[1] Both terms are used interchangeably, e.g., by Geminus (cf. the index in Manitius, p. 307f., p. 325) or by Vettius Valens (Anthol., ed. Kroll, p. 317, 1/2, p. 343, 8/17, etc.).
[2] E.g. in the Latin version of Ptolemy's Analemma (Opera II, p. 217, 17).
[3] In geometry κλίμα can denote the length of the generating line of a cone or the edge of a pyramid (e.g. Heron, Stereom. 14 and 30, Opera V, p. 12, 15–20 and p. 28, a4/b5).
[4] Cf. above IV A 2.
[5] An entirely different meaning of κλίμα is associated with the four cardinal directions, East, West, etc. (e.g. Heron, Geom., Opera IV, p. 176, 18f. or Isidorus, Etym. III 42, 1; XIII 1, 3) or with the four principal winds (Cramer, Anecd. gr. Par. I, p. 369, 3f.).
[6] Cf. above Table 2 (p. 44).
[7] Geogr. I, 23, ed. Nobbe, p. 45–47; trsl. Mžik, p. 65f. The same spacing is also found in the "Diagnosis" (cf. Diller [1943], p. 44, verso 12–p. 46).
[8] Cf. above I A 4, 1.

Ptolemy takes this list for granted in the arrangement of several of his tables.[9] But he not only may exceed these "seven" climata, as in the above mentioned lists of "parallels," but he also can restrict the number of climata to five in his "Phaseis"[10] where he begins with Syene as No. I.

The set (1) of the "seven climata" contains for $M=14^h$ (or $M:m=7:5$) the latitude of Alexandria. It does not single out, however, $M=14;24^h=3,36^\circ$ (or $M:m=3:2$), the parameters characteristic for Babylon.[11] But in Table 1 (p. 722) we have mentioned another sequence of seven climata which is obviously based on Babylon. These climata begin at $M=14;8^h=3,32^\circ$ – thus not very far from the value $M=14^h$ traditional for Alexandria – and proceed in steps of $0;16^h=4^\circ$ until $M=15;44^h=3,56^\circ$. Hence Alexandria is now "clima I" and "clima II" is exactly Babylon.

This second list of seven climata is well attested in the astrological literature, e.g. in the Anthology of Vettius Valens, a younger contemporary of Ptolemy.[12] This does not justify, however, a distinction between "astrological" and "geographical" climata.[13] Both sets are extensions of the same Babylonian pattern, one a little less drastic (hence older?) than the other. We shall see in the next section that there existed additional schemes, operating on the same principle of arithmetic progression for M, but, e.g., based either on System A or on B for the rising times. This multiplicity of methods alone eliminates the hypothesis of a simple alternative between two types of climata. And the fact that earlier and more primitive methods survive much longer in the astrological literature than in works based on theoretical astronomy proves nothing for an original distinction between astrological and astronomical climata.

A precise distinction has been postulated between "parallel" (as referring to a mathematical line) and "clima" (meaning a narrow zone for which M may be considered practically constant).[14] In fact, however, the ancient terminology is much less rigid; Ptolemy, e.g., uses the two terms interchangeably.[15] In mathematical context (e.g. in the computation of oblique ascensions or of shadow lengths) "clima" as well as "parallel" mean, of course, always a definite circle.

[9] Angles between ecliptic and meridian: Alm. II, 13 (cf. p. 50f.); diagram for ortive amplitudes: Alm. VI, 11 (cf. Fig. 32 below p. 1216), with explicit reference to the "seven climata" (ed. Heiberg, p. 538, 25/539, 1); tables for oblique ascensions in the "Handy Tables" (cf. V C 4, 2 A); tables for parallaxes, ibid. (cf. p. 990); values for φ in the nomogram of the "Analemma" (cf. Fig. 35, p. 1382).

[10] Opera II, p. 4, 3–20.

[11] Cf., e.g., above p. 366.

[12] Vettius Valens, Anthol., ed. Kroll, p. 24, 13–21 for seven climata, p. 157, 14 for Alexandria = No. I, p. 157, 22 for Babylon = No. II. Cf. also Honigmann SK, p. 42f. and Neugebauer-Van Hoesen, Greek Horosc. L82, L102, IVa and b; also p. 184.

[13] This unfortunate classification was introduced by Honigmann (SK p. 3 et passim). When Paulus Alex. in his astrological treatise calls the clima of Alexandria the "third" (ed. Boer, p. 3, 5 and p. 10, 18), thus following (1), then Honigmann simply speaks about an "unpassende Reminiszenz."

[14] Honigmann, SK, p. 11/12. The width of a zone of practically constant conditions is occasionally specified to be 400 stades (Geminus, Isagoge, ed. Manitius, p. 62/65, p. 170/171). Cleomedes I, 10 (ed. Ziegler, p. 98, 4f.) says that the gnomon near Syene casts no noon-shadow at the summer solstice in an area of 300 stades diameter. Pliny NH II 182 (Jan-Mayhoff I, p. 197, 5–7; Loeb I, p. 315) considers 300 to 500 stades the limits for practically equal shadow lengths, whereas Posidonius takes 400 stades as within observational accuracy in the determination of latitudes (Strabo, Geogr. II 1, 35, Loeb I, p. 330/1; Budé I 2, p. 44).

[15] Cf., e.g., Ptolemy, Opera II, p. 161, 24 and p. 164, 3.

In geographical context, however, "clima" can mean a zone between parallels, more or less accurately defined. In the Middle Ages the climata are usually contiguous, with boundaries at $M = ..;15^h$ and $..;45^h$, e.g., in Abū'l Fedā[16] ($\approx$A.D. 1300) or in the Alfonsine Tables[17] (late 13th cent.).

It is of fundamental importance for the understanding of the historical development of the concept "climata" to realize that it has its origin in problems of spherical astronomy, not in geography. Babylonian astronomers had discovered that the length of daylight is a function of the rising times of consecutive ecliptic arcs. When the Greeks became aware of the significance of the variability of the longest daylight it was natural, under the influence of Babylonian methods, to arrange the values of M in a sequence of constant difference. These procedures point to the time of early Alexandrian science, perhaps the second or third century B.C. To associate a single person with this extension of Babylonian methods is, of course, impossible.[18]

To utilize such a mathematical scheme as an ordering principle of empirical geographical material — lists of cities, ethnographic characteristics of zones, etc. — represents a secondary development of very little astronomical concern. In the extant literature, ancient as well as modern, this later accretion has almost completely obscured the initial purely mathematical structure. The shift of interest to the geographical aspects of the theory of the climata further contributed to the difficulties preventing us from reaching a secure chronology of its early stages.

Another problem which remains unsolved is the question of the origin of the number seven for the climata. From a purely arithmetical viewpoint a beginning at the equator ($M = 12^h$) and a convenient constant increment $\Delta M = 1/4^h$ or $\Delta M = 1/2^h$ require no special motivation. Indeed, we find both increments in Hipparchus and in Ptolemy.[19] The selection of exactly seven of these parallels, however, can hardly be explained by any practical requirements.[20] An astrological relation to the seven planets would not be surprising but textual evidence for it seems lacking, except for a vague remark by Eusebius ($\approx$A.D. 300) in refuting Bardesanes ($\approx$A.D. 200) who seems to have said that each clima is ruled by one of the planets.[21] But no such doctrine seems to be attested for the seven climata in hellenistic astrology.[22]

A. Climata and Rising Times

It is essential for our analysis of the concept "climata" that we realize that we are dealing with a problem of mathematical astronomy, not of geography. It is also clear that the basic method originated under Babylonian influence at an

[16] Geogr., trsl. Reinaud II, 1, p. 9–12.
[17] E 1 in the edition Venice 1521.
[18] Cf. above p. 721; also below p. 733, n. 28 for a possible connection with Eudoxus, i.e. evidence from the fourth century B.C.
[19] According to Strabo, Geogr. II 5, 36–42 (Loeb I, p. 509–517) Hipparchus singled out 10 parallels between $M = 13^h$ and 17^h for which ΔM is either $1/2^h$ or $1/4^h$, and once, at the end, 1^h. For Ptolemy see above p. 725.
[20] Entirely unfounded is the association of some Old-Babylonian mythology with this hellenistic invention (Honigmann, SK, p. 8).
[21] Eusebius, Praep. evang. VI 10 (278) [164], ed. Dindorf, Opera I, p. 321, 3–6.
[22] Perhaps there exists some relation to the geographical version of the doctrine of the "Heptomades"; cf. Boll, Lebensalter, p. 137ff.; also Kranz [1938], p. 139.

early period when Greek astronomy did not yet have any spherical astronomy at its disposal. Thus the longest daylight M was accepted as the fundamental parameter of the problem which consisted in constructing for any given M a proper sequence of rising times, either according to System A or to System B as known from the Babylonian prototype. We still have the treatise of Hypsicles[1] (about 150 B.C.) which gave in detail the solution for System A and for the M of Alexandria (14^h). Similar treatises must have existed for System B and for sequences of M. No doubt this is the nucleus from which the doctrine of the "seven climata" evolved; the number seven as well as $\Delta M = 4°$ seem to be associated with these arithmetical methods from the very beginning.

With the invention of spherical trigonometry, or of equivalent methods, perhaps in the time of Hipparchus, the linear schemes for the rising times moved more into the background, while the values of M became related to geographical latitudes φ. Nevertheless the number seven of the climata and their Greek names, however distorted, survived deep into the Middle Ages. The correct determination of φ to any M had been given in the Almagest or in the Handy Tables. The low level of medieval science can again be illustrated by the fact that the same canonical sequence of M-values was associated with a whole spectrum of more or less reasonable values for the latitudes φ. Obviously nobody dreamt of computing φ from M or vice versa and one dealt with these quantities as with independent parameters which could be individually modified to accommodate geographical requirements.[2]

In the following we shall only deal with the very early phase of the theory of the climata, i.e. when it was restricted to the arithmetical pattern of System A or B and concerned only with the rising times for the 12 zodiacal signs.

In order to simplify comparisons in all cases we display the values M, $M:m$, and d (always in degrees), regardless of the way these data appear in the sources, explicit or implicit.[3] We count the climata from I to VII, beginning with the southernmost as No. I, whatever the relation might be to the climata I to VII in (1), p. 725.

System A. Within the framework of our historical reconstruction it is not surprising to find a set of seven climata based on Alexandria and computing with System A. The table of rising times which belongs to "clima I" in this list is, of course, identical with the table (10), p. 717 derived in Hypsicles' treatise. The whole sequence of seven climata[4] is found in the Anthology of Vettius Valens (I, 7[5]); it is represented by the following set of parameters:

	I	II	III	IV	V	VI	VII		
M	3,30° $=14^h$	3,34° $=14;16^h$	3,38° $=14;32^h$	3,42° $=14;48^h$	3,46° $=15;4^h$	3,50° $=15;20^h$	3,54° $=15;36^h$	$\Delta M =$ $4° = 0;16^h$	(1)
$M:m$	7:5	107:73	109:71	37:23	113:67	23:13	13:7		
d	3;20°	3;46,40°	4;13,20°	4;40°	5;6,40°	5;33,20°	6	$\Delta d = 0;26,40°$	

[1] Cf. above p. 715.
[2] Cf., e.g., Millás-Vallicrosa, Est. Azar., p. 64 and p. 67.
[3] As we have seen the "System" in combination with M determines all parameters needed for the computation of a table of oblique ascensions (cf. p. 713).
[4] Note the remark "since there exist seven climata" (ed. Kroll, p. 24, 13 f.).
[5] Ed. Kroll, p. 24, 5–21.

In III, 16[6] Vettius Valens mentions the "Anaphorikos" of Hypsicles and calls the clima of Alexandria "first" clima. When Paulus Alexandrinus refers to the "clima through Alexandria" he calls it the "third clima";[7] he then follows the terminology of Ptolemy's list.[8] Again we see that the terminology is as flexible as the choice of the basic parameters.

The seven climata which follow the above given scheme (1) do not contain the table for Babylon for which $M=3{,}36^\circ$. Nevertheless its presence is felt in many of our sources. Vettius Valens, e.g., right after the reference quoted above to Alexandria's "clima I" gives for "clima II" not the data which belong to (1) but rising times for Babylon.[9] In I, 6 and I, 7, however, he cites Babylonian values as belonging to the "clima of Syria" which he again counts as No. II.[10]

No doubt there existed beside the scheme (1) another one for System A, based on $M=3{,}36^\circ$ for clima II and progressing as before with $\Delta M=4^\circ$. Indeed we shall find exactly such a list for System B[11] (which, incidentally, also calls clima II the clima of "Syria").

The values for M in (1) and in the restored scheme based on $M=3{,}36^\circ$ can be combined into a table of 14 equidistant values of M with $\Delta M=2^\circ$. Apparently Firmicus Maternus (≈ 350 A.D.) had such a combined pattern at his disposal from which he gave garbled excerpts. Table 2 shows the completed scheme; values in [ ] are omitted by Firmicus.[12] At his time the original meaning of the linear rising times had been so completely obliterated that he could assign the same table (II b, i.e. Babylon) to Alexandria and (!) Babylon. Obviously he should have started with I a, i.e. Alexandria, which would have provided him with seven climata instead of only six.

Firmicus is certainly not the inventor of the combined pattern of arithmetical rising times. Some three centuries earlier Pliny made use of the same set of "climata" in Book VI of his Natural History[13] where he assigns to specific geographical areas values for the longest daylight which agree exactly with six of the values M shown in Table 2. Pliny's seventh value is $M=15^h$ for the Hellespont, called clima V as is traditional with the trigonometric climata.[14] Alexandria and Babylon are of course correctly separated by Pliny; Rome and Ancona are placed one step of $0;8^h$ farther north than with Firmicus. Evidently there existed no fixed tradition how to incorporate Italian localities within the arithmetical patterns developed in an early period of hellenistic astronomy.[15]

[6] Ed. Kroll, p. 157, 12–17.
[7] Ed. Boer, p. 3, 5; p. 10, 18. Cf. also above p. 726, n. 13.
[8] Above p. 725 (1).
[9] Ed. Kroll, p. 157, 20–23. The same in I, 14 (Kroll, p. 28 f.).
[10] Ed. Kroll, p. 22, 23 and p. 23, 11/12.
[11] Cf. below p. 730.
[12] Math. II, 11, ed. Kroll-Skutsch I, p. 53–55. For details of the reconstruction cf. Neugebauer [1942, 2], p. 258 f.
[13] Pliny, NH VI, 211–218 (Jan-Mayhoff I, p. 517–521; Loeb II, p. 4 4–501). Cf. also below p. 747.
[14] Above p. 725 (1). Cf., however, for the special role of $M=15^h$ above p. 711 f.
[15] Since Pliny refers to Nigidius Figulus (1st cent. B.C.) in connection with the value $M=15;12^h$ for Rome it has been assumed that Nigidius was Pliny's direct source for the whole selection of "climata" (cf., e.g., Honigmann, SK, p. 31, p. 45, etc). Honigmann then appointed Serapion gnomonicus, supposedly a pupil of Hipparchus, as "ältesten Urheber" of the whole scheme. I see no gain in this web of hypotheses. Incidentally, $M=15;12^h$ is also in Ptolemy's Geography (VIII 8, 3 Nobbe, p. 205, 7 f.) the value given for Rome.

Table 2

Clima	I a	I b	II a	II b	III a	III b	IV a	IV b
M	[3,30° =14^h	[3,32° =14;8^h	[3,34° =14;16^h	3,36° =14;24^h	3,38° =14;32^h	3,40° =14;40^h	[3,42° =14;48^h	3,44° =14;56^h
$M:m$	7:5	53:37	107:73	3:2	109:71	11:7	37:23	28:17
d	3;20°]	3;33,20°]	3;46,40°]	4°	4;13,20°	4;26,40°	4;40°]	4;53,20°
Firmicus	[Alexandria]			Alex. and (!) Babylon	Rhodes	Athens		Hellespont
Pliny	1 Alexandria			2 Babylon	3 Rhodes	4 Athens		

V a	V b	VI a	VI b	VII a	VII b	Clima
3,46° =15;4^h	3,48° =15;12^h	[3,50° =15;20^h	3,52° =15;28^h	3,54° =15;36^h	[3,56° =15;44^h	M
113:67	[19:11	23:13	29:16	[13:7	59:31	$M:m$
5;6,40°	5;20°]	5;33,20°]	5;46,40°	6°]	6;13,20°]	d
Rome			Ancona			Firmicus
	6 Rome			7 Ancona		Pliny

System B. As we have remarked previously[16] System B is not very frequently attested in our sources. Fortunately, however, we have from P. Mich. 149[17] for this system a list for seven climata, similar to (1), p. 728, but based on Babylon. We have referred above to this text and gave the corresponding rising times in Table 1 on p. 722. For the sake of comparison we repeat here the basic parameters:

	I	II	III	IV	V	VI	VII	
M	3,32° =14;8^h	3,36° =14;24^h	3,40° =14;40^h	3,44° =14;56^h	3,48° =15;12^h	3,52° =15;28^h	3,56° =15;44^h	$\Delta M=$ 4°=0;16^h
$M:m$	53:37	3:2	11:7	28:17	19:11	29:16	59:31	
d	2;40°	3°	3;20°	3;40°	4°	4;20°	4;40°	Δd=0;20°
	Ethiopia	Syria	Rhodes	Asia, Ionia	Argos	Rome, Italy Mar. Gaul	Germany, Brit.[18]	

(2)

[16] Cf. above p. 721.

[17] For the details cf. Neugebauer [1942, 2], p. 256f.

[18] One more name is only partially preserved, perhaps A[sia] or A[rmenia].

The names of the zones are rather unusual; in particular "Ethiopia" for a zone north of Alexandria is strange. Perhaps the author knew Ethiopia to be the southern-most region of the "inhabited world"[19] and thus assumed it to represent the first clima. On the whole the geographical names give the impression of being borrowed from a list of climata with $\Delta M = 1/2^h$.

Note that one more step in (2) would lead to $M = 4{,}0° = 16^h$, $M:m = 2:1$, i.e. to the boundary $M = 16^h$ of Ptolemy's climata. The linear methods, however, seem never to have made this step beyond the number seven.

Additional Modifications. So far we have four types of "seven climata" which operate with arithmetical schemes: either based on Alexandria or Babylon as starting points and either System A or System B for the rising times. These four schemes have the increment $\Delta M = 4°$ in common, operating in typical Babylonian fashion with time degrees. Table 3, p. 732, shows the complete list of rising times.

It is with Hipparchus that we first find evidence for an adjustment to a reckoning with equinoctial hours by choosing $\Delta M = 1/4^h$ (or $1/2^h$, or 1^h), thus stepping outside the Babylonian numerical tradition by shifting to the Egyptian-hellenistic 24-hour system. Hipparchus also did not restrict himself to the number seven of parallels and the same holds for Ptolemy.[20] Nevertheless Ptolemy knows of a set of "seven" climata with $\Delta M = 1/2^h$ which he assumes, e.g., in the Handy Tables[21] and which became the standard during the Middle Ages, beginning with $M = 13^h$ at Meroe and ending with $M = 16^h$ at the Borysthenes.[22] We do not know when this pattern was introduced; only its Alexandrian origin seems evident and the number seven betrays the influence of the arithmetical predecessors.

The question arises whether the new scheme with $\Delta M = 0;30^h$ was related exclusively to trigonometrically determined oblique ascensions or whether it can also be associated with arithmetical methods. There exists indeed evidence for the second alternative, though only in relatively late sources. As we have seen[23] Cleomedes (4th cent. A.D.) gives rules which concern rising times of System B applied to $M = 15^h$, a parameter which does not belong to the linear sequences of Table 3. The same rules were also used by Martianus Capella (5th cent.) and, taken from him, with Gerbert[24] who applied them (not very skillfully) to $M = 18^h$ and $M = 15^h$. Finally we have from the late Middle Ages Syriac lists of length of daylight for each of the twelve months of the year, based on $M = 15^h$ and the corresponding set of rising times which correctly follow System B.[25] As usual the Near Eastern tradition is superior to the western.[26]

However late these sources may be it seems likely that the application of a linear pattern of rising times to $M = 15^h$ belongs to an early phase of Greek astronomy, in particular since we know that this value of M plays a role of its own,[27]

[19] Cf., e.g., Strabo, Geogr. II 2, 2 (Loeb I, p. 363); I 2, 24 (Loeb I, p. 111 ff.); etc.
[20] Cf. above p. 727, n. 19.
[21] Cf. above p. 726, n. 9.
[22] Cf. above p. 725 (1).
[23] Above p. 723; cf. also for Manilius, p. 722.
[24] Cf. above p. 724.
[25] Cf. above p. 723 (1). For earlier Syriac sources on rising times cf. above p. 720. See also Bar Hebraeus, L'asc. II, 3. ed. Nau, p. 143–157; on climata l.c. II, 1, 7–9, p. 125–129 and Candel. p. 583–590.
[26] Cf. below p. 745.
[27] Cf. above p. 711.

Table 3

Clima		I		II		III		IV		V		VI		VII	
		Alex.	b	a	Bab.	a	b	a	b	a	b	a	b	a	b
	M	3,30°	3,32	3,34	3,36	3,38	3,40	3,42	3,44	3,46	3,48	3,50	3,52	3,54	3,56
	m	2,30	2,28	2,26	2,24	2,22	2,20	2,18	2,16	2,14	2,12	2,10	2,8	2,6	2,4
	$M:m$	7:5	53:37	107:73	3:2	109:71	11:7	37:23	28:17	113:67	19:11	23:13	29:16	13:7	59:31
	ρ_1	21;40°	21; 6,40	20;33,20	20	19;26,40	18;53,20	18;20	17;46,40	17;13,20	16;40	16; 6,40	15;33,20	15	14;26,40
	ρ_2	25	24;40	24;20	24	23;40	23;20	23	22;40	22;20	22	21;40	21;20	21	20;40
	ρ_3	28;20	28;13,20	28; 6,40	28	27;53,20	27;46,40	27;40	27;33,20	27;26,40	27;20	27;13,20	27; 6,40	27	26;53,20
A	ρ_4	31;40	31;46,40	31;53,20	32	32; 6,40	32;13,20	32;20	32;26,40	32;33,20	32;40	32;46,40	32;53,20	33	33; 6,40
	ρ_5	35	35;20	35;40	36	36;20	36;40	37	37;20	37;40	38	38;20	38;40	39	39;20
	ρ_6	38;20	38;53,20	39;26,40	40	40;33,20	41; 6,40	41;40	42;13,20	42;46,40	43;20	43;53,20	44;26,40	45	45;33,20
	d	3;20	3;33,20	3;46,40	4	4;13,20	4;26,40	4;40	4;53,20	5; 6,40	5;20	5;33,20	5;46,40	6	6;13,20
	ρ_1	22;30	22	21;30	21	20;30	20	19;30	19	18;30	18	17;30	17	16;30	16
	ρ_2	25	24;40	24;20	24	23;40	23;20	23	22;40	22;20	22	21;40	21;20	21	20;40
	ρ_3	27;30	27;20	27;10	27	26;50	26;40	26;30	26;20	26;10	26	25;50	25;40	25;30	25;20
B	ρ_4	32;30	32;40	32;50	33	33;10	33;20	33;30	33;40	33;50	34	34;10	34;20	34;30	34;40
	ρ_5	35	35;20	35;40	36	36;20	36;40	37	37;20	37;40	38	38;20	38;40	39	39;20
	ρ_6	37;30	38	38;20	39	39;30	40	40;30	41	41;30	42	42;30	43	43;30	44
	d	2;30	2;40	2;50	3	3;10	3;20	3;30	3;40	3;50	4	4;10	4;20	4;30	4;40

quite independent of the sequence of the trigonometric climata. To associate with it an arithmetical scheme would be a quite natural step in the adaptation of Babylonian astronomy.[28]

B. Early Mathematical Geography

Early Greek geography is greatly influenced by a tendency to adjust empirical data to numerological speculations. It is the same tendency which we have seen at work in the arrangement of the climata[1] and which we shall meet again in the arithmetical schemes of the shadow tables.[2] In the field of geography the definition of the terrestrial zones in the "Introduction" by Geminus is an example.[3] Using the sexagesimal division of the circle according to which the circumference c contains 60 parts[4]

$$c = 1{,}0^{p} \tag{1}$$

he divides the earth's quadrant into three zones: hot, temperate, cold, covering 4^p, 5^p, 6^p, respectively, beginning at the equator (cf. Fig. 43). The northern limit of the hot zone at

$$4^{p} = c/15 \tag{2}$$

corresponds to the obliquity of the ecliptic. Locating the arctic circle[5] such that it separates arcs of exactly 5^p and 6^p may well be the real cause of the importance of $\varphi = 36°$ in later geography and astronomy, rather than any relation to Hipparchus or to Rhodes.[6]

The lucky accident that the obliquity of the ecliptic nearly fits the arithmetical pattern (2)

$$\varepsilon = c/15 \tag{3}$$

permits us to define ε as the side of the regular 15-gon inscribed in the celestial great circle through the solstices and the poles of equator and ecliptic. Obviously this numerological description had an appeal of its own. We find this definition of ε at the beginning of our era in Vitruvius,[7] in the second century in Theon of Smyrna,[8] in the third in Anatolius, Bishop of Laodicea,[9] in the fifth in Proclus.[10]

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[28] Hipparchus tells us (Ar. Comm., Manitius, p. 29) that Eudoxus applied in the "Phenomena" the ratio $M:m = 12:7$, presumably for a certain region in Greece, after having used 5:3 (i.e. $M = 15^h$) in the "Enoptron" for Greece in general (Manitius, p. 23; cf. also above p. 581, n. 8). The ratio $M:m = 12:7$ can hardly be correct, however, because it cannot be expressed in units of hours or degrees ($M \approx 15;9,28,\ldots^h = 3,47;22,\ldots^\circ$). It is tempting to emend the ratio to 11:7 which belongs as climate III b to the Babylon sequence, associated with "Athens" (cf. above Table 2, p. 730) or with "Rhodes" (above p. 730 (2)). If one accepts this emendation we would have here the earliest evidence for the arithmetical climata, indeed from prehellenistic times.

[1] Above IV D 1, 3 A.

[2] Below IV D 2, 1 A.

[3] Geminus, Intr. V, 45–48 (Manitius, p. 58, 18–60, 13); XVI, 6–12 (Manitius, p. 164, 22–168, 20).

[4] Greek *μέρη*: cf. above p. 590 and note 2 there.

[5] Cf. Fig. 1, p. 1351.

[6] Cf. above p. 275.

[7] Archit. IX 7 (ed. Krohn, p. 216, 6f.; Loeb II, p. 250/251; Budé, p. 27).

[8] Ed. Hiller, p. 151, 16–18 (Martin, p. 214/215; Dupuis, p. 246/247); p. 199, 7f. (Martin p. 324/325; Dupuis p. 320/321); p. 202, 12; p. 203, 10–14, etc.

[9] Cf. Ver Eecke, Dioph., Introd., p. XI, n. 2; also Heron, Opera IV, p. 168, 10–12 = Theon Smyrn., ed. Hiller, p. 199, 7.

[10] Comm. Euclid., ed. Friedlein, p. 269, 13–18 (trsl. Ver Eecke, p. 231); Hypotyposis III, 28 (ed. Manitius, p. 54, 1–5), VI, 13 (Manitius, p. 206, 6–8). Obviously wrong is a reconstruction by Mugler of a corrupt passage in Proclus, Comm. Rep., trsl. Festugière II, p. 152, n. 1 which leads to $\varepsilon = 20°$ (!).

The numerical value $\varepsilon = 24°$ is, of course, repeatedly mentioned as a convenient rounding.[11]

Above we mentioned a passage from Strabo,[12] based on Hipparchus, according to which Eratosthenes operated with the sexagesimal division of the circle in which $\varepsilon = 4^p$. This implies that he had assumed, at least in purely geographical context, $\varepsilon = 24°$. Ptolemy, however, describes in Alm. I,12 a much more refined determination.[13] After having explained his own instrument, constructed for the measurement of the noon altitude of the sun and reporting a variation between the solstices of at least 47 2/3° and at most 47 1/2 1/4° he declares these results to be essentially in agreement with a ratio found by Eratosthenes (and accepted also by Hipparchus)[14] of about 11/83 of the circumference of the meridian. Indeed

$$2\varepsilon = \frac{11 \cdot 6{,}0°}{1{,}23} \approx 47;42{,}39°$$

is in agreement with Ptolemy's limits. Thus he accepts

$$\varepsilon = 23;51{,}20°. \qquad (4)$$

The origin of the formulation by Eratosthenes and Hipparchus

$$2\varepsilon = 11/83\, c \qquad (5)$$

is unknown.[15]

The question of the actual measurement of the size of the earth does not concern us here. It will suffice to remark that the value arrived at by Eratosthenes (and accepted by Hipparchus)[16] of

$$c = 252\,000 \text{ stadia} \qquad (6)$$

is obviously adjusted so as to result in conveniently round numbers for the sexagesimal parts, i.e.

$$1^p = 4200 \text{ stadia}. \qquad (7)$$

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[11] "About" 24°: Hipparchus, Comm. Ar. I 10, 2 (Manitius, p. 96, 21); Plutarch, Moralia 590 F (Loeb VII, p. 464/465); Ptolemy, Planisph. (Opera II, p. 259, 13) or Geogr. VII 6, 7 (Neugebauer [1959, 1], p. 23, n. 6). Also without such specification $\varepsilon = 24°$ is common: e.g. Anonymus, Maass, Comm. Ar. rel., p. 132, 1 (≈ A.D. 500) or Anon., Logica et Quadr. (11th cent.), Heiberg, p. 104, 21 f.; Bar Hebraeus (13th cent.), L'asc. II 1, 1; II 2, 3 f. etc. (Nau, p. 113, p. 134 ff.) beside the accurate value 23;55 in II 1, 9 (Nau, p. 128).

[12] Strabo II 5, 7 (Berger, Geogr. Fr. Hipp., p. 111 f., II B, 15 and II B, 23; Loeb I, p. 436/437 f.). Cf. also above p. 590.

[13] Opera I, 1, p. 67, 17–68, 6. Theon's commentary (Rome CA II, p. 528/529) is, as usual, only a paraphrase of Ptolemy's text.

[14] By arguments which are bound to lead to absurd results Diller [1934] tried to show that Hipparchus assumed $\varepsilon = 23;40°$ "although this fact has disappeared entirely from the tradition and is not attested by any ancient author." Diller first computes latitudes φ from distances given for parallels of longest daylight M (i.e. he converts rounded numbers of stadia into accurate degree values) and then operates with a formula of modern spherical trigonometry to find ε from φ and M. This then is taken seriously to establish a deviation of 0;10°.

[15] Berger, Geogr. Fr. Erat., p. 131 tries to show that (5) does not belong to Eratosthenes. His arguments are much too pedantic in view of Ptolemy's clear text (cf., e.g., Thalamas, Erat., p. 121 f.). In any case there remains the problem of explaining the origin of the peculiar fraction 11/83 in (5).

[16] Strabo II 5, 7 (Loeb I, p. 437).

This sexagesimal divisibility remained useful also for the later degree division, resulting in the norm

$$1^\circ = 700 \text{ stadia} \tag{8}$$

in which form one usually refers to Eratosthenes' meridian measurement.

Eratosthenes' data were obtained from an estimate of the meridian distance from Syene to Alexandria.[17] If he assumed a northerly progress on the Nile[18] of about 5000 stadia and found from solar altitudes a change of latitude of about $1;12^p$ then $c = 50 \cdot 1;12 = 1,0 = 250\,000$ would be the result of which (6) is a convenient adaptation.[19] Neither great accuracy of measurements nor theoretical considerations beyond the most direct consequences of the concept of sphericity of the earth are required to reach this result.[20]

The existence in early Greek geography of any mathematically defined map projection is problematic. Obviously it is to Hipparchus that one would look for a systematic study of the representation of geographical coordinates in a plane map, but no such theory has yet been found.[21] The first trace of a general discussion of geographical mapping seems to be a passage in Strabo[22] where he says that the oikoumene can be most accurately represented on a colossal globe (as supposedly constructed by Crates, about 170 B.C.) — ten feet in diameter. For the oikoumene, however, a plane map should do (about 7 feet in length for 70,000 stades in longitude and less than 3 feet in width)[23] on which parallels and meridians could be represented as straight lines, either forming an orthogonal network or by drawing "slightly converging" meridians. Unfortunately no numerical data are mentioned for these constructions which look like a precursor of the mapping used by Marinus[24] (about A.D. 100).

Geographical mapping based on consistent mathematical principles was preceded by attempts to combine different aspects of the terrestrial sphere into one picture, just as the stereographic projection of the celestial sphere was preceded by figures which in a primitive way combine visible and invisible hemisphere in one diagram.[25] Indeed there exist geographical drawings in which a circular ring for the horizon surrounds a cross section of the terrestrial globe, showing the tropics, the ecliptic, etc. as straight lines; superimposed is a rectangle representing Egypt and in the lower half a topography of the nether world in the earth's interior.[26]

Another "map" represents the seven climata as parallel strips within a circle[27] and a marginal note associates them with numerical geographical latitudes (φ). As we have seen[28] Ptolemy had full control over the trigonometric relations which lead from the longest daylight M to φ and vice versa. The application to

[17] Actually Syene lies 3° to the east of Alexandria.

[18] It should not have been difficult to obtain in hellenistic Egypt a fair estimate of the reduction of sailing time to account for the huge bend of the Nile in the region of Tentyra-Diospolis Parva.

[19] A summary of the literature supporting this view is found in Prell [1959].

[20] Cf. above p. 653f.

[21] For the little we know about Hipparchus' methods in mathematical geography cf. above I E 6, 3.

[22] Geogr. II 5, 10; Loeb I, p. 449, Budé I, 2, p. 90.

[23] Indeed between $M = 12\,1/2^h$ and 16^h are about $40^\circ = 48000^{st}$.

[24] Cf. below V B 4, 1.

[25] Cf. below IV D 3, 2.

[26] Cf. for details Neugebauer [1975], based on Marc. gr. 314 fol. 222^r and several parallels.

[27] Vat. gr. 211 fol. 121^v.

[28] Above I A 4, 3 and I A 4, 7.

the special case of the climata is contained in Alm. II, 6[29] and once more in the headings of Alm. II, 8. The latitudes given for the climata in the scholion to the map, however, represent a much more primitive level than reached in the Almagest. The first two columns in the subsequent list

Cl.	φ	$\Delta\varphi$	Alm. II, 6
I	15°		16;27°
II	23	8	23;51
III	30	7	30;22
IV	36	6	36
V	41	5	40;56
VI	45	4	45;1
VII	48	3	48;32

are taken from the scholion to the map; the third and fourth column show that the values of φ form a difference sequence of second order which agrees for $\varphi=36$ with Ptolemy.[30] As we have seen Hipparchus made use of such sequences for shadow lengths at higher latitudes[31] and for terrestrial distances.[32] The new list underlines still further the importance of arithmetical methods also for early Greek mathematical geography.

§ 2. Shadow Tables

The theory of sun dials requires, of course, either spherical trigonometry or graphical methods of projection, e.g. of the "Analemma" type.[1] These methods lead to the determination of the direction and of the length of the shadow of a gnomon as function of day (i.e. solar longitude), hour, and geographical latitude.

As in the case of the variation of the length of daylight and of the oblique ascensions of ecliptic arcs the exact trigonometric or graphic solutions of gnomon problems were preceded by a much more primitive phase in which simple arithmetical patterns provided reasonably close approximations of the actual variation of shadow lengths during the year and during the day. For the problems concerning the length of daylight it is obvious that the arithmetical schemes for the rising times were of Babylonian origin since exactly the same numerical parameters were found in cuneiform and in Greek sources.[2] No such parallelism has yet been discovered for the shadow tables[3] and we shall find elements in the Greek texts

[29] Cf. above I A 4, 7, Table 2 (p. 44).

[30] Alm. II, 8 has for clima VII only $\varphi=48°$ (in MS D expressly 48;0). This may be a residue of the simple arithmetical pattern.

[31] Cf. above I E 3, 2, Table 30 (p. 304).

[32] Cf. Table 32 (p. 305).

[1] Cf., e.g., below V B 2, 6 E.

[2] Cf. p. 712ff.

[3] A Babylonian shadow table belongs to the second tablet of the "series" ^{mul}Apin (cf. above p. 598), published by Weidner [1912], p. 198f. The length s of the shadow (measured in cubits) and the time t after sunrise (measured in time degrees) are related through the formula $s \cdot t=c$ with $c=1{,}0$ at the summer solstice, $c=1{,}15$ at the equinoxes, $c=1{,}30$ at the winter solstice: cf. for details above p. 544. The Greek scheme, described below p. 738 (1), is obviously unrelated to this Babylonian approach.

that suggest a purely Greek origin, perhaps as early as in the fourth century B.C.[4]

We shall discuss the Greek gnomon theory under two different aspects: first the primitive, purely arithmetical, approach[5] which is still unaware of the importance of the geographical latitude; secondly the role of shadow lengths in Greek mathematical geography at a time when the concept of geographical latitude (or the equivalent variation in the length of the longest daylight) had long become familiar.[6] Ironically, the primitive, geographically inflexible methods survived all scientific progress, being handed down deep into the late Middle Ages and ranging over the whole mediterranean area and as far as Armenia or Ethiopia.[7]

1. Arithmetical Patterns

A. Greek Shadow Tables

For the sake of easy reference I give here the list of texts which constitute the foundation upon which the subsequent discussion rests. With the exception of one early papyrus ((n), around 200 B.C.) and one inscription from a Nubian temple ((c), probably Roman) all sources are from Byzantine codices, written sometime between the 12th (f) and the 16th century.

(a) Berol. 173, fol. 117^r–118^v; published: CCAG 7, p. 188–190
(b) Paris, suppl. gr. 1148, fol. 90^r–90^v; unpublished, copy of (a); cf. CCAG 8, 3, p. 83, cod. 60
(c) Taphis, South Temple, inscription; published: CIG III. No. 5038; better: Borchardt, Zeitm., p. 29
(d, A) Ambros. gr. 325, fol. 304^r–305^v; published with (d, V)
(d, P) Par. gr. 854, fol. 196^v–197^r; published with (d, V)
(d, V) Vat. gr. 1056, fol. 44^v, 65^r: published CCAG 5, 3, p. 76–78
(e) Par. gr. 2243, fol. 663^v; published: Schissel [1936], p. 114f.; cf. also CCAG 8, 3. p. 18 cod. 39
(f) Par. gr. 22, fol. 1^r–2^r; unpublished: cf. CCAG 8, 3, p. 3, cod. 27
(g) Vindob. philos. gr. 190, fol. 72^r–73^r; published: Schissel [1936], p. 115–117; cf. CCAG 6, p. 51, cod. 7
(h) Matrit. Bibl. Nat. 4681, fol. 150^v–151^v; unpublished; cf. CCAG 11, 2, p. 83, cod. 37
(k) Par. gr. 1630, fol. 110^r–111^r; partial publ. (no tables): Cramer, Anecd. Par. I, p. 382, 18–34; cf. CCAG 8, 3, p. 10, cod. 32
(l) Par. gr. suppl. 1148, fol. 186^v–187^r; unpublished, copy of (m); cf. CCAG 8, 3, p. 86, cod. 60
(m) Berol. 173, fol. 176^r; unpublished; cf. CCAG 7, p. 62, cod. 26
(n) Pap. gr. 1, Inst. Oesterr. Gesch. Forsch.; published: Wessely [1900]; Neugebauer [1962, 3], p. 31–33, p. 42–44
(o) Par. suppl. gr. 652, fol. 168^v, cf. below p. 742; unpublished; cf. CCAG 8, 4, p. 81, cod. 112
(p) Cambr. Trin. Coll. R. 15, 36, fol. $23^{r/v}$; unpublished; cf. CCAG 9, 2, p. 51, cod. 64
(q) Athos, Docheiariou, Lambros, Catal. 2934 (260) No. 6, fol. 404^r; unpublished
(r) Athos, Dionysiou, Lambros, Catal. 3599 (65) No. 24; lost
(s) Athos, Dionysiou, Lambros, Catal. 3823 (289) No. 5, fol. 75^r–76^v; unpublished
(t) Bodleian. Savile 51, fol. 79^v–80^r (unpublished, identified by D. Pingree)

As in the case of the parapegmata[1] the irregular fluctuations of the Greek lunar calendars must have suggested as entries for shadow tables the consecutive

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[4] Cf. below p. 739.
[5] Below IV D 2, 1 A.
[6] Below IV D 2, 2.
[7] Below IV D 2, 1 B.
[1] Cf. above IV A 3, 3.

zodiacal signs of the solar year. As an alternative the use of the months of the (slowly rotating) Egyptian calendar existed, or later of the months of the Alexandrian or julian calendar. Accordingly we distinguish a "Type Z" (zodiacal signs) from a "Type M" (months, Egyptian or julian) of shadow tables.

The basic pattern (e.g. for a table of Type Z) of all these texts is as follows

Hour	♑	♐	♏	♎	♍	♌	♋
		♒	♓	♈	♉	♊	
1	28	27	26	25	24	23	22
2	18	17	16	15	14	13	12
3	14	13	12	11	10	9	8
4	11	10	9	8	7	6	5
5	9	8	7	6	5	4	3
6	8	7	6	5	4	3	2
7	9	8	7	6	5	4	3
8	11	10	9	8	7	6	5
9	14	13	12	11	10	9	8
10	18	17	16	15	14	13	12
11	28	27	26	25	24	23	22

(1)

The hours are obviously seasonal, not equinoctial, hours. The numbers must refer to the end of the hour since we have a finite shadow length at 1. Consequently "6" belongs to the noon shadow and "11" gives the last finite entry, being the same as the shadow at the end of the first hour.[2]

Several of our texts give a simple rule: a man standing upright marks the end of his shadow on the ground and measures its length by setting one foot in front of the other. He then can find the hour from the corresponding number in the table (1). It is important to note that this procedure avoids all problems of ancient metrological norms: each person measures the shadow by his own feet.

The shadows for the hours before and after noon increase by 1, 2, 3, 4, and 10 $(=1+2+3+4)$ feet in each month (or sign). The noon shadows, and hence every line in (1), form a linear sequence of difference 1. The extrema for the noon shadows are always

$$u=2, \qquad U=8 \tag{2}$$

thus the equinoctial noon shadow $s_0=5$.

For tables of Type M the column entries in (1) are in the case of the julian calendar:

$$\text{Dec.}\quad \begin{matrix}\text{Nov.} & \text{Oct.} & \text{Sept.} & \text{Aug.} & \text{July}\\ \text{Jan.} & \text{Febr.} & \text{March} & \text{Apr.} & \text{May}\end{matrix}\quad \text{June} \tag{3}$$

or for the Alexandrian calendar (I = Thoth, etc.):

$$\text{IV}\quad \begin{matrix}\text{III} & \text{II} & \text{I} & \text{XII} & \text{XI}\\ \text{V} & \text{VI} & \text{VII} & \text{VIII} & \text{IX}\end{matrix}\quad \text{X}. \tag{4}$$

[2] In text (c) one finds an entry for the 12th hour, always with πο(δες) πληρ(εις) "full feet"(?). Text (a) and its copy (b) gives α at the 12th hour, probably a misreading of δ for δ(ύσις ἡλίου) "sunset" in (f).

Several of our texts of Type M mention with each month the corresponding length of daylight and night,[3] now to be understood, of course, in equinoctial hours. These hours always vary linearly between $m=9^h$ and $M=15^h$, limits well-known to us from Hipparchus, being assumed by his predecessors as characteristic for "Greece."[4] Thus we have the following arithmetical pattern that underlies our tables:

$$\begin{array}{rccccccccl} \text{length of daylight:} & 15 & 14 & 13 & 12 & 11 & 10 & 9 & \text{hours} \\ \text{noon shadow:} & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \text{feet.} \end{array} \tag{5}$$

Both sequences are the simplest linear progressions with difference 1 and it is obviously futile to search for some accurate observations beyond a rough estimate valid for the general area of Greece. In particular one should not be misled into considering the ratio $M:m=15:9$ as proving Byzantine influence, only because this ratio is in later times (e.g. in the Almagest) the norm for the latitude of the "Hellespont."

If we show in Fig. 44 those geographical latitudes φ which correspond to the noon shadows listed in (5) it is only to demonstrate that such a linear scheme is a reasonable approximation for, say, $\varphi=38°$ (Athens) but excludes definitely, e.g., Alexandria ($\varphi\approx31°$) or Babylon ($\varphi\approx32;30°$). Fig. 44 is constructed under the assumption that the shadow casting person represents a "gnomon" of $g=6$ feet. In a later shadow table, however, which is trigonometrically computed[5] the standing person is given a height of 7 feet.[6] Under this condition Fig. 44 would have to be replaced by Fig. 45 which would make Babylon or Rhodes possible choices.

There exists, however, evidence which speaks for a Greek origin of the shadow tables. In the presumably earliest tables which operate with a zodiacal scheme it seems evident that the shadow lengths refer to the whole of a sign, which implies that the cardinal points are located at the midpoints of the signs. This would indicate adherence to the Eudoxan norm for the vernal point.[7]

Furthermore in the introductions to several of the shadow tables one finds a dedication or a reference to a "King Philip."[8] It is pointless to ask whether Alexander's father was meant[9] or Philip Arrhidaeus (of the "Era Philip") since we have a person of this name much more likely to be connected with shadow tables. Our earliest text, (n), a Ptolemaic papyrus, contains not only a shadow table but also weather prognostications, apparently related to a work ("De signis") which goes under the name of Theophrastus. Rehm has suggested[10] considering Euctemon as the real author, well-known, e.g., through references in Ptolemy's "Phaseis."[11] In

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[3] Texts (d, V), (e), (f), (g), (p).

[4] Cf. above p. 581, notes 7 to 10. Only in text (q) do we find $M:m=14:10$.

[5] Ambros. C 37 sup. fol. 137ᵛ-139ʳ (unpublished); cf. CCAG 3, p. 7, cod. 11.

[6] This text also operates with $M:m=15:9$ but refers it correctly to a latitude $\varphi=41°$ (cf. Almagest II, 8 where $\varphi=40;56°$ corresponds to $M=15^h$). For a gnomon of 7 feet cf. also below p. 744.

[7] Cf. above p. 599.

[8] Texts (a), (b), (d, V), (l), (m); cf. also the Ethiopic texts V_1 and V_2 below p. 742.

[9] This is certainly the case with Bar Hebraeus (13th cent.) because Macedonia is mentioned in this context: cf. below p. 744.

[10] Rehm, Parap., p. 122-140.

[11] Cf. below p. 929.

the parapegmata Euctemon is often associated with a certain Philip[12] and Ptolemy names the Peloponnesus, Locris, and Phocis as regions of his observations.[13] The Middle Ages made Philip a king and the same happened to Ptolemy.[14] For us, however, this name points to a possible origin of the shadow tables within the earliest period of Greek astronomy, i.e. the fourth or fifth century B.C.[15]

None of the Byzantine codices which contain our Greek shadow tables is earlier than the 12th century A.D. As to be expected these texts abound in scribal errors, the most characteristic of which consists in the change of the 7-column pattern (1), (3), or (4) into a pattern of six pairs[16] in which, consequently, one whole column is missing. Excepting two cases[17] it is always the column with the noon shadow of 6 feet which is suppressed. There is only one table from an Athos monastery (q) correctly arranged in 12 columns, and one inscription in the Nubian temple in Taphis (c). Two texts, (g) and (h), are correct only for the columns with the noon shadows 2, 3, and 4: then the text skips to 6, 8, 10, 13 or to 6 and 10, respectively. In particular the scribe of (g) did not understand that the shadow lengths were referred to seasonal hours while he had for the lengths of daylight the usual linear sequence of equinoctial hours. Thus he made a disastrous attempt at adjusting the single columns in (l) to equinoctial hours by introducing arbitrarily several absurd differences.[18] The text (q) shows 12 tables for the julian months, with the usual hour differences in each table, and the entry *δύσις* for the 12th hour. The noon shadows are (from June to December)

2 3 4 6 8 10 13

a sequence also found in (g).

In the texts (a) and (b) King Philip is addressed by a "horocrator" Sextus but nothing is known about the man or his title. In (d, V) and by Bar Hebraeus[19] a Dionysius is related to King Philip; an author of this name who wrote on winds[20] is found in the same manuscript in which a Theodorus writes to a Theophilus about shadow tables.[21] It seems senseless to try to identify these people with more or less known astronomical or astrological writers.

B. Late Ancient and Medieval Shadow Tables

All the tables described in this section are ultimately derived from the Greek prototype (1), p. 738. We begin again with a bibliographical list of our source material.

[12] Generally identified as Philip of Opus; cf. RE 19, 2 col. 2351, 67–2352, 5 [v. Fritz]. For his association with Euctemon see, e.g., Ptolemy, Phaseis, ed. Heiberg, p. 17, 15; 18, 5 etc. Rehm says in Parap., p. 99, n. 3 (and similarly in RE Par. col. 1346, 13–41): "So ist das Parapegma des Philippos tatsächlich sicher nichts anderes als eine noch dazu sehr wenig selbständige Bearbeitung des euktemonischen," referring to Griech. Kal. III [1913], p. 36 where he represents in a diagram Euctemon and Philip as independent (!) sources of Ptolemy's Phaseis.

[13] Ptolemy, Phaseis, p. 67, 5, ed. Heiberg. Also Hipparchus, Comm. Ar., Manitius, p. 29, 13–18.

[14] Cf. below p. 834, n. 8.

[15] Also for meteorological data a schematic transfer to Alexandria from Greece has been established; cf. Hellmann [1916], p. 332–241.

[16] The same error we noticed above (p. 724) with Gerbert (10th cent.) for the length of daylight.

[17] Texts (g), (h), and perhaps (m).

[18] Cf. Schissel [1936], p. 115–117 (who did, however, not grasp the simple background).

[19] Cf. below p. 744. [20] Cf. CCAG 8, 4, p. 5, fol. 192. [21] Texts (d, A), (d, P), and (k).

Coptic:	(C, A)	Morgan, Catal., p. 137
	(C, G)	Crum, Copt. Mon., p. 13
	(C, MF)	Bouriant [1898]; also including the two preceding texts: transcription. p. 578–584, French transl., p. 586–590
Ethiopic:	Ab	Abbadie, Catal., also Chaine, Catal., No. 37; Conti Rossini [1913], No. 121
	B_1, B_2	Brit. Mus., Wright, Catal., No. 397, 15 = or. 816 fol. 43^r–44^v
	D_1, D_2	Berlin, Eth. 84 fol. 15^v and fol. 16, respectively
	V_1 to V_4	Vat. Aeth. 119 fol. 56; 128 fol. 140f.; 123 fol. 66^v; 171 fol. 91^v, respectively
	W	Rhodokanakis [1906] No. 25 (= Aeth. 6) fol. 22f.
Arabic:	(A, A)	Sobhy [1942], p. 187 (French trsl.), [1943], p. 250
	(A, B)	Ibn al-Bannā', Col., p. 22 (French tabulation)
Syriac:	(S, A)	Budge, Syr. Anat. II, p. 607/608 (Engl. trsl.)
	(S, B)	Nau, Bar Hebr., p. 159 (Fench trsl.)
Armenian:	(R, A)	Tumanian, Arm. Astr., p. 193 (Engl. tabulation)
Latin:	(L, A)	Cod. Scaliger. lat. 28 fol. 2^r
	(L, B)	Palladius, Agric., ed. Schmitt, p. 68, p. 114, etc., also Bilfinger, Zeitm., p. 55–57
	(L, C)	Wandalbert of Prüm, horologium. Migne Pl 121. cols. 633/634; also Bilfinger, Zeitm., p. 73–78.
	(L, D)	MS Cotton, Titus D 27, fol. 12^v (unpublished)
	(L, E)	Exeter Cathedral: Warren, Leofric Missal, p. 58; text also in Cabrol-Leclercq, Mon. eccl. lit. V col. 533
	(L, F)	Cabrol-Leclercq, Mon. eccl. lit. V col. 530–532. Un horologium Mozarabe
	(L, G)	Missal given around 1500 by Robert, Bishop of London, to the Abbey of Jumièges (west of Rouen). Cf. Ducange, Gloss. (1845), Vol. V, p. 223a, (1886), Vol. VI, p. 291, s.v. Pes
	(L, H)	Beda(?), Libellus de mensura horologii, Migne PL 90, cols. 951–956; also Bilfinger, Zeitm., p. 27f.; similar in gloss to De temporum ratione, Chap. 23, Migne PL 90 col. 447/448
	(L, I)	MS Berlin, Staatsbibl. 2°, 307 fol. 34^r (unpublished. communicated to me by Mr. G. J. Toomer)

We have three Coptic shadow tables, one (C, A) from an inscription in the Byzantine monastery of Anbā Sim'ān (on the West bank of the Nile near Asuān), two, (C, G) and (C, MF) from parchment booklets.[1] The monastery was abandoned in the 13th century, hence (C, A) was written before this time, not being a visitor's inscription. The two other documents are simply "late" (i.e. between 1000 and 1500?).

All three texts are of "Type M," based on the Coptic-Alexandrian calendar; the shortest shadow is given for month X (Paōne ≈ June), the longest for month IV (≈ December). None of these texts combines two months in one table but (C, MF) seems to be derived from such a prototype (see below). Exactly as in the Greek texts the shadows increase before and after the 6th hour by 1 foot, 2, 3, 4, and 10 feet, respectively. (C, G) gives the shadow lengths for 11 hours in each month, (C, MF) only for the hours 1, 6, 9, 10, and 11, (C, A) only for 6^h, 9^h and 10^h.[2] Trivial scribal errors (and misprints) abound in all tables; short headings say that the shadows are measured in feet while (C, G) has a pious postscript addressed to "the beloved brother Stauros" by the "sinner Shenoute."

The noon shadows in (C, MF) are

$$8 \quad 7 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1. \tag{1}$$

[1] All three texts are edited and translated in Bouriant [1898]; an added astronomical commentary by Ventre Bey is without value.

[2] This selection is probably determined by the hours of prayer.

The omission of 6 is reminiscent of a Greek type described in the preceding section (p. 740). By spreading out such a six-column text again to 12 columns the scribe was one column short and instead of restoring the suppressed column 6 he expanded his count back to 1.

In (C, A) the noon shadow 6 is preserved (in month V) and $u=1$ is given in X. Hence one can restore as noon shadows

$$[7] \quad 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1. \tag{2}$$

Perhaps one may see in this sequence an attempt of adaptation to a region with shorter noon shadows than foreseen in the original Greek tables.

A more drastic change in the noon shadows is found in (C, G). As the text stands the noon shadows are

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
4	5 1/2	7	9	7	5	4	2 1/2	1	1/2	1	2 1/2.

On the basis of symmetry 5 in VI can be changed to 5 1/2. Then, in order to restore a linear sequence we must emend 9 to 8 1/2 and interpret the 1/2 in X as $-1/2$, i.e. as shadow falling toward south as would be possible in a region south of Asuān ($\varphi=\varepsilon$). Hence we may perhaps restore as noon shadows

$$[8\ 1/2] \quad 7 \quad 5\ 1/2 \quad 4 \quad 2\ 1/2 \quad 1 \quad [-]\ 1/2.$$

Certainly derived from a Coptic original is the Greek text (o) of our list p. 737, extant in a copy written in the 16th century. The shadow lengths are given in 12 tables for the 12 months of the year, beginning with September, which reflects the Coptic origin, since September corresponds to Thoth. The noon shadows are

IX	X	XI	XII	I	II	III	IV	V	VI	VII	VIII	
4 1/2	6	7 1/2	9	7 1/2	6	4 1/2	3	1 1/2	1/2	1 1/2	3.	(3)

Excepting the minimum 1/2, obviously chosen only in order to avoid "zero," we have here the data which underlie most of the Coptic and Ethiopic tables: $u=0$, $U=9$, and thus the difference 1 1/2 per month.

As direct comparison of the Ethiopic shadow tables with the Coptic ones is simplified by the agreement between the two calendars.[3] The hourly variation of the shadow lengths is again the traditional one; hence a table is fully defined by the list of monthly noon shadows (all texts being of Type M). The texts known to me are represented in the following list[4]

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	
V_1, V_2	3	4	5	7	8	5	5	4	3	2	2	2	
D_2	3 1/2	4	5	7	8	5	5 1/2	4 1/2	3 1/2	2 1/2	2	2 1/2	(4)
B_1, V_3, V_4	4	6	7	9	7	6	4	3	1	0	1	3	
Ab, B_2, D_1, W	4 1/2	6	7 1/2	9	7 1/2	6	4 1/2	3	1 1/2	0	1 1/2	3	

[3] Maskaram = Thoth ≈ September.

[4] For details cf. Neugebauer [1964], p. 62–67, p. 69. All our texts (as Ethiopic texts in general) are of very recent date (last three centuries).

The first two types still show the Greek extrema $u=2$, $U=8$. The last line represents correctly the type (3). The tables B_2, V_3, and V_4 are actually arranged in seven columns with IV and X "single" for the solstices; the other texts have single columns for all 12 months.

The foreign origin of these schemes was vaguely remembered. In one manuscript (D_1) the tables are called "Measure of the hours of Egypt" while two texts (V_1 and V_2) make "Philip, the King of the Greeks" the author. The reference to Egypt is not surprising in view of the supremacy which the Alexandrian patriarch exercised over the Ethiopian church. The association with "King Philip," however, must go back to the Greek tradition.[5] Also the ratio 15:9 of longest to shortest daylight found in Ethiopic texts[6] is probably of early Greek origin[7] rather than the result of Byzantine influence on Ethiopia.

The change of the extremal noon shadow lengths from $u=2$, $U=8$ to $u=0$, $U=9$ cannot be explained as the result of systematic observations since the amplitude of the noon shadows should decrease, not increase, as one moves south. It may have been noticed, however, that $u=0$ had to be accepted for regions where the sun reaches the zenith. In order to preserve as far as possible the traditional linear sequence a difference of 1 1/2 may have seemed to be a proper solution.

A badly distorted list of noon shadows is preserved in an Arabic text (A, A), written for the use of Copts and ascribed to Demetrius, the 12th patriarch of Alexandria who died in A.D. 230.[8] A little table[9] gives the following noon shadows for the 12 months beginning with Tybi (=V):

$$\begin{array}{cccccccccccc} V & VI & VII & VIII & IX & X & XI & XII & I & II & III & IV \\ 9 & 7 & 5 & 3 & 2 & 1 & 1 & 2 & 4 & 6 & 8 & 10. \end{array} \tag{5}$$

Obviously two arithmetic progressions are contaminated here, one with the difference 1, the other with 2. Perhaps this is a degenerate form of the table of difference 1 1/2 in (3) or (4). A remark is added that for every month the shadow increases 7 feet from noon to the afternoon (*'aṣr*).

This last term denotes the Islamic afternoon prayer. In the calendar of Ibn al-Bannā (≈1300) this increment varies between 6 and 7 feet[10] as is evident from the following table[11]

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
noon shadow:	7	6	5	4	3	2	3	4	5	7	9 1/5	11
									emend:	[6]	[7]	[8]
shadow at *'aṣr*:	14	13	11	10	9	8	8	10	10	11	15	15
						emend:	[9]		1[2]	1[3]	1[4]	
increment:	7	7	6	6	6	6	[6]	6	[7]	[7]	[7]	[7]

[5] Cf. above p. 739.

[6] Found, e.g., in Berlin, Eth. 84 fol. 41ᵛ and in Vindob. Aeth. 6 fol. 32ᵛ II, 16–33ʳ I, 16.

[7] Cf. above p. 739.

[8] Chaine, Chron., p. 251. Actually our manuscript was written in 1768 (cf. Sobhy [1942], p. 169).

[9] Sobhy [1942], p. 187, Arabic text [1943], p. 250.

[10] The increment of 6 feet corresponds exactly to the 3rd hour after noon.

[11] (A, B): I = January, etc.

It is interesting to note that the noon shadows (accepting the obvious emendations for the last three months) are exactly the shadow lengths of the original Greek scheme.

Quite unexpected is the existence of the accurate Greek pattern of Type Z[12] in a Syriac work written in 1279 by Bar Hebraeus.[13] Only in the list of increments do we find one incorrect number where he says that the shadow increases per hour by 1, 3, 5(!), 10, and 20 feet beyond the noon shadow (instead of 6 feet for the third hour). And Dionysius from the court of King Philip is mentioned as the authority behind the whole method which is valid for Macedonia.[14]

In the same chapter[15] we also find definite measures for the gnomon: either 7 "feet," or 12 "fingers," or a sexagesimal division into 60 "degrees." The values 7 and 12, respectively, are also known to us from a Greek text for shadow lengths computed with trigonometric methods.[16]

Another Syriac text which gives shadow lengths exactly following the Greek scheme (Type M, correctly in 7 columns) is found in a "Syrian Anatomy."[17] Though the list of shadow lengths is incomplete and some numbers garbled it suffices to mention the data for the first hour:

28 27 26 25 24 23 22

which agree exactly with the classical pattern (1), p. 738.

A list of Type M in 12 columns from mediaeval Armenia (date and author unknown[18]) also agrees with the Greek prototype, excepting some trivial scribal errors.[19] Hence we can say that Armenia, Syria, and Nubia (Taphis[20]) best preserved the original Greek pattern.

Derived from a composition from the pre-Islamic period we have in a Pahlavi work, the Shāyest nē-shayest,[21] a shadow table based on arithmetical progressions. The lengths of the noon shadows are given for the beginning and the midpoint of each zodiacal sign, the afternoon shadows only for the endpoints of the signs. The shadow lengths in the latter case are assumed to form a linear zigzag function with 6;10 feet as minimum, 14;10 feet as maximum,[22] thus with a difference $d=1;20$ per sign (cf. Fig. 46). The noon shadows vary between $u=0;30^f$ at ♋ 0° and $U=10^f$ at ♑ 0°; the difference is $\pm 0;30^f$ for each 15° of solar longitude between ♉ 0° and ♌ 30° and between ♐ 15° and ♑ 15°, but $\pm 1^f$ otherwise.

[12] Above p. 738 (1).
[13] (S, B) trsl. Nau, p. 159; cf. above p. 740.
[14] Cf. above p. 739, n. 9.
[15] Nau, p. 157.
[16] Cf. above p. 739, notes 5 and 6.
[17] (S, A) trsl. Budge, p. 608.
[18] (R, A), Tumanian, p. 193.
[19] Beyond some wrongly placed numbers in the first two columns all numbers for 3^h (and thus for 9^h) are one unit too low.
[20] Cf. above p. 740.
[21] I.e. a moral and social code. Our text is Chap. XXI in a supplementary section; cf. West, Pahl. T., p. 397–400; also Kotwal [1969], p. 86–89. The arithmetical pattern makes it very easy to correct the few doubtful passages in the text.
[22] The text uses "parts" as smaller units such that 1 foot = 12 parts.

The dotted lines in Fig. 46 represent the corresponding data for the Greek scheme[23] for noon and for 3 (seasonal) hours (before or) after noon. The noon shadows agree exactly for ♎ 15° and ♓ 15° at $s=5;30^f$; near the extrema the slopes are the same, 1^f per sign. For the afternoon shadows the slope of the Greek function is only 1^f against $1;30^f$ in the Iranian scheme. The increase since noon at 3 p.m. is in the Greek scheme always $1+2+3=6^f$ as compared with $5;40^f$ in the Shāyest in the section from ♉ to ♌, but reduced to $4;10^f$ at ♑ 0°. It seems a plausible assumption that the Iranian table was modelled after the Greek prototype with essentially the same equinoctial noon shadow s_0. The small change from $s_0=5^f$ to $s_0=4;30^f$ (or $5;30^f$?) is certainly not the result of observations but the consequence of the arithmetical pattern as a whole.

Two Latin shadow tables share with the Byzantine tables the basic error of arranging the material in six columns for two months each. Scaliger's manuscript[24] is at least consistent in so far as the noon shadows (and hence each sequence of hour shadows) form an arithmetic progression with difference 1, the noon shadows from $u=2$ to $U=7$. The differences between the hours before and after noon[25] are given as 1, 2, 4 (!), 3 (!), 10; consequently all entries for 3^h (and thus for 9^h) are one unit too high.

Three Spanish "Mozarab" (Latin) manuscripts of the 11th century[26] give a table which corresponds exactly to the Byzantine tables with six pairs of months, the noon shadows increasing with constant difference 1 from $u=2$ to $U=8$ but omitting 6.[27] These tables also mention the correct difference pattern for the hours before and after noon.

Also Palladius[28] (4th to 5th century A.D.) clearly depends on a six-column table in which the 6-foot noon shadow was omitted. But instead of ending the table with 5, 7, 8 he (or his source) also gave the last difference the value 2. In this way he obtained the following table:

	XII	XI	X	IX	VIII	VII
	I	II	III	IV	V	VI
noon shadow:	9	7	5	4	3	2.

All increments for the hours before and after noon are correct.

Additional distortions in western medieval tables made the original pattern almost unrecognizable. Shadow tables in missals of the 10th or 11th century[29] are based on six noon shadows at constant difference 2 from $u=1$ to $U=11$. The shadows for the third and ninth hours are correctly $1+2+3=6$ feet longer.

[23] Above p. 738 (1).

[24] Cf. p. 741 (L, A), later than the 9th century. The codex was probably written in the monastery of Flavigny (on the Moselle, south of Nancy) in the 9th century but our table is "manu recentiore."

[25] In this table also the hours are paired: "hora 1 et 11," etc.

[26] Above p. 741 (LF); cf. also the calendar of Ibn al-Bannā (above p. 743).

[27] Cf. above p. 740.

[28] Above p. 741 (LB).

[29] Exeter Cathedral and a Cotton MS (LD); probably also in the missal of Jumièges (west of Rouen) of 1150 (LG). A similar, also slightly corrupt pattern, is found in the manuscript (L, 1). The hourly increments are said to be 1, 2, 2, 5, 10 feet, i.e. 2 and 5 instead of 3 and 4. The longest shadows run with the difference 2 between 19 and 29 instead of between 21 (i.e. $u=1$) and 31 (i.e. $U=11$).

The same sequence of noon shadows is found in (Pseudo-)Beda's Libellus de mensura horologii[30] but now the difference 2 is also applied between the hours, excepting the traditional 10 at the ends. Thus we have, e.g., for the two months I and XII the following shadows for the hours from 1^h (and 11^h) to 6^h:

$$29 \quad 19 \quad 17 \quad 15 \quad 13 \quad 11 \quad (=U).$$

Even worse is the scheme which underlies a poem by the monk Wandalbert of Prüm,[31] of about 850, in which the difference 2 runs from $u=3$ to $U=13$ and between all hours, excepting the first one which has become 11 instead of 10. As usual the western tradition represents the lowest level in our material.

2. Shadow Lengths in Greek Geography

The earliest chronologically secure information about the geographical use of gnomon shadows comes from Hipparchus' Commentary to Aratus. There we read that the ratio of the length g of the gnomon to the length s_0 of the equinoctial noon shadow is for "Greece," and in particular for Athens,

$$\begin{gathered} g:s_0=4:3 \\ \text{corresponding to a geogr. lat. } \varphi\approx 37^\circ \\ \text{and a longest daylight } M=14\,3/5^h. \end{gathered} \tag{1}$$

This is said in criticism of a statement made by Aratus that for Greece $M:m=5:3$ and of the assumption made by Eudoxus that $M:m=12:7$ (?).[1]

Computing accurately it would follow from $g:s_0=4:3=1;20$ that $\varphi\approx 36;52^\circ$ in agreement with the rounded value 37° given by Hipparchus. Though mathematically correct this result nevertheless does not agree very well with the actual latitude of Athens which is 38°. Again we see that the ratio 4:3 is only a convenient estimate and not the result of careful observation. Such primitive data were apparently never checked; at least Vitruvius, some 150 years later, still quotes the same ratio.[2]

Strabo tells us[3] that Hipparchus assumed for Byzantium as longest daylight $M=15\,1/4^h$ and for the noon shadow s_1 at the summer solstice

$$g:s_1=120:(42-1/5)$$

or the ratio $s_1:g=\dfrac{41;48}{2,0}=0;20,54$. Ptolemy, in Alm. II, 6 gives for $g=1,0$ and

[30] Above p. 741 (LH).
[31] West of Koblenz, north of Trier; cf. above p. 741 (LC).
[1] Hipparchus, Ar. Comm. Manitius, p. 27, p. 35, and p. 29. Cf. for the last mentioned ratio above p. 733, n. 28.
[2] Vitruvius, Archit. IX, 7 ed. Krohn, p. 215, 8f.; Loeb II, p 248/249. Similar data in the same section (for Rhodes, Tarentum, and Alexandria) are not transmitted securely enough to be usable as independent evidence.
[3] Strabo, Geogr. II 5, 41; Loeb I, p. 513. Of little interest, because isolated, is another remark by Strabo (II 5, 38; Loeb I, p. 511) according to which Hipparchus (if he is meant) assumed for Carthage $g:s_0=11:7$. This would lead to $\varphi\approx 32;28^\circ$, almost 4 1/2° less than in reality.

$M = 15;15^h$ the ratio 20;50.[4] This shows again that Hipparchus' data are essentially consistent.[5]

In the early periods of Greek geography the terrestrial latitude is, at best, a parameter of only secondary importance. Obviously Hipparchus is referring to this period when he says that the followers of Philip observed shadow data in Greece but committed errors in the determination of latitudes.[6] Philip, no doubt, is the same person we know as "king" from the arithmetical shadow tables.[7] When Hypsicles (in the 2nd cent. B.C.) declares that he obtained the ratio $M:m = 7:5$ for Alexandria from the solstitial shadows[8] we may perhaps not only think of analemma methods but also consider the possibility of some arithmetical patterns.[9]

As far as we know the early Greek shadow tables are associated with the fixed ratio $M:m = 15:9$, characteristic for Greece.[10] Based on the experience with the tables of rising times it could be expected that the shadow tables would also be eventually extended over a sequence of different climata. Vestiges of such a process seem indeed to be extant in a chapter of Pliny[11] where he speaks about the Greek theory "exquisitissimae subtilitatis" of the seven parallels ("circuli"). We displayed above in Table 2, p. 730 the hours of longest daylight which he associated with these climata. Six out of Pliny's seven parallels coincide exactly with one of the arithmetically defined climata, either of System A or of System B, combined into one sequence of 12 equidistant tables. Only one of Pliny's parallels does not belong to this set of arithmetical climata, his "fifth," with $M = 15^h$, through the Hellespont. This, however, can also be considered a link with the early shadow tables.

Returning to Pliny's six parallels which do belong to the arithmetical climata we excerpt from Pliny's text the following data:

Clima			
Pliny	arithm.	Locality	$g:s_0$
I	I a	Alexandria	7:4
II	II b	Babylon	35:24
III	III a	Rhodes	?
IV	III b	Athens	21:16
VI	V b	Rome	9:8
VII	VII a	Ancona	35:36(?)

Unfortunately only four out of these six give us reliable data for the ratios. In all cases, excepting III, both g and s_0 are given in feet. In III, however, the units are

[4] Cf. above Table 2, p. 44.
[5] The correct determination of φ from M or vice versa suggests the use of analemma methods by Hipparchus (cf. above I E 3, 2).
[6] Hipparchus, Ar. Comm. I, III 10, Manitius, p. 28/29.
[7] Above p. 739, p. 743 and p. 744. Cf. also p. 929.
[8] Hypsikles, Anaph., ed. De Falco-Krause-Neugebauer, p. 48.
[9] Of the type (5) in p. 739.
[10] Cf. above p. 711 and p. 739.
[11] Pliny, NH VI, 211-218 (Jan-Mayhoff I, p. 517-521; Loeb II, p. 494-501).

"inches" (unciae) and the numbers are corrupt.[12] The ratio 35:36 for Ancona, although given twice, cannot be correct since only to the north of Ancona, at Venetia, is equality of g and s_0 reached.[13]

Excepting Rome[14] the above given ratios do not agree very well with the actual geographical positions:

Locality	$g:s_0$	thus φ	φ actual	Δ
I Alexandria	7: 4 = 1;45	29;45°	31;13°	−1;28°
II Babylon	35:24 = 1;27,30	34;26	32;33	+1;53
IV Athens	21:16 = 1;18,45	37;18	38; 0	−0;42
VI Rome	9: 8 = 1; 7,30	41;38	41;53	−0;15

These numbers are too few to detect or to deny the existence of some underlying common pattern. But it is interesting to see that a man in Pliny's position had no better data at his disposal for important parameters which are easy to measure. This is a telling illustration for the absence of any scientific organization in antiquity.

§ 3. Spherical Astronomy before Menelaus

1. Authors and Treatises

> Euclid showed in Theorem I of the "Phaenomena" in a few words that the earth is in the midst of the cosmos, as a point or a center, and the students trust the proof as if it were two and two is four.
>
> Galen (from Euclid, Opera VIII, p. XXXII)

The six treatises discussed in the following go under the names of three authors, Autolycus, Euclid, and Theodosius:

Autolycus:	On the moving sphere; henceforth abbreviated:	Rot. Sph.[1]
	On risings and settings (in 2 books)	Ris. Set.
Euclid:	Phaenomena	Phaen.
Theodosius:	On localities	Hab.[2]
	On days and nights (in 2 books)	Dieb.
	Spherics (in 3 books)	Sph.

[12] Readings and emendations vary between 100:77 (≈ 1;17,55), 105:77 (= 15:11 ≈ 1;21,4,51), and 100:74 (= 50:37 ≈ 1;21,49). Using a sequence of constant second difference 0;0,35 for all six climata from Ia to IIIb one finds for IIIa the value $g:s_0 = 1;22,50 = 1,46;18,10:1,17 \approx 106:77$ and a corresponding $\varphi \approx 35;55°$.

[13] Pliny NH II, 182 and VI, 218 (Jan-Mayhoff I, p. 197, 11–13; p. 521, 18f. Loeb I, p. 317; II, p. 501).

[14] The ratio 9:8 is also found in Vitruvius, Arch. IX 7, 1 (ed. Krohn, p. 215, 6f., Budé, p. 26, Loeb II, p. 249).

[1] "Rotating" sphere conveys a better idea of the meaning of this title than "moving" sphere.

[2] From "*De Habitationibus*," as usually quoted.

The first two authors wrote in the second half of the fourth century B.C. while Theodosius may be placed near 100 B.C.[3]

Combined, these treatises represent a compendium of elementary spherical geometry and astronomical applications. Their pedantry and dullness characterizes them as favored textbooks in which capacity they survived not only in Byzantium but also in Arabic, Hebrew, and Latin translations until deep into the Renaissance. They imitate the style of Euclidean rigor though the real assumptions are rarely named and the proofs are often not much more than a slightly different formulation of the assertions. They usually conceal any connection with astronomical applications and numerical data, excepting perhaps a reference to fictitious observations with a diopter[4] (to show that a semicircle of the ecliptic is always exactly above the horizon), or a solemn proof that it takes four years until the fixed star phases repeat themselves in case the length of a year exceeds 365 days by 1/4 day, while no periodicity exists if the excess is an irrational fraction of a day.[5] Theodosius' Spherics only once (in II, 19/20) denotes the poles of his sphere as "visible" and "invisible" and none of the 437 scholia (to a total of 60 theorems) offers an astronomical interpretation.

Yet, it is the interest of the schoolmasters that has preserved for us the remnants of early treatises[6] which otherwise would have left no traces. Ptolemy, e.g., never mentions these tracts in any of his works, simply because both in contents and methodology they were totally useless for the astronomy of his time. It only characterizes the decline of scientific level when in the fourth century Pappus seriously occupied himself with the completion of some theorems of Autolycus.[7] But the interest which Pappus, and similarly Theon, showed in antiquated treatises undoubtedly enhanced the latter's reputation and recommended them to the attention of Byzantine scholars as well as to Arabic translators.[8]

There is little doubt about the dates of Autolycus and Theodosius. For Theodosius references in Vitruvius and Strabo give us as upper limit the beginning of our era while Suidas credits him with a commentary on Archimedes' "Method."[13] Strabo, enumerating famous men from Bithynia, mentions Theodosius after Hipparchus.[14] Accepting as generally valid the rule that Strabo names persons in chronological order one assumes for Theodosius a date in the first century B.C. As evidence for a dependence on Hipparchus could also be cited the use in the "De habitationibus"[15] of Hipparchian parameters for the length of the summer, were it not for the question whether this section is genuine

[3] For the general background of these treatises cf. Tropfke GEM V, p. 118–121 and Heath, GM I, p. 348–353; II, p. 245–252.
[4] Euclid, Phaen. 1.
[5] Theodosius, Dieb. II, 19; a similar discussion of irrational quantities is found in Theod., Sph. III, 9 (ed. Heiberg, p. 146, 10) and in Pappus, Coll. VI, 8 (ed. Hultsch, p. 484).
[6] The number of preserved copies is relatively high, e.g. some 40 manuscripts of Autolycus (none older than about A.D. 900, i.e. not less than 12 centuries of transmission).
[7] Cf. below IV D 3, 6.
[8] At the end of the 14th century Ibn Khaldûn still names the Spherics of Theodosius as preparatory for the study of Menelaus (trsl. Rosenthal III, p. 131).
[13] Cf. for details Fecht, Theod., Introduction, and Ver Eecke, Theodosius, Introduction.
[14] Strabo, Geogr. XII 4, 9 (Loeb V, p. 466/467).
[15] Theorem 10; of course Hipparchus' name is not mentioned in the text.

or not.[16] Strabo's passage secures, however, Theodosius' origin from Bithynia; that he is often named "from Tripolis"[17] is caused by a confusion with a namesake in Suidas.[18]

A little more intricate is the question of the relative chronology of Euclid and Autolycus. For the writing of the "Elements" one can probably accept a date near 330 B.C.[19] but we cannot relate the "Phaenomena" to the "Elements." As to Autolycus we know that he came from Pitane[20] and that he was the teacher of Arcesilaus who was born around 315 and died about 240.[21] Obviously this is not sufficient material for a clear relation to the lifetime of Euclid. The generally accepted argument in favor of Autolycus' priority is singularly naive. Theorem 2 of Euclid's Phaenomena consists of four propositions with proofs for only three of them while the missing one is replaced by the remark "(that this is the case) has been shown (elsewhere)"; indeed theorem and proof are found as Theorem 10 in Autolycus' "Rotating Sphere." That a remark of this kind should be genuine in any Greek mathematical treatise, Euclidean or not, seems to me utterly implausible; I would assume the obvious, i.e. that a scholion replaced, perhaps in a damaged copy,[22] the first of four proofs by a simple reference to generally known theorems. In fact I see no reason why Euclid (presumably in Alexandria) and Autolycus (presumably in Athens) should not have written independently, and perhaps even simultaneously, on the mathematical theory of astronomical phenomena.

Much ingenuity has been expanded to establish the contents of a "pre-euclidean" spherics on which supposedly all three of our authors, and particularly Theodosius, depend[23]; to suggest Eudoxus as the author[24] only extends the flimsiness of the argument. The main argument seems to be that Theodosius often agrees very closely with his predecessors, a fact indicating to me not more than that he was familiar with their writings. I do not deny that it is quite obvious that the treatises of Euclid and of Autolycus were not the first studies of spherical geometry but it seems to me pointless to postulate the existence of one specific work which exactly fills all the gaps in the extant proofs and propositions noted by the 19th century historians. I think it is high in time to forget about the search for "Urschriften."

For the modern editions of Theodosius and Euclid see the Bibliographies in VI D 2. The earliest secure reference to Euclid as the author of the "Phaenomena" is the remark by Galen (2nd cent. A.D.) quoted at the beginning of this section.[25]

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[16] Cf. below p. 757.

[17] E.g. still in Czwalina's translation of the Spherics (1931) who overlooked Heiberg's "*Tripolites deleatur ubique*" ("Corrigenda", p. XVI).

[18] Suidas, ed. Adler I, 2, p. 693; cf. also Konrat Ziegler in RE VA (1934) col. 1931.

[19] Date suggested by Vogt [1912].

[20] On the coast of Asia Minor, opposite Lesbos.

[21] For the chronological details cf. the introduction in Mogenet, Autol. p. 5–19.

[22] The beginning of a roll is always particularly exposed.

[23] Nokk [1850]; Heiberg, Stud. Eukl. (1882); Hultsch [1886] etc.; Björnbo [1902]; Heath, GM I, p. 348–352 (1921); Mogenet [1947].

[24] For the underlying argument cf. below p. 761.

[25] Above p. 748; for the testimonia in general see Euclid, Opera VIII, ed. Menge-Heiberg, p. XXXII–XXXIV.

The separation of the "Phaenomena" into at least two different versions could have taken place long before Pappus.[26]

Autolycus' treatises were edited twice in modern times, once by Hultsch (1885) with a Latin translation, once (without translation) by Mogenet (1950), based on a huge text-critical apparatus. The German translation by Czwalina (1931) is based on Hultsch's text, the poor English translation by Bruin-Vondjidis (1972) on Mogenet's. The earliest reference to Autolycus as the author of the "Rotating Sphere" is found in a commentary to Aristotle by John Philoponus (6th cent.).[27] It was only recently that it was noticed that the two "Books" of the "Risings and Settings" were actually only two versions of the same treatise (Schmidt [1949]). Our Fig. 47 illustrates the relations between the theorems in the two "Books." Obviously all of the material from Book I is also contained in Book II, excepting two statements about obvious relations between "True" and "apparent" phases. The inverse, however, is not true; in particular the second half of "Book II" (Theorems 9 to 18) is a quite consistent discussion of the order of occurrence of the fixed star phases depending on the star's position with respect to the ecliptic.[28] In general Book II is better organized than Book I, e.g. by stating at the very beginning (II, 1) the basic assumptions made for the visibility limits of stars[29] with respect to the sun and for the sun's motion in the ecliptic. Thus one could think of "Book II" as a revised edition of an earlier version,[30] reminiscent of the duplication of theorems in Euclid's "Phaenomena."

2. Figures in the Texts

Before turning to the mathematical and astronomical contents of our six treatises we must describe the character of the illustrations which constitute an important part of the text since each theorem is accompanied by at least one figure to explain the relative positions of points and circles on the sphere.[1]

Unfortunately, classical scholars have maltreated illustrations in an extraordinary fashion. Instead of recognizing that variants and errors in diagrams belong as much to the evidence for the interrelations between manuscripts as spellings or omissions, drawings are usually replaced in our editions by some modern

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[26] Euclid, Phaen. 2 concerns three cases: $\varphi \gtreqless \varepsilon$, while the version used by Pappus contained only the discussion for $\varphi > \varepsilon$. This is taken as evidence for a date of the complete version later than Pappus (cf. Menge-Heiberg, Prolegomena, p. XXXII) as if six centuries between Euclid and Pappus would not suffice for the loss of two short sections.

[27] Cf. Mogenet, Autol., p. 160.

[28] Cf. below IV D 3, 4. It is only in Book II (Theorem 1 and all Theorems from 9 to 18) that the phrase *κρύψιν ἄγειν*, "cause hiding (of phase)," is used. On the other hand only Book I says *κατὰ συζυγίαν ἀνατέλλει καὶ δύνει* for diametrically opposite phases. Also the drawings follow different patterns: inclined ecliptic in Book I, horizontal ecliptic in Book II.

[29] In Book I only mentioned at the end of the proof of Theorem 10, though used implicitly long before.

[30] Such a "revised edition" could go back to the author himself; cf., e.g., the preface of Apollonius to Book I of his "Conics."

[1] Occasionally explicit references to figures are found in the texts, e.g. in Euclid, Phaen. 12b (p. 74, 1, ed. Menge), Autolycus, Rot. Sph. 2 (p. 199, 16, ed. Mogenet), Theodosius, Dieb. I, 1 (p. 58, 5/6) and Hab., Scholion 13 (p. 46, 17, ed. Fecht). Cf., however, below n. 3.

figures (usually of inept execution) which give us no information about the actual appearance. Only A. Rome began, around 1930, in his editions of the commentaries to the Almagest, to reproduce the figures also as accurately as possible and to provide them with a critical apparatus. His pupil J. Mogenet did the same for the treatises of Autolycus, thus providing us for the first time with a secure basis for the discussion of figures in Greek spherical astronomy.[2]

What treatment modern editors are capable of inflicting on diagrams may be shown by one example from Fecht's edition of Theodosius, De Diebus (1927). Five consecutive propositions, II 10 to 14, show complex but similar configurations. The core of the proofs concerns two arcs (here simply called AB and CD) intersecting in X-shaped fashion on a meridian. In Fig. 48 I have shown only what concerns this crucial part of the proofs. Figure **d** represents the arrangement for II, 13 as given by Fecht (p. 138) without apparatus or explanation of any kind; particularly noticeable are the dotted lines which one would not expect to find in an ancient drawing. The puzzle is solved by (unpublished) figures in one of the underlying manuscripts, Vat. gr. 191. There one finds on the margin[3] of fol. 58ʳ two figures, here partially reproduced as **a** and **b**. Figure **a** is incorrect insofar as it places both points B and C on the parallel II; the error is caused by thoughtlessly repeating the arrangement in the preceding theorem. Figure **b**, drawn on the margin directly below **a**, repairs the error by placing C on I and indicating that the arc AB should contact the parallel I, such that A belongs to III, B to II. My own figure **c** makes a small correction to **b**, since the contact of AB with I has nothing to do with the intersection of I and DB. Finally figure **d** shows how Fecht concocted from two figures, one wrong and one correct, the absurd drawing in his edition.[4]

Figures for spherical astronomy fall into two classes: in many cases a configuration on the sphere is shown as it seems natural to us, e.g., an ellipse for the ecliptic within a circle that represents the outline of a sphere. Of this type are almost all figures in Autolycus, Ris. Set. Book II (cf. Fig. 49, taken from II 10). But even there the situation is not quite so simple. The outer circle is not some outline of the sphere but represents specifically the horizon on which a star ε is located at a given moment when the ecliptic has a definite position with respect to the horizon. Hence, even such a "visual" representation is not some more or less general picture of the sphere but must be "read" like a map that explains a specific geometric configuration. Nevertheless this type of figures can be interpreted as showing the sphere as seen from a direction perpendicular to the horizon.

Such a "visual" interpretation fails, however, in a large second group of figures for which Pl. VIII may give examples.[5] In the simple drawing at the end of the right page the main circle again represents the horizon; the two small

[2] Cf., e.g., the contrast between the ancient figure Mogenet, Autol., p. 207 and its absurd counterpart in Hultsch's edition, p. 31.

[3] Normally figures are inserted in spaces left free in the text (and often remaining blank) but this is not the case in Vat. gr. 191. Hence it seems possible that the figures on the margins are later additions.

[4] For another drastic case of total disrespect for the importance of diagrams for the understanding of a text (Heron) cf. Neugebauer [1938, 2] II.

[5] Euclid, Phaen. 2 and 3 from Vat. gr. 204 fol. 61ᵛ/62ʳ (for this manuscript cf. Mogenet, Autol., p. 145 and p. 187).

circles tangential to it are the greatest always visible and always invisible circles, while the larger upper circle is the parallel on which a star performs its daily rotation, in part above (inside), in part below (outside) the horizon. The two other figures shown on our plate concern much more complex situations, but one simple principle remains valid: circles and arcs above the horizon are shown inside of it, sections below the horizon outside. Actual contacts are shown as contacts at their proper side, even at the price that circular arcs must be twisted into S-shaped curves.

This way of representing spherical configurations has one great advantage over the "visual" form: when curves (i.e. circles) intersect in the diagram we always deal with real intersections, not with apparent crossings of arcs which actually belong to different hemispheres. This is a consequence of the guiding principle in the non-visual representations: upper and lower hemispheres are mapped onto the interior and the exterior of the horizon circle, respectively. All that is inside is actually visible, all outside invisible.

I have no doubt that we have uncovered here a basic principle of spherical representation. One had only to approach it from a strictly mathematical standpoint to be led to the method of "stereographic projection" which not only separates the two hemispheres but also carries with it the wonderful quality of representing circles as circles.[6]

It is clear that this insight should also influence the form of our modern commentaries on treatises of spherical astronomy. Indeed, in many cases it is misleading to explain an ancient theorem with the help of a perspective drawing. If one wishes, as is often necessary, to replace a more or less distorted diagram, found in a manuscript, by a consistent and correct drawing then it is often stereographic projection which best reveals the core of the argument.[7] The proofs for many theorems in our material become almost self-evident when one follows a discussion with stereographic projection whereas a picture of the three-dimensional appearance is often extremely difficult to arrange in such a fashion that none of the arcs on one side interfere with arcs on the other side. Our subsequent discussion will furnish ample evidence for the close similarity of stereographic projection with the representations in the texts, though still removed from mathematical consistency. It is mainly the naive depiction of the greatest always invisible circle as a little appendix to the horizon (instead of surrounding from outside all possible positions of the horizon) that is basically wrong from the viewpoint of stereographic projection.

Seen in a wider historical perspective, however, it is certainly not mathematical convenience which initially motivated the separation of visible and invisible hemispheres in the diagrams of spherical astronomy. Surely we meet here only a tendency which is common to many aspects of "primitive" pictorial art, best known, and in details studied by H. Schaefer, in Egyptian drawings.[8] This does not mean that I wish to suggest any dependence of our material on Egyptian prototypes – on the contrary, I am sure that no direct parallel could be un-

[6] Cf. below V B 3, 1.
[7] This has been first observed and widely utilized by A. Rome (CA I, p. 141 note).
[8] Heinrich Schäfer, Von ägyptischer Kunst, besonders der Zeichenkunst, Leipzig, Hinrichs, 1919; this first edition is the best one (in my opinion).

covered for our specific diagrams outside Greek scientific treatises.[9] One should realize that in this field the archaic strata of civilization exercise a powerful influence far into periods which seem to have long outgrown their primitive origins. This even remains true in a very technical sense: the way of representing spherical configurations which we found in the treatises of early Spherics remains in use long after the discovery of exact methods, e.g., analemma or stereographic projection. Neither the Almagest nor its Commentaries[10] ever use mathematically consistent diagrams.

No mathematical principle should be sought for in the lettering of figures. Letters are assigned to points more or less in alphabetical order as the text proceeds. When the alphabet is exhausted, including the episemata, one begins again with α′, etc. The order in which letters are associated with parts of a figure is usually not significant; AB and BA can correspond to the same arc without any directional implications.[11] A sequence ABC, however, usually implies that B lies between A and C, but on a circle ACB may mean the same; even sequences out of order can occur.[12] Occasionally the same letter may denote a fixed point and a movable object, e.g. in a statement of the form "the point D, starting from D, traverses the arc DE and arrives at E."[13]

We have found in Theodosius, Dieb. II 13,[14] that errors occur in diagrams. Letters are easily misplaced[15] or sometimes an arc may be missing,[16] but by and large figures are well drawn.[17] In many cases the extant diagrams show an axial symmetry which is not wrong but which is not required by the theorem or proof in question. Such symmetries (e.g., placing the equinoxes on the horizon) detract from the general validity of the proposition. It is impossible to tell whether such symmetrizations, caused either by the greater simplicity of construction or by its aesthetic appeal, belong to the archetype or are copyist or editorial "improvements." In a modern commentary it seems advisable to represent configurations in the generality required by the theorem. hence avoiding strictly symmetric arrangements.[18]

The concluding theorems in Theodosius, Dieb. (II, 15 to 19) are suspected to be later additions since they differ greatly from the rest of the treatise by discussing the influence of fractions of days in the length of the year on the periodicity of fixed star phases.[19] The figures add to the suspicion; they are all of the type shown in our Fig. 50a in which the upper circle represents the horizon, the lower the ecliptic which is tangential to the summer-tropic. But neither

[9] Cf. also the figures in the "Eudoxus Papyrus," above p. 689.
[10] Cf., e.g., Rome CA II and his notes to the figures.
[11] E.g. Euclid, Phaen., p. 118, 19/20 (ed. Menge).
[12] Theodosius, Dieb. II, 19: ΕΓΔΖ instead of ΕΓΖΔ (p. 154, 12, ed. Fecht).
[13] Euclid, Phaen. 9, 11, etc.
[14] Above p. 752.
[15] E.g. Theodosius, Dieb. Lemma II, 10 has the letters T and Y (and the corresponding arc) misplaced (ed. Fecht, p. 122).
[16] Theodosius, Dieb. II 10 and 11: the arc representing the winter tropic is missing.
[17] I do not know what kind of drawing instruments were used that could produce with high accuracy constructions of great complexity (cf., e.g., the astrolabic figures 1 to 8 in Delatte AA II).
[18] Cf. below Figs. 52a and 52b; or Euclid, Phaen. 14, Versions (a) and (b) (ed. Menge, p. 86/87).
[19] Cf. above p. 749. Also the references to Callippus and to Meton and Euctemon look strange in this context.

horizon nor solstices are needed for these (utterly trivial) theorems and the circle of the ecliptic alone would suffice as illustration of the text. In corresponding figures in the same treatises, however, when ecliptic, horizon, and summer-tropic are depicted the figures always look like Fig. 50b with a Λ-shaped ecliptic, never shown as full circle or with a smooth contact. It seems evident that these representations come from different sources.[20]

3. Spherics

> Quoi qu'il en soit, ses trois Traités ont peu fait pour l'avancement de l'Astronomie; ils sont aujourd'hui presqu'inutiles même à l'histoire de la science. Ils ne prouvent guère que le goût des Grecs pour les subtilités métaphysiques, qu'ils portaient jusque dans la Géométrie.
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> Delambre, HAA I, p. 243

The title of the work of Theodosius to which we refer for short as "Spherics" should probably be understood to mean "On spherical (surfaces)" or, in modern terminology, "On spherical geometry." In fact, however, Theodosius comes nowhere near recognizing the fundamental importance of the great-circle triangle and his theorems rarely go beyond the geometrically obvious in the relations between a few special great circles and their parallels, without ever mentioning that one is dealing with configurations of interest only to astronomy. Philoponus, in the sixth century, quite correctly describes[1] the differences between the formal mathematical approach of Theodosius, the more realistic discussion in Autolycus' "Rot. Sph.," and the open admission of the astronomical background in Euclid's "Phaenomena," a background that in fact is common to all these treatises.

The little that is strictly mathematical in these treatises nowhere reaches the level of real Greek mathematics, from Archytas or Euclid's "Elements" to Archimedes or Apollonius. For example, it is nowhere clearly stated whether or not the interior of the sphere is considered to be accessible or not. In Theodosius, Sph. I, 2, e.g., the midpoint of a sphere is found by using interior distances which would also suffice to determine the diameter. For no apparent reason, however, I, 19 gives another construction to find the diameter, now operating with lines inside a small polar cap. In many cases there suddenly appear chords of spherical arcs[2] though in general arcs are treated as curves on the surface.[3] In Theodosius Sph. I, 16 and 17 it is shown that the distance from the points of a great circle to its pole is the diagonal of the square that has the radius of the sphere for its side[4]

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[20] It also may be noted that the concluding theorems to Theodosius, Hab. (10 to 12), do not fit the rest of the treatise very well; cf. below p. 757.

[1] For a summary cf. Mogenet, Autol., p. 160; text: ibid. note (3).

[2] Examples: Euclid, Phaen. 12 (ed. Menge, p. 72, 6–74, 1); Theodosius, Sph. II, 11 and 12; III, 2 and 3, etc.

[3] "Angles" subtending an arc never occur; arcs are either "similar" (ὅμοιος) or "equal" if located on the same circle (e.g. Theodosius, Dieb. 9, p. 122, 11 f., ed. Fecht). Otherwise arcs are "greater than" or "less than" similar (e.g. Euclid, Phaen., p. 48, 4; p. 66 (b), 8; Autol., Rot. Sph., p. 34, 2).

[4] We would say: $R\sqrt{2}$.

and I, 20 makes use of this fact in order to find the great circle that connects two points on a sphere. The procedure is unnecessarily complicated if one has access to the interior, but inadequate if one is supposed to remain on the surface.

The formal definitions of the basic concepts (diameter, axis, poles) already occur in Euclid's "Elements"[5] and are still repeated in more or less the same words by Heron, four centuries later.[6] The cinematic character of these definitions again underlines the astronomical origin of the whole topic.[7] But even Autolycus in his "Rot. Sph." does not mention that his concluding theorem applies to horizon and ecliptic.[8]

There are, of course, also purely physical assumptions made, without saying so. For example the stability of the poles and of the axis of the universe with respect to a given horizon is inferred from the experience that the fixed stars always rise and set at the same spot, on the horizon – an argument explicitly mentioned in the Eudoxus Papyrus.[9] Our treatises, however, turn it the other way and derive a "theorem" about the stability of risings and settings from the tacitly assumed stability of the axis of rotation.[10] The most important assumption, the absence of a fixed star parallax, is only obliquely alluded to in the form "similarly it can be shown that whatever point on the earth is chosen, it is the center of the cosmos."[11] On the other hand some implausible forms of the universe are discussed[12] (cylinder and cone) and properly demolished by some pseudo-optical arguments[13] for the sphericity of the world.

The question of definitions is related to the problem of authenticity of the introductions to our treatises as we have them. The introduction to Theodosius' Dieb. with all the definitions is undoubtedly only a scholion[14]; the introduction to Euclid's "Phaenomena" not only describes the basic circles (including the galaxy) but also contains proofs for the greatcircle quality of ecliptic, equator, and horizon (based on mutual bisecting). Most of this material does not fit the remaining text or is repeated in it. Similarly the definitions in the Rot. Sph. of Autolycus contain assertions about the character of uniform motion which are actually the equivalent of Theorem 2 in the text.[15] This mixture of descriptive definitions, astronomical concepts, and elementary assertions, including proofs, is reminiscent of similar definitions and statements in the Eudoxus Papyrus[16] though the terminology is not the same.

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[5] Elements XI, Definitions 14 to 17 (Opera IV, p. 4, 21-6, 3; trsl. Heath, Vol. III, p. 261).

[6] Heron, Definitions 76 to 81 (Opera IV, p. 52-55).

[7] Cf. also Mogenet [1947].

[8] Theorem 12, which states that both these circles are great circles. Also Theorem 11 does not mention the ecliptic.

[9] Tannery, HAA, p. 287f., No. 21.

[10] Autolycus, Rot. Sph. 7; Euclid, Phaen. 3.

[11] Euclid, Phaen. 1 (p. 12, 9, ed. Menge).

[12] Euclid, Phaen. Introd. (p. 6, 5, ed. Menge); cf. for this topic also above p. 576f.

[13] A reference "as shown in the Optics" (Phaen., p. 2, 8, ed. Menge) seems to have no basis in the extant works on optics.

[14] References to "Theodosius" and phrases like "he says," etc. (ed. Fecht, p. 54, 11 etc.).

[15] The definition of axis and poles appears only in one of the Greek manuscripts; cf. Mogenet, Autol., p. 195, 9-11 appar.

[16] Tannery, HAA, p. 287f., Nos. 15 to 21.

The terminology in our treatises is in some important cases very different from the later astronomical usage as we know it, e.g., from the Almagest. We shall discuss in the following sections some of these cases in greater detail: (A) a peculiar use of "day" and "night" for the period of invisibility or visibility of stars; (B) the use of "up" and "down" for directions in the ecliptic; (C) the concept of non-intersecting ("asymptotic") semicircles; and (D) the "interchange" of hemispheres, related to the determination of the rising- and setting-time of ecliptic arcs.

A. Polar Days

Theodosius in his little treatise that is euphemistically called "On Habitations" discusses in the first six and the last three of its twelve theorems conditions at, or near to, the north pole ($90-\varepsilon \leqq \varphi \leqq 90$) or near to the equator ($0 \leqq \varphi \leqq \varepsilon$),[1] leaving only Theorem 9 (with its preparatory theorems 7 and 8) for the zone which Greek geography conventionally calls "inhabited."

The last three theorems (10 to 12) concern the length of the "day" in the polar region. Theorem 10 states that at the north pole the day lasts longer than seven months, hence the night is about five months, a ratio that decreases as one moves farther south (Theorem 11) until one reaches the circle (at $\varphi = 90-\varepsilon$) where at the summer solstice "the day is 30 days long" (Theorem 12). Only by reading the proofs one finds what lies behind such an absurd formulation. "Night" is understood here to mean the time in which stars are visible, "day" being the opposite, and it is assumed that stars become visible when the sun in the ecliptic is at least 15° distant from the horizon. This visibility condition which we shall find again in the treatise of Autolycus on "Risings and Settings"[2] is, of course, absurd for polar regions where the sun can move nearly parallel to the horizon. Nevertheless, 15° before and 15° after the summer solstice, i.e. for 30 days, the sun (considered to be a point) is so near the (ideal) horizon that, on the basis of the above mentioned criterium, no star can be seen; hence the "day" is 30 days long.[3]

At the pole itself the time between vernal and autumnal equinox is assumed to be 187 days — this follows (without saying so) from Hipparchus' estimates for the lengths of the seasons[4] — and the "day" is therefore $15+187+15=217$ days long, i.e. more than 7 months[5]; hence the "night" is less than 5 months.

I am not sure that these rather absurd theorems are not a later addition. In contrast to Theodosius' systematic disregard for the solar anomaly[6] the use of the Hipparchian parameters involves precisely this concept. Furthermore the last three theorems repeat statements about the polar regions that have been made before in the Theorems 1, 3, and 4. On the other hand the discussion concerning the length of "day" and "night" is not extended beyond the polar

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[1] Of course no value for the obliquity of the ecliptic is ever mentioned in any of our treatises.

[2] Cf. below IV D 3, 4; also V A 3.

[3] Also the author of scholion No. 146 to Autolycus, Ris. Set. II, 1 (Mogenet, p. 273) begins the "day" when the sun is still 15° distant from the horizon.

[4] Cf. above p. 58 (1).

[5] Here, as always in such general estimates, "month" means 30 days; cf., e.g., Geminus, Isag. VI, 5 (Manitius, p. 70, 7).

[6] Theodosius, Dieb. I, 1 (Fecht, p. 56, 26f.; also p. 54, 2).

area, as one might expect in analogy to the other theorems. It is, of course, impossible to say how far inconsistencies may nevertheless be part of the original version.

B. Directional Terms

An interesting terminology is found in Theodosius Dieb. I, 9 and 10 describing the solar motion in the ecliptic as a progress from "higher" to "lower,"[1] similar, but opposite in direction, to the later expressions "leading" and "following."[2] For example I, 9 deals with points of sunrise (↑) and sunset (↓) on the semicircle of the ecliptic from ♑ to ♋. Then it is shown that, when E↑ (at one time) is higher than Z↑ (cf. Fig. 51), also the setting H↓ that follows E↑ will be higher (at some other time, e.g. in the next year) than the setting Θ↓ that follows Z↑. The proof runs as follows: nothing remains to be shown (cf. Fig. 51 upper half) when H↓ is in a position between E↑ and Z↑ (the latter included). Hence we may assume that H↓ is lower than Z↑ (Fig. 51 lower part). Then the interval from E↑ to H↓ corresponds to a daytime when the sun is nearer to the winter solstice than from Z↑ to Θ↓. Hence this second interval is longer than the first one and thus Θ↓ is in a lower position than H↓, q.e.d.

The same terms are also found in Dieb. II, 16[3] and in the scholia Nos. 41 and 45. In the scholion 35 to Hab. 8 azimuths are distinguished in the same way.[4] The earliest known occurrence comes from Eudoxus who uses ἄνωθεν repeatedly to describe a northerly direction.[5] Chalcidius in the 4th century uses *superior/inferior* for longitudinal amplitudes of Venus with respect to the sun.[6]

C. Non-Intersecting Semicircles

Autolycus, Sph. 8, formulates an important auxiliary theorem: if at sphaera obliqua a great circle touches the same parallels as the horizon (i.e. the greatest always visible and invisible circles) it will in the course of its rotation coincide with the horizon. The principle of the figure in the text is shown in our Fig. 52a in which AHBΔΓΘ represents the given horizon, AΔ and ΔE the greatest always visible/invisible circles which also are touched by the circle ΔBEΓZ. The semicircles AHBΔ and ΔZΓE are called "non-intersecting,"[1] a term which has received, thanks to Apollonius' theory of conic sections, the totally different meaning of "asymptote."

The significance of this concept in the present context can best be explained when we replace Fig. 52a by its equivalent in stereographic projection (and thus are able to avoid the strictly symmetric arrangement of the figure in the text[2]). Then we see (cf. Fig. 52b) that the "non-intersecting" quality of the two semicircles simply means that ΔΓE is obtainable from ABΔ through rotation around the

[1] *ἀνώτερος* and *κατώτερος* in the direction of increasing longitudes.
[2] *προηγούμενος* and *ἑπόμενος* in the direction of the daily rotation.
[3] Fecht, p. 148, 16.
[4] Fecht, p. 50, 2.
[5] Quoted by Hipparchus, Comm. Ar.; cf. ed. Manitius, index p. 310.
[6] Cf. Neugebauer [1973, 1].
[1] *ἀσύμπτωτοι*; the restriction to semicircles is, of course, essential since complete great-circles would always intersect on the sphere.
[2] Cf. also above p. 754.

north pole, i.e. the center of the always visible circle $A\Delta$. The proof in the text is in effect nothing more than this statement, expressed in terms of similar arcs on a concentric circle on which H moves into the corresponding position Z.

The insight obtained in this theorem is perhaps the most important result of early spherical astronomy and it is not surprising to find it applied in the proofs for a variety of theorems. Its contents can also be formulated in the following form: for a given locality all possible positions of the horizon ($A\Lambda$) with respect to the greatest always visible circle ($A\Delta$) can be represented by all congruent circles of diameters $A\Lambda$ that simultaneously touch the circles $A\Delta$ and ΛE. Hence different situations during one revolution of the cosmos can always be represented by turning the horizon by the proper amount. The plane astrolabe, based on stereographic projection, is the mechanical realization of this insight. The "non-intersecting semicircles" represent identical arcs of the horizon in different positions occupied during the axial rotation of the cosmos.

As one of many examples for the application of this method we may mention another repeatedly used theorem, formulated in Theodosius Sph. II, 13. There it is shown that arcs on "parallels" (i.e. circles perpendicular to the axis) between "non-intersecting" semicircles of horizon positions are "similar" (i.e. are subtended by equal angles); thus (cf. Fig. 53):

$$AB = \Gamma\Delta \sim EZ = H\Theta \sim K\Lambda$$

and corresponding arcs of the horizon between the parallels are equal:

$$KH = KE = \Lambda\Theta = \Lambda Z$$
$$EA = ZB = H\Gamma = \Theta\Delta.$$

All these relations are evident in stereographic projection since they are the direct result of the rotation of the horizon about the center M of the parallels. Theodosius, however, does not yet have command of this simple technique (although his diagrams[3] come quite close to stereographic projection) and thus his proof occupies almost three pages in the printed text. The same configurations appear later on in III, 11 to 14 (both for sphaera recta[4] and sphaera obliqua) in the discussion of rising times of ecliptic arcs — as always without the slightest hint of astronomical meaning and with endless proofs which could have been reduced to a few sentences by referring to the rotation of the celestial sphere.

D. "*Interchange*" *of Hemispheres*

Euclid's Phaenomena and Theodosius' Dieb. share a terminology which seems to be unknown from any other source: they speak about "interchange[1] of the visible/invisible hemisphere" in the following sense: suppose an arc AB (e.g. of the ecliptic) rises in the east such that A is exactly at the horizon and sets in the west until B is at the horizon (cf. Fig. 54a); then the arc AB is said to have "interchanged" the visible hemisphere. Fig. 54b illustrates, in the same way,

[3] Theodosius, Sph. ed. Heiberg, p. 67.

[4] The theorem for sphaera recta that corresponds to II, 13 is formulated in II, 10.

[1] *ἐξαλλαγή* and *ἐξαλλάσσειν*. Menge translates it by *permutatio*. Nokk says an arc "durchwandert" the hemisphere; this covers perhaps the factual but not the literal meaning.

the interchange of the invisible hemisphere by an arc BC. It is difficult to say how this terminology originated. The formal definitions[2] seem to mean "interchange of an arc of the ... hemisphere" but in the actual use it is always the arc that causes the interchange of the hemisphere. In both cases, as Fig. 54 shows, the arc in question begins and ends being completely within the same hemisphere; of course a large part of the hemisphere which was at the beginning above the horizon will be at the end below it, and vice versa.

Euclid's Phaenomena do not contain any application of the concept "interchange" to an astronomical problem but it can hardly be doubted that Euclid had the same questions in mind which we find in Theodosius' Dieb. connected with it. There the arc in question is an arc of solar travel in the ecliptic during daytime or night. We shall return to this aspect of the problem in a later section.[3]

Autolycus does not use the terminology in question but one could perhaps assume that he knew of it. His statement[4] that the dodecatemorion (arc of 30°) diametrically opposite to the sun's dodecatemorion is visible some time during the night can be expressed in the form that the time of visibility of the dodecatemorion is the time of interchange of the invisible hemisphere by the dodecatemorion in question opposite the sun.[5] We have no proof, however, that such a formulation was implied here or in similar cases.

4. Fixed Star Phases

We have seen how Theodosius determined the time of visibility of fixed stars[1] by applying the rule that the sun should be 1/2 sign away from the horizon, measured on the ecliptic. It seems likely that this visibility limit, 1/2 sign ahead or behind the sun, is related to the old measurements of arcs by multiples and fractions of signs, a norm of which the "steps" of 15° are a remnant.[2] Nowhere in our treatises appear measurements in degrees which would allow for less schematic estimates.

From an astronomical viewpoint a universal 15° visibility limit is a rather crude simplification of facts which obviously are much more complex. It cannot have escaped notice that not all stars appear or vanish simultaneously or that the eastern and the western parts of the horizon are not the same in darkness near sunrise or sunset. Nevertheless the 15° limit — or the equivalent 15-day limit for the solar motion — was generally accepted. Thus we find, e.g., in

[2] Definitions are given in the introduction to Euclid, Phaen. (p. 10, 3-10, ed. Menge), in the introduction to Theodosius, Dieb. (p. 54, 7-16, Fecht), and in the scholia Nos. 106 and 114 to Euclid, Phaen. (p. 147 and 150, ed. Menge). In spite of variants in the formulations these definitions are obviously derived from a common source.

[3] Cf. below p. 765.

[4] In Ris. Set. II, 1.

[5] The sun at the midpoint of the arc AB (cf. Fig. 55) is 15° distant from the horizon; CD is the dodecatemorion diametrically opposite to AB. While AB interchanges one hemisphere CD interchanges the other. The arc CD is visible almost all night.

[1] Theodosius, Hab. 10 to 12.

[2] Cf. above IV B 5.

Autolycus[3] the following estimates for the visibility of fixed stars that are located on the ecliptic[4]:

$\Omega \rightarrow \Gamma$:	30 days; during this time neither risings nor settings are visible	(1)
$\Gamma \rightarrow \Theta_1$:	5 months; only risings are visible	
$\Theta_1 \rightarrow \Theta_2$:	30 days; neither risings nor settings are visible	
$\Theta_2 \rightarrow \Omega$:	5 months; only settings are visible.	

That such simple patterns can coexist with much more refined theoretical schemes is shown, e.g., by the Babylonian planetary texts which give subdivisions of the synodic arc,[5] very similar to (1), while we know from the same period of the existence of sophisticated mathematical devices in computing ephemerides for sequences of planetary phases.

The assumption of a fixed elongation Δ (of 15°) of the sun from the horizon as necessary and sufficient condition for the fixed star phases leads to an important symmetry relation. Since the intersections between ecliptic and horizon are always diametrically opposite to one another (cf. below p. 1368, Fig. 56) it takes $(180-2\Delta)^\circ$ of solar travel to come from the position which causes the phase Γ of a star to the position which causes Θ_1; similarly there lies again an arc of $(180-2\Delta)^\circ$ between the sun's position at Θ_2 and at Ω. Therefore, assuming, as always, solar mean motion, the same number of days should elapse between Γ and Θ_1 and between Θ_2 and Ω.

In reality such a general scheme does not exist and its appearance in a parapegma for the dates of the phases points to the influence of a scheme based on $\Delta = $ const., rather than to an origin from actual observations. Tannery, who drew this conclusion[6], investigated the "Geminus-Parapegma"[7] and found that only for the phases ascribed to Eudoxus some evidence exists for symmetry between the intervals $\Gamma \rightarrow \Theta_1$ and $\Theta_2 \rightarrow \Omega$. This observation by Tannery is the sole basis for the widely accepted hypothesis that the work of Autolycus is derived from a similar work on spherics by Eudoxus, or, in other words that Eudoxus is the author of the "lost pre-Euclidean textbook on spherics."[8] In fact, all that Tannery's observation implies, is that Eudoxus also assumed a fixed visibility limit for all phases – as is not surprising. For the existence of a treatise on spherics it proves absolutely nothing.

The interest in the fixed star phases, risings and settings, appearances and disappearances in the course of the year, undoubtedly belongs to the earliest strata of Greek astronomy. The treatise of Autolycus on "Risings and Settings" is the first attempt, known to us, to bring these phenomena into a mathematical scheme. As we mentioned before[9] the extant text consists of two versions of which the second one (now called "Book II") is the better one. In its second half, the

[3] Autolycus, Ris. Set. II, 6. Expressly formulated in II, 1 and then consistently applied in II, 3 and II, 9 to 18; implicitly used in I, 2 and 3; casually mentioned in the proof of I, 10, a theorem that is parallel to II, 15.

[4] Cf. for the notation below VI B 5, 2 or VI D 3, 4.

[5] Cf., e.g., ACT II, p. 312.

[6] Tannery, Mém. Sci. II, p. 225–255.

[7] Cf. above IV A 3, 3.

[8] Cf. above p. 750.

[9] Above p. 751.

Theorems 10 to 18 represent a rather systematic survey of the sequences of the phases, depending on the position of the star in relation to the ecliptic (the coordinate "latitude" is never mentioned). The basic concept consists in the coordination of the star and a point of the ecliptic that rises or sets simultaneously with the star.[10] If the sun is located at exactly this point one speaks of the "true" phase, if it is 15° distant from it one has the "visible" or "apparent" phase. The theorems here under consideration concern only the visible phases.

In order to be able to summarize the contents of Autolycus' theorems in a simple fashion we represent in Fig. 56 the situation in three typical cases in stereographic projection.[11] The correctness of the assertions made in the Theorems II, 10 to 18 is then easily verified on these diagrams. The proofs in the text are simply descriptive timetables for the motion of the sun from phase to phase, repeated without mercy for all nine theorems.[12]

Explanation to Fig. 56.

The small circle in the middle represents the greatest always visible circle, its center is the north pole. The larger concentric circle is the equator, the skew circle which carries the sun is the ecliptic. The circular arcs which are tangential to the always visible circle are horizons; S is the star at the moment of rising or setting at one of the two horizons, respectively. The shaded sides of the horizons belong to the lower hemisphere.

The positions of the sun $\odot$ are always 15° distant from the intersection of the ecliptic with the horizon, thus producing the apparent phases for S. The tabulations give the visibility situation during the solar travel between the phases. For example the $++-$ in Table a, second line, means: when the sun is somewhere on the arc between Γ and Θ_2 on the ecliptic, then the star (column $*$) is visible (during some part of each night), its rising ($\uparrow$) is always visible, its setting ($\downarrow$) is always invisible. The last line in each table gives the sequence of the phases as the sun traverses the ecliptic. In the Tables a and c the assumption is made that S is sufficiently far away from the ecliptic. For stars nearer to the ecliptic, but still to the south of it, Θ_1 and Θ_2 can coincide or interchange their order; similarly for Γ and Ω for stars to the north of the ecliptic.

The main topic of Autolycus' Theorems II, 10 to 18 concerns the differences in the order of the phases for stars north or south of the ecliptic. For a star on the ecliptic itself the postulate of the 15° visibility limit results in the above listed[13] simple pattern (1) of II, 6. Now stars to the north and to the south must be considered. The points of the ecliptic which rise and set simultaneously with the star are the solar positions at the "true" phases. Hence we denote these points by the same letters as the corresponding visible phases which take place 15° before or after, but adding a bar to the letter, e.g. $\bar{\Gamma}$ for the point 15° before Γ. Hence

[10] For the terminology (συνανατέλλειν, συνδύνειν, etc.) see Hultsch, Autol., Index p. 255f.; for the astrological usage (παρανατέλλοντα, etc.) cf. Boll, Sphaera, p. 75–90.

[11] Using the south pole as center of projection. The figures in the text are much more primitive than in the Rot. Sph. or in Euclid and Theodosius and show no trend toward stereographic projection.

[12] Theorem II, 9 is also related to this group but at least the proof seems to be corrupt since it introduces the meridian which has no connection with the phases.

[13] Above p. 760.

we can say that we have for a star on the ecliptic

$$\bar{\Gamma}=\bar{\Omega}=\mathrm{S} \quad \text{and} \quad \bar{\Theta}_1=\bar{\Theta}_2. \tag{1}$$

For a star to the south of the ecliptic (Fig. 56a) the arc $\bar{\Omega}\to\bar{\Gamma}$ increases with the distance of S from the ecliptic and the same holds for the diametrically opposite arc $\bar{\Theta}_2\to\bar{\Theta}_1$. For the visible phases, however, $\Omega\to\Gamma$ increases whereas $\Theta_1\to\Theta_2$ decreases until Θ_1 and Θ_2 coincide and thereafter invert their order. Instead of saying that the order changes after $\Theta_1=\Theta_2$ we can also say that the change happens after $(\bar{\Theta}_2\to\bar{\Theta}_1)=(\bar{\Omega}\to\bar{\Gamma})=15°+15°=1^s$. This explains the way how four theorems describe the phases for stars to the south of the ecliptic:

Case a	$\bar{\Omega}\to\bar{\Gamma}$	sequence	⊙ on arc	ris. and sett.	
II, 11	$<1/2^s$				
II, 16	$<1^s$	$\Theta_1\to\Theta_2$	$\Omega\to\Gamma$	invisible	(2)
II, 12 and 17	$=1^s$	$\Theta_1=\Theta_2$			
II, 18	$>1^s$	$\Theta_2\to\Theta_1$	$\Theta_2\to\Theta_1$	visible	

Figs. 56a and 56b tell us that not only risings and settings but also the star itself remain invisible as the sun moves from Ω to Γ, while in the final stage, i.e. for stars sufficiently far to the south, risings, settings, and the star become visible when the sun travels on the arc $\Theta_2\to\Theta_1$. It should be noted, however, that the statements in the text, using the expression "cause hiding"[14] concern only the visibility of risings and settings, not of the star itself.[15]

For stars north of the ecliptic we have

Case c	$\bar{\Gamma}\to\bar{\Omega}$	sequence	⊙ on arc	ris. and sett.	
II, 10	$<1/2^s$	$\Omega\to\Gamma$	$\Omega\to\Gamma$	invisible	
II, 13	$<1^s$				(3)
II, 14	$=1^s$	$\Omega=\Gamma$	$\Gamma\to\Omega$	visible	
II, 15	$>1^s$	$\Gamma\to\Omega$			

These two lists reveal clearly the principle of composition of the second part of Book II. For a modern reader the inclusion of cases for distances $<1/2^s$ seems superfluous since the same phenomena take place as long as the condition $<1^s$ is valid. Similarly it is unnecessary to discuss both rising and setting of the star separately (e.g. in II, 12 and II, 17) since the relative position of all circles remains the same. But such clumsiness matters little and on the whole we have a useful derivation of qualitative visibility conditions for the risings and settings of fixed stars in the course of a year.

5. Rising Times, Length of Daylight, Geographical Data

On several occasions our treatises formulate theorems which concern the time of rising or setting of arcs of the ecliptic. The lack of any numerical data, however, restricts the results to some rather obvious inequalities and symmetries, much

[14] κρύψιν ἄγειν; cf. above p. 751, note 28.

[15] The translations by Czwalina and by Bruin-Vondjidis repeatedly err by adding the word star where only the phases are meant.

too general to be useful for any practical application. The Babylonian arithmetical schemes and their hellenistic adaptations to variable geographical latitudes are far superior in usefulness and theoretical significance to these trivialities disguised as mathematical theorems.

The two short treatises which we have from Autolycus do not deal with the concept of rising times of ecliptic arcs. The "Phaenomena" of Euclid, however, discuss in six theorems the risings and settings of the zodiacal signs:

Theorem 8 establishes symmetries and inequalities between rising and setting amplitudes,[1] noting the decrease in length of the arcs of the horizon which are crossed at rising and setting by the signs of the zodiac between the equinoxes and solstices.[2]

Theorem 9: similar symmetries and inequalities for the rising times of semicircles of the ecliptic. Since the length of daylight is the rising time of the semicircle between $\lambda_{\odot}$ and $\lambda_{\odot}-180°$ we have a theorem concerning the variation of the length of daylight. There is no mention, however, anywhere in our treatises of such a connection. We shall see presently[3] how the concept of length of daylight got entangled with the recognition of the daily motion of the sun. This may well be the reason that so little attention was paid to the oblique ascensions of semicircles. What Theorem 10 has to add to Theorem 9 is of little interest.

With Theorem 11 opens the discussion of setting times by stating that the rising time (ρ) of an arc equals the setting time ($\bar{\rho}$) of the diametrically opposite arc. Then Theorems 12 and 13 say that

$$\bar{\rho}(♋)=\bar{\rho}(♐)>\bar{\rho}(♌)=\bar{\rho}(♏)>\bar{\rho}(♍)=\bar{\rho}(♎) \tag{1a}$$

and

$$\rho(♑)=\rho(♊)>\rho(♒)=\rho(♉)>\rho(♓)=\rho(♈) \tag{1b}$$

while Lemma 13 states that

$$\rho(\alpha)=\bar{\rho}(\beta) \tag{2}$$

when α and β are arcs of equal length, located symmetrically to the equinoxes.

The discussion of rising times ends here and no essential progress was made during the next four centuries until Menelaus opened the way to an exact determination of the rising time of any given arc of the ecliptic and for any geographical latitude.

The remaining five theorems of Euclid's Phaenomena deal with the time of "interchange of the visible (or invisible) hemisphere"[4] by arcs of the ecliptic of unspecified length, depending on their position relative to the solstices and equinoxes. The purpose of these investigations becomes clear only through the treatise of Theodosius "On Days and Nights" as we shall see presently.

Of independent geometrical interest are quite sophisticated considerations (in the proof of the main theorem, Phaen. 14) concerning the great-circle distances

[1] Cf. for the continued interest in these quantities above p. 38.

[2] The proof more or less assumes what should be demonstrated. Scholion 81 (ed. Menge, p. 143) rightly refers for the necessary lemma to Theodosius, Sph. III, 7.

[3] Below p. 765.

[4] Above IV D 3, 3 D.

from a point of the ecliptic to the horizon.[5] We shall also see that the proof in question implies limits to the geographical parameters of the problem.[6]

We shall now turn to the use Theodosius makes throughout his "On Days and Nights" of the concept "interchange of ... hemisphere." Let A be the point of the ecliptic occupied by the sun at its rising, B at its setting (cf. below p. 1367, Fig. 54a), while $\tau(AB)$ denotes the time of interchange of the visible hemisphere by the arc AB. Then $\tau(AB)$ is the length of daylight in a strict mathematical sense, i.e. the time between exact sunrise and sunset, taking the solar motion AB during this interval into consideration. Similarly (cf. Fig. 54b) the length of night between sunset at B and sunrise at C is the time $\bar{\tau}(BC)$ in which the arc BC interchanges the invisible hemisphere.

The definition of "daylight" and "night" by the exact moments of sunrise and sunset, however, leads to great troubles. This becomes evident if one follows the proof of Dieb. I, 1 where it is stated that the length of daylight always decreases as the sun moves from the summer- to the winter-solstice (and similarly for the other semicircle). The proof runs as follows (cf. Fig. 57): let $Z\uparrow$ to $H\downarrow$ be the solar travel during a certain daytime after the summer-solstice, let $\Theta\uparrow$ be the sun's position at sunrise at a later day and make $\Theta K = ZH$. Then ZH and ΘK are traversed by the (mean) sun in equal times. Since ZH is nearer to the solstice than ΘK the first mentioned arc interchanges the visible hemisphere in a longer time than the second arc:

$$\tau(ZH) > \tau(\Theta K) \tag{3}$$

(no proof is given for this crucial lemma but a scholion refers correctly to Euclid's Phaenomena [7]). Thus, while ΘK interchanges the visible hemisphere the sun will reach the point Λ which is below the horizon when K sets. Hence the actual sunset will take place still nearer to $\Theta\uparrow$, e.g. at $M\downarrow$. Consequently the daylight $\Theta\uparrow - M\downarrow$ will be shorter than $Z\uparrow - H\downarrow$, q.e.d.

An obvious result of this approach is, e.g., the theorem that the sum of daylight and night is not constant. Theodosius does not get tired of deriving similar seemingly paradoxical consequences of his definitions. For example, daytime + night is different from night + daytime[8]; or at "equinox" daytime and night are not equal unless sunrise or sunset occur exactly at the moment when the sun is at the equator.[9] All this is, of course, the consequence of minute changes in rising- or setting-time of consecutive arcs of the ecliptic (order of magnitude $1/2°$) and causes only effects far below the accuracy of ancient time measurement — not to mention that all numerical data were missing.

Similarly without any practical meaning are theorems which state that on the semicircle between Cancer and Capricorn the sun is always to the east of the meridian at mid-day[10] and at mid-night.[11] All this is based on the consideration

[5] Similar discussions are also found in Theodosius, Dieb., Lemma II, 10 and in its applications in II, 10 to 14.

[6] Cf. below p. 766f.

[7] Scholion 7 (ed. Fecht, p. 156), but calling it Phaen. 18 instead of 14.

[8] Dieb. II, 5 and 6.

[9] Dieb. I, 11.

[10] "Mid-day" is the exact midpoint between sunrise and sunset and not identical with "noon."

[11] Dieb. II, 10.

of the mean motion of a mathematical point, called "sun." in relation to an ideal horizon. Alone the introduction of the solar anomaly would upset the whole structure. At best one could say that these investigations may have prepared the way for more convenient definitions and for the concept "equation of time" which introduced a clear distinction between purely geometric effects and parameters of the real solar motion.[12] Historically the work of Theodosius is an excellent example of illustrating the great progress that consists in abandoning definitions in the theory of time reckoning that are based on the naive popular description of the phenomena.

The "Spherics" of Theodosius is less removed from astronomical usefulness than his theory of time-intervals since he is dealing with geometric qualities of spherical astronomy — of course without ever saying a word about the astronomical significance of the circles he is considering. In the present context we have only to mention a group of theorems in Book III that relate (in fact) equal ecliptic arcs to arcs on other great circles by different spherical projections:

in III, 5: through parallel circles to the equator onto the horizon, i.e. ortive amplitudes at sphaera recta
in III, 6 and 9: by circles of declinations onto the equator, i.e. rising times at sphaera recta; III, 9 considers incommensurable lengths of intervals[13]
in III, 7: by parallels to the equator onto the horizon, i.e. ortive amplitudes at sphaera obliqua
in III, 8: by horizons onto the equator, i.e. rising times at sphaera obliqua
II, 10 to 14 concern additional inequalities for ratios between rising times.

Without numerical data these inequalities and statements about general trends remain rather removed from any connection with observations at given localities.

In view of this situation it is not surprising that no attempt was made to connect the rising times (or the length of daylight) with geographical latitude.[14] Nevertheless many phenomena of geographical importance are discussed in our treatises. In this respect the two works of Theodosius, the De Hab. and the De Dieb. are complementary. The first one concerns phenomena almost exclusively in the polar region ($90-\varepsilon \leqq \varphi \leqq 90$) and in the equatorial zone ($|\varphi| \leqq \varepsilon$), while the De Dieb. assumes (tacitly) intermediary latitudes.

In order to avoid certain difficulty in the proof of a theorem[15] Euclid assumes in Phaen. 14 that the pole of the horizon (i.e. the zenith) lies between the summer tropic and the greatest always visible circle; this is equivalent to assuming

$$\varepsilon < \varphi < 45°. \tag{4}$$

For similar reasons the same condition (4) is also introduced by Theodosius in Dieb. Lemma to II, 10 and its applications in II, 11 to 14. In Theodosius, Sph. II, 22

[12] Cf. above I B 2.
[13] Cf. above p. 749, n. 5.
[14] Or, to use the terminology of our treatises: with the inclination between horizon and equator; or with the radius of the greatest always visible circle.
[15] Its assertion concerns the monotonic decrease of the time of interchange of the visible hemisphere as function of the distance from the summer solstice (cf. above p. 765).

and 23 inclinations between great circles are investigated (horizon, equator, and ecliptic) and again a condition leading to (4) is imposed.[16]

In Phaen. 4 to 7 Euclid investigates rising and settings of fixed stars on the ecliptic, depending on whether or not the latter is reaching the interior of the greatest always visible circle. In modern terms this means the separating of cases with respect to

$$\varphi \gtreqless 90 - \varepsilon. \tag{5}$$

Similarly Phaen. 2 distinguishes cases for

$$\varphi \gtreqless \varepsilon. \tag{6}$$

Hence we already find with Euclid the concern with geographical zones which are basic for Theodosius' De Hab. The emphasis on the equatorial zone survives as the interest in geographical treatises and maps in the zenith position of the sun,[17] based on theorems about the verticality of the ecliptic to the horizon found in our treatises.[18]

6. Later Developments

The fact that the early treatises on spherical astronomy by Autolycus, Euclid, and Theodosius became what we today call "textbooks" gave later writers an occasion for commentaries and amplifications. Pappus, in the early fourth century, assembled such notes in the sixth book of his "Collections." In the Introduction he refers to the "Phaenomena" of Euclid, to the "Sphaerics" and the "Days and Nights" of Theodosius,[1] and summarizes later[2] the "Rotating Sphere" of Autolycus; neither the "Risings and Settings" of Autolycus nor the "Habitations" of Theodosius are ever mentioned.

It has long been observed that Pappus' notes which constitute Book VI are of a rather modest quality, the outcome of a superficial reading of his sources to which he added, for some formal omissions or lacking case distinctions, proofs without much interest.[3] Even in the time of Euclid or Autolycus Pappus' notes would not have contributed any new insight or method, but in the fourth century A.D. they were at best of antiquarian interest.

The Sect. 32 to 34 of Book VI are a good example of the way in which Pappus deals with mathematical and astronomical topics. He refers to the opinion of "some" who say that it is obvious that the sun traverses a smaller arc of the ecliptic than rises in the same time since it takes the sun a whole year to move through the ecliptic, whereas it takes only one nycthemeron for the ecliptic to rise completely. Pappus correctly remarks that this argument is mathematically inconclusive since one can conceive of motions with velocities $v_1(t)$ and $v_2(t)$, respectively, such that $v_1 < v_2$ for small t whereas the opposite inequality holds for larger t. To show

[16] It may be noted that (4) accidentally covers the space of the five climata from Syene to Mid-Pontus to which Ptolemy restricts his discussion in the "Phaseis" (cf. above p. 726).
[17] Cf., e.g., above p. 43f.
[18] Euclid, Phaen. 2; Theodosius, Hab. 3 and 5.
[1] Pappus, ed. Hultsch II, p. 474, 10, 6f., and 12f., respectively.
[2] Book VI, Sect. 27 (Hultsch, p. 518–524).
[3] It suffices to refer to Mogenet's discussion (Autol., p. 167–170) of Pappus' attitude toward Autolycus.

this he constructs a cinematic example[4] which, in modern terms, simply means that one can find a constant c such that $ct > \tan t$ for small t, whereas $\tan t$ increases for larger t much faster than ct.

This example then induces Pappus to a long digression on the behavior of quantities which may or may not indefinitely increase or decrease (the latter term meaning decrease toward zero). He proceeds to give examples,[5] displaying an impressive mathematical apparatus for such trivia as to say that the length of a chord in a circle has the diameter as upper bound but may be chosen as small as one desires; or that the area of a rectangle adjacent to a given side has a lower bound but is unlimited in the opposite direction; etc.

Even when one ignores such digressions it is obvious that Book VI is not more than an unsystematic collection of notes. After the first 26 sections, which deal mainly with material from Theodosius' Sphaerics comes a long section (27) summarizing Autolycus, to be followed by discussions again related to Theodosius (Sect. 28 to 36). In Sect. 37 Pappus suddenly turns to Aristarchus' treatise on the "Sizes and Distances of Sun and Moon."[6] Aristarchus' statement that less than a hemisphere is visible from a point outside a sphere brings Pappus again to a long geometric discussion,[7] ending in investigations of the projection of a circle into an ellipse and related topics (Sect. 38 to 52). It has often been said that these sections are a commentary to Euclid's "Optics" because of a reference to Euclid in a scholion[8], the contents, however, do not justify such an attribution.

The concluding sections of Book VI (53 to 60) return to spherical astronomy, formally supplementing some theorems in Euclid's "Spherics" but in fact mainly concerned with rising- and setting-times of zodiacal signs in relation to geographical latitude. For no visible reason Pappus raises the question of finding a region for which the rising times of Cancer and Leo are equal. This, once more, leads to a digression,[9] this time of historical interest: we are told that Hipparchus has shown "by arithmetical methods" that there exist regions for which an ecliptic arc beginning at ♋0° has a shorter rising time than an arc of equal length ending at ♎0°.[10]

Pappus' own problem of finding a region for which $\rho(♋) = \rho(♌)$ has, of course, no answer in the linear schemes (excepting the trivial solution $\varphi = 0°$). But Pappus had Ptolemy's tables of rising times (Alm. II, 8) at his disposal to which he refers in Section 57 in order to show that $\rho(♋) > \rho(♌)$ for $\varphi < \varepsilon$ while the opposite inequality holds for $\varphi > \varepsilon$.[11] Here Pappus clearly moved beyond the limits of early spherics.

A scholion to the beginning of Book VI, saying "It contains the solution of difficulties in the little astronomy"[12] has been taken as evidence for the existence

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[4] Hultsch, p. 536, 8–540, 25.
[5] Hultsch, p. 540, 26–546, 2.
[6] Translated by Heath. Arist., p. 412–414.
[7] Cf. also above p. 640.
[8] Cf. Hultsch, apparatus to p. 568, 12.
[9] Sect. 55, Hultsch, p. 600, 9–11; German translation in Björnbo, Menelaos, p. 70.
[10] Cf. above p. 301.
[11] Hultsch (p. 622, 19–27) has maltreated the text because he was unfamiliar with the geographical terminology; cf. Honigmann SK, p. 80f.
[12] Hultsch, p. 474, 2. The expression "ἐν τῷ μικρῷ ἀστρονομουμένῳ" is rather unusual.

of a collection of works called "Little Astronomy," supposedly assembled to facilitate the study of the "great astronomy," i.e. the Almagest – a story invented by Vossius in the early 17th century and later included by Fabricius in his Bibliotheca Graeca (late 18th cent.),[13] to be repeated ever since.

To me it seems extremely doubtful that there ever existed a definite collection of treatises known as "Little Astronomy." In fact this term is only attested once more in an anonymous commentary (perhaps by Eutocius,[14] about A.D. 500) to Book I of the Almagest. There, in connection with a study of isoperimetric figures, a reference is made to a proof "given by Theon in his Commentary to the little astronomy."[15] Which one of Theon's many commentaries is meant, I do not know, but I see no connection with the contents of Pappus VI or any special collection of treatises.[16]

All that factually underlies the story of the "Little Astronomy" is the experience that the relevant Byzantine codices always contain more or less the same hodgepodge of treatises on elementary astronomy, mathematics. and optics, but without any specific title or logical arrangement – it suffices that this material was useful for the schoolmasters who added new and different treatises in the course of time. It is not surprising that the same process went on also within the Arabic translations of these and similar treatises. Finally, in the 13th century, Naṣīr ad-Dīn aṭ-Ṭūsī gave to a large group of various treatises[17] the title "Intermediate Books." I do not see how this implies the existence of a collection of treatises on spherics under a different title and a millennium earlier.

It is probably an early phase of Greek astronomy that is reflected in some remarks by Geminus in his "Isagoge" (first century A.D.[18]) in which he criticizes assumptions made by the "old ones" (*ἀρχαῖοι*) concerning the "pairing" (*κατὰ συζυγίαν*) of the zodiacal signs such that ♋ and ♑ stand by themselves while ♌ and ♊, ♍ and ♉, etc., are associated.[19] Geminus objects[20] against the underlying assumption that the whole signs ♋ and ♑ rise and set farthest toward north and south, respectively, while in fact it is only the first degree of these signs that reach an

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[13] Cf. Hultsch, p. 475, n. 1.

[14] As suggested by Mogenet [1956].

[15] Hultsch, Pappus III, p. 1142, 10f.: "*τοῦ μικροῦ ἀστρονόμου*".

[16] D. Pingree, Gnomon 40 (1968), p. 15f. has enumerated the passages which are commonly invoked to support the hypothesis of the existence of a collection under the name "Little Astronomy." Among these sources I consider the following ones entirely unrelated to our problem: Theon's commentary on the "little astrolabe" (mentioned by Suidas) is well-known to be concerned only with the "planisphaerium" (cf. below V B 3, 7 F); Cassiodorus' distinction between Ptolemy's "minor" and "major" astronomy concerns only Ptolemy and has nothing to do with a collection; Philoponus discusses specifically the methodology of Theodosius, Autolycus, and Euclid from the viewpoint of philosophical classification, without any reference to a larger "collection" (cf. Mogenet, Autol., p. 160; cf. also above p. 755).

Boll [1916], p. 72 states that a work of Ptolemy is referred to as *μικρὸς ἀστρονομούμενος* by the "Anonymous of 379" (CCAG 5, 1, p. 197, 23 and p. 205, 18 – the latter repeated by "Palchus" CCAG 1, p. 81, 2) and thus adds the "Phaseis" to the well-known "collection." In fact, however, the word *μικρὸς* is not in the text, being Boll's own arbitrary addition.

[17] Steinschneider [1865]; Pingree, Gnomon 40, p. 16.

[18] Cf. for this date above IV A 3, 1.

[19] In astrological parlance such signs are called "seeing each other"; cf., e.g., Bouché-Leclercq, AG, p. 159–162; also P. Mich. 149, XII 27 (Mich. Pap. III, p. 104).

[20] Isag. II, 27–45; VI, 44–50; VII, 18–31.

extremal position. This criticism is, of course, correct when one locates the solstices at ♋ 0° and ♑ 0° but one can hardly doubt that the "old ones" had the Eudoxan norm in mind according to which the cardinal points lie at the midpoints of their respective signs.[21]

Geminus unfortunately does not mention any numerical data for the rising times accepted by the adherents to the objectionable pattern. It seems reasonable, however, to assume that any numerical scheme at that early period would be of a linear arithmetical type. Since we know that the same arrangement of the zodiacal signs existed for Greek shadow tables as well,[22] based on the ratio $M:m=15:9$ for the extremal lengths of daylight, it makes sense to reconstruct a similar table of rising times, based on the arithmetical rules valid for the same geographical situation. Hence, in order to follow the Eudoxan norm, we call ρ_1 the rising time for the arc from $\lambda=345°$ to $\lambda=15°$ (in Hipparchian longitudes), and similarly for $\rho_2, \dots, \rho_{12}$. For these oblique ascensions one finds the following simple pattern:

$$\begin{aligned} \rho_1 &= 15° \\ \rho_2=\rho_{12} &= 20 \\ \rho_3=\rho_{11} &= 25 \\ \rho_4=\rho_{10} &= 30 \\ \rho_5=\rho_9 &= 35 \\ \rho_6=\rho_8 &= 40 \\ \rho_7 &= 45. \end{aligned} \tag{1}$$

From (1) we obtain the corresponding longest daylight by computing the rising time for the quadrant from an equinox to the summer solstice. Thus

$$M=2(1/2\,\rho_4+\rho_5+\rho_6+1/2\,\rho_7)=2(15+1{,}15+22;30)=3{,}45°=15^h \tag{2}$$

and similarly $m=2{,}15°=9^h$, as it should be.

Hence we see that the Eudoxan norm in combination with a "System A" pattern produces a simple linear zigzag function for the rising times. It is the change to the Babylonian way of pairing the signs that gives a better accounting of the geometrically evident symmetries[23] and thus results in a refinement of the theory of rising times and of the length of daylight. It is perhaps not accidental that Geminus formulates geometrical symmetries in this context.[24] On the other hand the linearity of the accepted pattern is essential for the convenience of these rules.[25]

Concerning fixed star phases Geminus has nothing to say[26] that is not general knowledge since Autolycus, describing the four (true and visible) morning and evening phases and their mutual relations depending on the position of the star

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[21] This has been observed, long ago, by Smyly (Hibeh Papyri I, p. 141).
[22] Cf. above IV D 2, 1 A.
[23] This does not imply the assumption of observational improvements.
[24] Isag. VII, 32–34.
[25] For the trigonometrically computed oblique ascensions (Alm. II, 8) one must use the smaller intervals of 10° in order to obtain proper results by simple linear interpolation.
[26] Isag. XIII.

with respect to the ecliptic.[27] It is only from a chapter in the Almagest[28] that we hear about a considerably longer list of stellar phases. Its origin is unknown; Ptolemy's discussion does not give the impression that it was a novelty in his time. In his own work on the "Phaseis" of fixed stars he makes use only of the commonly considered (visible) risings and settings.

This enlargement of the list of stellar phases is caused by taking the meridian into consideration and expanding the traditional case distinctions in a purely formal fashion. Let us denote by $\odot_3$ three possibilities for solar positions: slightly below, exactly in, slightly above the horizon, either in the east (E) or west (W); let $*$ indicate the position of the star exactly in the horizon (E or W); let finally $+$ denote the upper, $-$ the lower culmination. Then we can tabulate Ptolemy's 26 cases as follows:

No.	E	M	W	cases
1	$\odot_3 *$			
2	$\odot_3$	$* +$		
3	$\odot_3$		$*$	9
4	$*$	$\odot \pm$		
5		$\odot \pm * \pm$		
6		$\odot \pm * \mp$		
7		$\odot \pm$	$*$	8
8	$*$		$\odot_3$	
9		$* +$	$\odot_3$	
10			$\odot_3 *$	9

This scheme contains in Nos. 1 and 3, 8 and 10 the conventional phases of heliacal rising and setting. What remains is hardly of any practical interest, not even for the "aspects" of astrological doctrine. It is obviously the work of pedants whose role is so conspicuous in the works on spherical astronomy.

§ 4. Plane Trigonometry

Trigonometry originated as a tool for the solution of astronomical problems which are basically of a numerical character. In this sense trigonometry bridges the gap between "rigorous" methods (in Greek terminology "διὰ τῶν γραμμῶν," i.e. proved by geometric constructions) and arithmetical procedures ("δι' ἀριθμῶν").[1] As far as we know the geometrical approach to the solution of

[27] Cf., e.g., above p. 761 f. and Fig. 56 (p. 1368).

[28] Alm. VIII, 4.

[1] It was P. Luckey ([1927], p. 29–31) who first understood that διὰ τῶν γραμμῶν means "by rigorous methods," in contrast to merely numerical results. Luckey started from the passages in Ptolemy's "Analemma," adding occurrences in the Almagest (which can easily be multiplied). Chronologically the earliest occurrence of this expression for "rigorous," known to me, is found in Hipparchus' commentary to Aratus (ed. Manitius, p. 150, 14–17). For its continued use can be quoted (without any claim to completeness): (Continuation of footnote on p. 772)

numerical problems is of Greek origin. At least no such combination has yet come to light in Babylonian astronomy which seems to proceed on purely numerical grounds without any support by geometrical concepts.[2]

We know very little about the history of trigonometry in Greek science. By the end of the third century B.C. Aristarchus and Archimedes make use of one important inequality, obviously considered well known, the equivalent of

$$\frac{\sin\alpha}{\sin\alpha'}<\frac{\alpha}{\alpha'}<\frac{\tan\alpha}{\tan\alpha'},\qquad \alpha>\alpha'. \tag{1}$$

By the second century A.D. trigonometry had reached its final form (before Islamic developments) in Book I of the Almagest.[3] Here we have typically trigonometric relations, e.g. the equivalent of formulae for $\sin(\alpha\pm\beta)$, and numerical tabulations which remained characteristic for the whole field. With it goes a definite system of norms, e.g. the reckoning of angles in degrees, the norm $R=60$ for the basic circle, and in general the sexagesimal system of computation. But always the "chord" is the only recognized (i.e. tabulated) trigonometric function.

We know practically nothing about the details of the process which led from Aristarchus to Ptolemy. The customary, if tempting, association of Hipparchus with the transition from lengthy discussions of special cases to a uniformly applicable method finds, however, as we have seen,[4] little support in the available sources. Hipparchus' terminology is occasionally as clumsy as Aristarchus' when he operates, e.g., with the "24th parts of the circumference" or when he reckons any arc in "zodiacal signs."[5] This is still reminiscent of Archimedes and Aristarchus

(Footnote 1 – continued)

Pappus, Comm. to Alm. VI, ed. Rome, p. 171, 16–17 (≈A.D. 320).

Theon, Comm. to Alm. I, ed. Rome, p. 451, 11–12 (≈A.D. 370); in the Great Commentary to the Handy Tables he uses the comparative (γραμμικώτερον) which can only mean "more accurate" (quoted Tihon [1971] I, p. VII, note 1).

Basil of Caesarea, Homily III 57 B, ed. Giet, Sources Chrétiennes [1950], p. 198, ridiculing "their proofs ... as exact and artificial nonsense"; Basil died 379.

Proclus, Comm. to Plato's Rep., ed. Kroll II, p. 27, 16–17 (≈450).

"Heliodorus", Comm. to Paulus Alex., ed. Boer, p. 92, apparatus (≈560).

Theodoros Metochites, Logos 14, 35, ed. Ševčenko, Métoch., p. 263, 32–33 (≈1300).

A Latin equivalent is found in Pliny. NH II, 63 (Jan-Mayhoff I, p. 147, 3f.): *ratione circini semper indubitata.*

It should be mentioned, however, that ψηφοφορία γραμμικῶς means "graphical method" (Ptolemy, Introd. to the Handy Tables, Opera II, p. 163, 13 and 23, p. 162, 22) in contrast to ψηφοφορία ἀριθμητικῶς (p. 166, 19; p. 169, 6).

[2] This does not mean that "trigonometric" problems are absent from Babylonian mathematics. On the contrary, we know of typical trigonometric topics ("chord" and "arrow") in Old-Babylonian mathematical texts (cf., e.g. Neugebauer, MKT I, p. 180) but we find no trigonometry in connection with astronomical problems. Also the general approach is not the same as in Greek trigonometry: in the cuneiform texts a numerical answer is sought to geometrically formulated examples whereas Greek trigonometry uses general geometrical theorems to obtain specific numerical results.

[3] Cf. above I A 1.

[4] Cf. above I E 3, 1.

[5] Cf. above p. 302; p. 299. Also "four-sixtieths of a great circle" (i.e. 24°) for the obliquity of the ecliptic (Strabo Geogr. II 5, 43, Loeb I, p. 520/521).

who measure angles in fractions of a right-angle,[6] or of half right angle,[7] or of the circumference,[8] or of zodiacal signs.[9]

The earliest preserved evidence for the approach to specifically "trigonometric" problems is found in the treatise "On the sizes and distances of the sun and moon" by Aristarchus,[10] written about 250 B.C. In this work he makes use of a theorem which is the equivalent of (1), p. 772 and which is expressly formulated a few decades later in the "Sand-Reckoner" of Archimedes[11] in the following form[12] (cf. Fig. 58): for two right triangles with one side (a) in common but with $\alpha > \alpha'$ (thus $b < b'$) one has the inequalities[13]:

$$\frac{c'}{c} < \frac{\alpha}{\alpha'} < \frac{b'}{b}. \tag{2}$$

With the help of these inequalities Aristarchus computes, or rather estimates, the numerical values of trigonometric functions in some specific cases of small angles. These computations confirm what one would have conjectured a priori, viz that at his time no ready made numerical tables existed for trigonometric functions, a level which is not accessible to us before Ptolemy's table of chords. The idea of committing numerical data once and for all to a tabular form may well be the result of contact with Babylonian mathematics and astronomy and could thus go back to the time of Hipparchus.[14]

No proof of (2) is given by Archimedes. Accepting the modern form (1) of these inequalities and assuming a configuration as shown in Fig. 59 we can write for the right half of (1)

$$\frac{\alpha}{\alpha'} < \frac{a}{a'}, \quad \alpha > \alpha'. \tag{3}$$

In this form the inequality is used by Aristarchus in his Proposition 4. He is interested in an angle α' which is 1/45 of half of a right angle (i.e. 1°); thus he compares $\alpha' = \frac{1}{45} \cdot \frac{(R)}{2}$ with $\alpha = (R)/2$. The sides in question are the lunar radius[15] $r_m = a'$ and (since $\alpha = 45°$) $E_0T = a$; hence from (3)

$$\alpha/\alpha' = \frac{(R)}{2} \Big/ \frac{1}{45} \cdot \frac{(R)}{2} = 45 < E_0T/r_m, \quad (R) = \text{right angle}$$

or

$$r_m < 1/45\, E_0T. \tag{4}$$

Since the lunar distance $R_m = E_0M > E_0T$ we have a fortiori

$$r_m < 1/45\, R_m. \tag{5}$$

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[6] Archimedes, Sand-Reckoner, Opera II, p. 226, 19–20; p. 228, 12 and 18; etc. Aristarchus, Heath, p. 352, 11–12; p. 380, 16–17; etc. Once, p. 376, 22, a right angle is divided in 60 parts but only to show that the ratio $1/4(R):1/30(R) = 15:2$.

[7] Aristarchus, Heath, p. 366, 6; p. 368, 9–10.

[8] Aristarchus, Heath, p. 366, 2–3; p. 380, 17; etc.

[9] Aristarchus, Heath, p. 352, 14–15; p. 364, 21–366, 2; etc.

[10] Cf. for this treatise above IV B 3, 1.

[11] Cf. above p. 645 (8 b).

[12] Archimedes, Opera II, p. 332, 3–10; trsl. Ver Eecke, p. 361.

[13] Equivalent to (1) since $c' = a/\sin\alpha'$, etc.

[14] Cf. above I E 3, 1.

[15] Cf. above p. 640 and Fig. 11 there; the present angle α' is the angle α in Fig. 11.

Because, using modern terminology, $r_m/R_m = \sin 1°$, $r_m/E_0T = \tan 1°$, one may say that Aristarchus gave the correct estimates[16]

$$\sin 1° < \tan 1° < 1/45. \tag{6}$$

The relation (3) based on Fig. 59 is used once more in Aristarchus' Proposition 7 where it is his goal to show that the distances of the sun and of the moon, R_s and R_m, respectively, satisfy the inequalities

$$18\, R_m < R_s < 20\, R_m. \tag{7}$$

In the first part of the proof he applies (3) to the complement $\bar{\eta}(=3°)$ of the moon's elongation η at quadrature and to an angle θ which can conveniently be constructed (1/2 of 45°). Thus he operates with

$$\bar{\eta} = \frac{1}{15}\cdot\frac{(R)}{2}(=\alpha'), \qquad \theta = \frac{1}{2}\frac{(R)}{2}(=\alpha), \qquad \mathrm{GK} = a, \qquad \mathrm{HK} = a'$$

(cf. Fig. 60). Consequently

$$\theta/\bar{\eta} = 15/2 < \mathrm{GK/HK}, \tag{8}$$

a relation which will be used presently.

In the second part of the proof the left-hand side of (1) or (2) is used, but now expressed as a theorem for chords. As the larger angle serves the angle of the regular hexagon, i.e. $(C)/6\,(=60°)$; the smaller angle is $2\bar{\eta} = (R)/15 = (C)/60$. Thus

$$\frac{(C)}{6}\Big/\frac{(C)}{60} = 10 > \frac{\mathrm{EO}}{\mathrm{EN}}, \qquad (C) = 4(R)$$

(cf. Fig. 61). This readily leads to the upper bound in (7) since

$$\frac{R_s}{\mathrm{EN}} = \frac{\mathrm{ED}}{\mathrm{EN}} = 2\frac{\mathrm{EO}}{\mathrm{EN}} < 20 \quad \text{and} \quad \frac{\mathrm{ED}}{\mathrm{EN}} = \frac{\mathrm{ES}}{\mathrm{EM}} = \frac{R_s}{R_m}$$

hence $R_s/R_m < 20$.

For the left-hand side of (7) Aristarchus observes that[17]

$$2 = \mathrm{EF}^2/R_s^2 = \mathrm{FG}^2/\mathrm{GL}^2 = \mathrm{FG}^2/\mathrm{GK}^2 > 49/25$$

(cf. Fig. 60), thus

$$\frac{\mathrm{FG}}{\mathrm{GK}} > \frac{7}{5} \quad \text{and} \quad \frac{\mathrm{FG}}{\mathrm{GK}} + 1 = \frac{R_s}{\mathrm{GK}} > \frac{12}{5}$$

and with (8)

$$\frac{R_s}{\mathrm{HK}} = \frac{R_s}{\mathrm{GK}}\cdot\frac{\mathrm{GK}}{\mathrm{HK}} > \frac{12}{5}\cdot\frac{15}{2} = 18.$$

But

$$\frac{R_s}{R_m} = \frac{\mathrm{EH}}{\mathrm{HK}} > \frac{R_s}{\mathrm{HK}} > 18 \tag{9}$$

q.e.d.

[16] Indeed $\sin 1° \approx \tan 1° \approx 0;1,2,50 < 1/45 = 0;1,20$. Note that $0;1,20 \approx \sin 1;16° \approx \tan 1;16°$.

[17] In order to find small integers, m and n, such that $2 > m^2/n^2$ one can try to solve step by step $2 = (m^2+1)/n^2$. The only solution $(m>1)$ is $m=7$, $n=5$. Hence $\sqrt{2} > 7/5 = 1.4 = 1;24$.

In modern terminology (7) is the equivalent of finding estimates for

$$R_m/R_s = \cos\eta = \cos 87^\circ \ (\approx 0;3,8,25)$$

in the form

$$0;3 = 1/20 < \cos 87^\circ < 1/18 = 0;3,20. \tag{10}$$

Aristarchus was, of course, not at all interested in these ratios for angles in general.

Once more it is the left-hand side of (1) or (2), applied to chords, that is used in the Propositions 11 and 12 which concern, as before Proposition 4, the small angle of 1° and its complement of 89°. In combination with (6) one can say that Aristarchus established the bounds

$$1/60 < \sin 1^\circ < 1/45 \tag{11}$$

and independently[18]

$$89/90 < \cos 1^\circ. \tag{12}$$

The preceding survey of the trigonometric problems encountered in Aristarchus' treatise shows that all cases are based on the inequalities (2) or its equivalent (1), stated, of course, for chords and not for sines. The earliest preserved proof for (1), and this for the left-hand side only, i.e.

$$\frac{\operatorname{crd}\alpha}{\operatorname{crd}\alpha'} < \frac{\alpha}{\alpha'}, \qquad \alpha > \alpha' \tag{13}$$

is found in the Almagest,[19] used by Ptolemy to find an estimate for crd 1/2°. Since it is likely that this proof is taken from older sources we reproduce here its essential steps (cf. Fig. 62).

Let the point D be determined in such a fashion that CD bisects the angle BCA. Consequently BD = AD = b and therefore DOFG perpendicular to AB. Since $a > a'$ also BE > EA and the circular arc with center D and radius DE will meet DF in G above AB, and DA in H below it. Therefore we have for the areas

$$\text{area DFE} < \text{area DGE}, \qquad \text{area DEH} < \text{area DEA}$$

and thus

$$\frac{\mathrm{FE}}{\mathrm{EA}} = \frac{\text{area DFE}}{\text{area DEA}} < \frac{\text{area DGE}}{\text{area DEH}} = \frac{\gamma}{\delta}$$

or

$$\frac{\mathrm{FE}}{\mathrm{EA}} + 1 = \frac{\mathrm{FA}}{\mathrm{EA}} < \frac{\gamma+\delta}{\delta} = \frac{1}{2}\cdot\frac{\varepsilon}{\delta}$$

and thus

$$\frac{\mathrm{AB}}{\mathrm{EA}} < \frac{\varepsilon}{\delta}.$$

Consequently (using Euclid VI, 3 and VI, 33)

$$\frac{\mathrm{AB}}{\mathrm{EA}} - 1 = \frac{\mathrm{BE}}{\mathrm{EA}} = \frac{a}{a'} = \frac{\operatorname{crd}\alpha}{\operatorname{crd}\alpha'} < \frac{\varepsilon}{\delta} - 1 = (1/2\,\varepsilon + \gamma)/\delta = \frac{\alpha}{\alpha'}$$

q.e.d.

As we have seen, the "Ptolemaic Theorem" for sides and diagonals of a quadrilateral, inscribed in a circle, allows one to compute crd $(\alpha \pm \beta)$ from crd α and

[18] Actually $\cos 1^\circ \approx 0;59,59,27$ while $89/90 = 0;59,20 \approx \cos 8;30^\circ$.

[19] Alm. I, 10; cf. above p. 24.

crd β.[20] Bīrūnī, however, has preserved[21] a "Lemma of Archimedes" that can be used to solve the same problem in a very elegant and simple fashion. We have no evidence, however, for any practical application of this method for the computation of tables of chords, nor does Ptolemy mention such an alternative possiblity. For our present purposes it will therefore suffice to present the mathematical steps on which the Archimedean lemma rests, without describing the actual historical versions and alternative proofs.[22]

Repeatedly use is made of the fact that, if $s = \mathrm{crd}\,\alpha$, then s is seen from all points C on the circle under the angle $\alpha/2$ (cf. Fig. 63). Consequently (cf. Fig. 64): for any position of C on the arc BA the angle at C is the sum of the angles at A and B. Finally, for the sake of convenience, we shall use the function $\cos\theta$ instead of writing historically correct $1/2\,\mathrm{crd}\,(180-2\,\theta)$.

Fig. 65 shows that the problem for the sum of two angles is trivial since

$$AC = AH + HC = AB\cos\frac{\beta}{2} + BC\cos\frac{\alpha}{2}$$

i.e.

$$\mathrm{crd}\,(\alpha+\beta) = 1/2\,(\mathrm{crd}\,\alpha\,\mathrm{crd}\,(180-\beta) + \mathrm{crd}\,\beta\,\mathrm{crd}\,(180-\alpha)). \tag{1}$$

The lemma of Archimedes is the key for the case of subtraction. First one has to construct (for $\alpha > \beta$) the angle $\alpha - \beta$. Fig. 66 shows that $CD = \mathrm{crd}\,(\alpha-\beta)$ if D is located in such a fashion that $AB = BD = \mathrm{crd}\,\alpha$. Hence we make in Fig. 67 the same assumption concerning D and construct $BF = BC$; hence also $AF = CD$. If G on CA is chosen such that $BG = BF = BC$ then (cf. Fig. 64)

$$\frac{\alpha}{2} = \frac{\beta}{2} + \gamma;$$

but in the triangle ABG holds

$$\frac{\alpha}{2} = \frac{\beta}{2} + \gamma'.$$

Thus $\gamma = \gamma'$ and consequently $AG = AF = CD$ and therefore

$$AG + GH = AH = HC + CD. \tag{2}$$

This relation is formulated as the "Lemma of Archimedes": if arc $AB = \mathrm{arc}\,BC$ and $BH \perp AC$ then $AH = HC + CD$.

The formula (2) solves our problem because it is equivalent to

$$AH = \mathrm{crd}\,\alpha\cos\frac{\beta}{2} = HC + CD = \mathrm{crd}\,\beta\cos\frac{\alpha}{2} + \mathrm{crd}\,(\alpha-\beta)$$

or

$$\mathrm{crd}\,(\alpha-\beta) = 1/2\,(\mathrm{crd}\,\alpha\,\mathrm{crd}\,(180-\beta) - \mathrm{crd}\,\beta\,\mathrm{crd}\,(180-\alpha)). \tag{3}$$

Thus we see that Archimedes had the exact equivalence to the relations $\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ at his disposal. As previously stated, we know nothing about its practical application.

[20] Cf. above p. 23 (C) and (A).

[21] In the "Book on the determination of the chords in a circle (Boilot [1955], p. 197 No. 64; translated in Suter [1910]) and in a chapter of the Qānūn (Boilot [1955], p. 211 f., No. 104; summary in Schoy, Bir.). Cf. also Toomer [1973], p. 20-23.

[22] Cf. for details Tropfke [1928].

Book V

Astronomy During the Roman Imperial Period and Late Antiquity

Nous avons la ferme persuasion de n'avoir rien omis qui puisse avoir la moindre importance

Conclusion of Delambre,
Histoire de l'Astronomie Ancienne.
1817 (Vol. II, p. 639)

Introduction

The period between Hipparchus and Ptolemy probably saw a great deal of astronomical activity, directed at the computation of lunar and planetary positions. The period of ingenious mathematical models of the Eudoxan type and of "pythagorean" speculations definitely came to a close while the enormous spread of astrological practices created an ever increasing need for numerical tables. Even today the scattered fragments of papyri clearly reflect this process (cf. for horoscopes Fig. 1 and for tables the chronological list below). On the other hand Ptolemy does not quote a single observation from the two centuries between Hipparchus and Menelaus (cf. V B 1 Fig. 16, p. 1375). It seems as if no serious theoretical progress was made during this period, except, of course, the creation of spherical trigonometry by Menelaus and, to a lesser degree, the mathematical geography of Marinus.[1] Two other names on our list, Vitruvius and Heron, can be associated with competent theoretical discussions on spherical astronomy[2] but this hardly reflects progressive work of their own.

In the first section (A) of Book V an attempt is made to get some insight in the type of planetary and lunar theory that prevailed in the period between Hipparchus ¬d Ptolemy. Section B discusses what we know about Ptolemy's astronomy yond the Almagest. The third and last section (C) deals with the period after ›lemy, e.g. with its most important bequest to Byzantine and Islamic astronomy, "Handy Tables," edited by Theon.[3]

Chronological Summary

Continued from p. 575; for continuation cf. below p. 943. Most dates are only ·oximately known, even if not expressly queried. For the papyri cf. also below 'f.

·ıs	≈0	P. dem Berlin 8279 (planet. tables)	[−17] to 12
's	≈0	P. Vienna D 4876 (moon)	≈0
,:lus	d. 36	P. Strasb. Inv. 1097 (plan. t.)	?
ıeca	d. 65		
Pliny	23 to 79	P. Flor. Inv. 75 D (plan. t.)	48 to 52
Geminus	50	P. Berlin Inv. 21226 (Jupiter)	50 to 56
Dorotheus	50		
Teukros	50		
Balbillus	30 to 80	P. Oxy. 303 (treatise, lunar theory)	≈50
Critodemus	50(?)		
Periplus maris Eryth.	50	P. Nelson (Mars)	86 to 88

[1] Cf. above I A 2 and below V B 4, 1.
[2] Cf. below V B 2, 1 to 4.
[3] Cf. below V C 4.

Heron	eclipse: 62	Stobart Tablets (plan. t.)	[63] to [140]
Plutarch	50 to 120	P. Flor. 44 (treatise, planets, moon)	≈ 100
Manetho	born 80	Bodl. Gr. Class. F 7 (plan. t.)	99 to 102
Menelaus	100	P. Tebt. 274 + P. Lund Inv. 35b (plan. t.)	107 to 120
		O. Bodl. 2177 (Jupiter)	120 to 125
Marinus	100	P. Lund Inv. 35a (lunar t.)	100 to 150
Hyginus, geom.	100	P. Carlsberg 9 (lunar cal.)	≈ 145
Adrastus	100	P. Carlsberg 31 (lunar latitude)	150
Theon Smyrn.	100	P. Carlsberg 32 (Mercury)	150
Ptolemy	100 to 170	P. Mich. 149 (treatise)	150
Pseudo-Plutarch, Plac.	150	PSI 1493 (moon)	150
Vettius Valens	160	PSI 1492 (Saturn)	150

Notes

Geminus: cf. for his date above IV A 3, 1.

Balbillus: it is often assumed that he is a son of Thrasyllus; cf., however, Gagé, Basileia p. 76ff.

Heron: cf. for the date: Neugebauer [1938, 2] p. 23f.

Ptolemy: cf. below p. 834f.

Theon of Smyrna: cf. for his date below V C 2, 3 B.

Vettius Valens: cf. Neugebauer [1954, 2].

P. dem Berlin 8279 and Stobart Tablets: [ ] indicates restored dates, based on the arrangement of the tables.

P. Oxy. 303: 1st century on paleographical grounds. Small fragment; cf. Neugebauer [1962, 1], p. 386f., No. 26.

A. Planetary and Lunar Theory before Ptolemy

In the first three Books we were on comparatively secure grounds: Ptolemy's own outstanding work and the wealth of original Babylonian sources still preserved made this possible; Egypt hardly deserves to be mentioned.

With Book IV we entered an entirely new situation where a later period had effaced all but vague and confused reports of its prehistory. This condition prevails right down to the time of Ptolemy; without his historical remarks we would know almost nothing about the astronomy of Hipparchus or Apollonius.

What is inevitable for the time before Ptolemy should not hold for the Byzantine period. That we know extremely little about a period which has left us an abundance of easily accessible documentation is to be blamed on the ridiculous provincialism of classical scholarship which is willing to compile tomes listing every appearance in Homer of the names Achilles or Agamemnon but has not yet found the time to produce, e.g., a reliable edition of Ptolemy's "Geography," a work whose influence on the civilization of the Middle Ages cannot be overestimated. And the fate of the Geography is not the exception but the rule. Over and over again attempts to see more clearly into the transmission of scientific knowledge within the Roman-Byzantine world and beyond its boundaries are made impossible by the absence of published texts. Were it not for a handful of men, foremost of whom are Heiberg, Manitius, Hultsch, Rome, Cumont, Boll, and, in spite of his faults, Halma — we would today still be where Scaliger (d. 1609), Bainbridge (d. 1643), Salmasius (d. 1658), van der Hagen (d. 1739), and Halley (d. 1742) left off. It is obvious that at present any attempt at writing a historical narrative would be utterly unsatisfactory. The chances are slim that the future will be much better.

§1. Planetary Theory

1. Introduction

The main source for our knowledge of planetary theory during the period between Hipparchus and Ptolemy are the papyri, Greek as well as demotic. Many horoscopes testify by their mere existence to the need of determining planetary positions at given moments while extant tables give us at least a general idea about the tools one had at ones disposal to solve such problems.

Beyond these original documents we also have some literary references to planetary theory, e.g. in Pliny, Vitruvius, or Vettius Valens. Unfortunately most references in these works are made for the sole purpose of displaying the author's

learning, omitting all observational data or theoretical reasoning. For example it would be interesting to hear some early discussions on planetary latitudes. Both Pliny[1] and Cleomedes[2] mention latitudes but give only extremal values without any additional information, e.g. about the corresponding longitudes or nodes. All we get are the following bare numbers, in the case of Pliny's text not even very securely defined:

Pliny		Cleomedes	
Zodiac and moon	$\pm 6^\circ$	moon	greatest[3]
Venus	$\pm(6^\circ+2^\circ)$	Venus	$\pm 5^\circ$
Mercury	$+5^\circ, -3^\circ$	Mercury	$\pm 4^\circ$
Mars	$\pm 2^\circ$	Mars and Jupiter	$\pm 2\ 1/2^\circ$
Jupiter	$\pm 2\ 1/2^\circ$(?)		
Saturn and sun	$\pm 1^\circ$	Saturn	$\pm 1^\circ$

Not only is the agreement between these two lists not very good but also the actual facts are not well represented[4] – Saturn, e.g., can reach almost $\pm 3^\circ$.

Of primary importance are, of course, the sidereal and synodic periods of the planets. In our sources one can distinguish three major groups: (a) the universally known round values for the sidereal periods, Saturn 30 years, etc.[5]; (b) periods taken from Babylonian astronomy, in two versions: the smaller numbers from the "goal-year-texts"[6] and the refined parameters underlying the computation of ephemerides[7]; (c) the astrological concoctions which arbitrarily interpret as "years" the numbers of degrees assigned to each planet as its "terms." These new "periods" are then combined with the periods of the above mentioned group (a) to new "mean periods," of course without any real astronomical significance.[8]

Beyond these three major groups of planetary periods one finds many more parameters mentioned whose origin in most cases, however, cannot be securely established. The following is a collection of these more or less isolated data. For the sake of comparison with the correct order of magnitude numbers from the tables in Alm. IX, 4 are also listed for the corresponding mean motions in longitude or anomaly.

Saturn

10741 1/4 days ($=29$ eg.y. $+ 156;15$ days): sidereal period. P. Flor. 44 (demotic).[9] Alm. IX, 4: $\Delta\bar{\lambda} = 359;43^\circ$.

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[1] Pliny NH II, 66f. (Loeb I, p. 212/215; Budé II, p. 29). For the solar latitude cf. above IV B 2, 2.
[2] Cleomedes II, 7; cf. below p. 964.
[3] No number given, presumably $\pm 6^\circ$.
[4] The data in the Almagest and particularly in the Handy Tables are far superior to the estimates here given; cf. below p. 1015 and note 2 there.
[5] Cf., e.g., above p. 606 (3), last column.
[6] Cf., e.g., above p. 605 (1).
[7] Above p. 605 (2).
[8] Above p. 606 (3), first column.
[9] Published Neugebauer-Parker, EAT III, p. 250–252. This text also mentions, for unknown reasons, an interval of 2281 1/4 days which is exactly 1/4 of the length of the 25-year lunar cycle (cf., e.g., above III, 2).

29 (eg.) years 160 days (=10745 days): Vitruvius IX, 1 (Budé, p. 13). Alm. IX, 4: $\Delta\bar{\lambda}=359;50°$.

370 months (=11 100 days): P. Vienna, Wessely[10]; perhaps copyist error?

378 days: synodic period. Cleomedes II, 7[11]; PSI 1492.[12] Alm. IX, 4: $\Delta\alpha=359;55°$.

256 years: misinterpretation of 256 synodic periods in 265 years with 9 sidereal rotations[13]: CCAG 5, 3 p. 132, 32.

Jupiter

140 months: crude sidereal period, either of 4200 days or of 11 eg.y. 8 months = 4250 days. P. Vienna, Wessely.[14] Alm. IX, 4: $\Delta\bar{\lambda}=349;32°$ or 353;41°.

11 (eg.) years 323 days[15] (=4338 days): Vitruvius IX, 1 (Budé, p. 13). Alm. IX, 4: $\Delta\bar{\lambda}=0;35°$.

398 days: synodic period. Cleomedes II, 7.[16] Alm. IX, 4: $\Delta\alpha=359;12°$. P. Flor. 44[17] mentions besides the 12-year period also intervals of 61 2/3 days and 740 days $(=12\cdot 61\ 2/3)$ but it is not certain that they belong to Jupiter.

Mars

The two most commonly mentioned round periods, 2 years and 15 years, are approximations both of sidereal and synodic periods: for 2 years $\Delta\bar{\lambda}\approx 22°$, $\Delta\alpha\approx 337°$; for 15 years $\Delta\bar{\lambda}\approx 351°$, $\Delta\alpha\approx 9°$. Censorinus mentions[18] a period of "about 9 years," perhaps originating from a mean value $1/2(15+2)$ years, of course astronomically meaningless. Cicero's[19] "24 months − 6 days" (=714 days) is a slight improvement over the 2-year period ($\Delta\bar{\lambda}\approx 14°$).

63 years: error for the astrological period of 66 years[20]: Firmicus Maternus, ed. Kroll-Skutsch I, p. 74.

294 years: error for the Babylonian period of 284 years[21]: Lydus, de mens., ed. Wuensch, p. 57, 2.

683 days: sidereal period: Vitruvius IX, 1 (Budé, p. 13). $\Delta\bar{\lambda}\approx 358°$.

780 days: synodic period: Cleomedes II, 7.[22] $\Delta\alpha\approx 0°$.

Astronomically meaningless are the following parameters:

1 year 4 months (=485 days), probably error for $45\cdot 12=540$ days $=1^y 6^m$: Maass, Comm. Ar. rel., p. 427, 28–428, 1.

18 months (=540 days): CCAG 4, p. 116, 19 variant + p. 183.

[10] Cf. above p. 737 text (n).
[11] Cf. below p. 964.
[12] Cf. below p. 790 f.
[13] Cf. above note 7.
[14] Cf. above note 10.
[15] Commonly quoted as 313 days, which is, however, only an emendation. The majority of manuscripts has 323, one 324, none 313.
[16] Probably mistake for 399 days; cf. below p. 964.
[17] Cf. above note 9.
[18] Ed. Hultsch, p. 58, 17–19 (fragm., Chap. III, 4).
[19] De natura deorum II, 53 (ed. Ax, p. 69, 27f.; Loeb, p. 174/175).
[20] Cf. above note 8.
[21] Cf. above note 7.
[22] Cf. above note 11.

2 years 5 months (=880 days): CCAG 12, p. 109, 1; Cleomedes I, 3 (ed. Ziegler, p. 30, 24–26); cf. Maass, Comm. Ar. rel., p. 427, apparatus 28, 9.

2 1/2 years (=910 days): CCAG 7, p. 218, 10; Psellus, Omnif. doctr. § 137, 6f. (ed. Westerink, p. 71).

Venus

The only parameter (beyond the Babylonian periods of 8 years and 1151 years) that makes sense astronomically is a synodic period of 584 days ($\Delta\alpha = 0;2°$),[23] found in Cleomedes II, 7[23a] and in Chalcidius CXII.[24]

The remaining periods are all astronomically meaningless. It is difficult to understand how such numbers got into literary or astrological works.

10 months (=300 days): CCAG 4, p. 117, 8+p. 184. Cf. Martianus Capella VIII, 882 (Dick, p. 465, 18).

336 days: Maass, Comm. Ar. rel., p. 428, 3.

348 days: Pliny, NH II, 38 (Loeb I, p. 193; implausible explanation Budé II, p. 135); Maass, Comm. Ar. rel., p. 273, 11 f. and p. 601 (No. 43); Migne PL, Vol. 83, col. 985.

13 months (=390 days): CCAG 7, p. 220, 23 f.

485 days: Vitruvius IX, 1 (Budé, p. 12).

84 years: error for the astrological period of 82 years[25]: Firmicus Maternus II, 25 (Kroll-Skutsch I, p. 74, 9–11).

Mercury

Again, there is only one parameter that makes sense: the synodic period of 116 days (Alm. IX, 4: $\Delta\alpha = 0;23°$) in Cleomedes II, 7 and (in part restored) in the Eudoxus papyrus.[26] About twice this period is mentioned by Isidore of Seville: 229 days[27] (Alm. IX, 4: $\Delta\alpha \approx 251°$).

Meaningless parameters are:

6 months (=180 days): CCAG 4, p. 117, 15.

6 months 6 days (=186 days): CCAG 12, p. 109, 10f.

11 months (=330 days): CCAG 7, p. 221, 27.

339 days (?=348−9 days): Pliny NH II, 39 (Budé II, p. 19).

360 days: Vitruvius IX, 1 (Budé, p. 12+p. 95). Cf. Martianus Capella VIII, 879 (Dick, p. 464, 2).

79 years and 108 years: Firmicus Maternus II, 25 (Kroll-Skutsch I, p. 74, 11–13) calls 79 years incorrectly a "medium" period; actually 79 is a mistake for 76, the large astrological period for Mercury.[28] Similarly 108 is actually the astrological period of the moon[29]; Firmicus' list got slightly garbled toward its end.

[23] A synodic period of 579 days from the Eudoxus papyrus is only based on a emendation (cf. above p. 687 (1)). Martianus Capella VIII, 883 (Dick, p. 466, 19) gives 19 months as synodic period which is a fairly correct rounding.

[23a] Cf. below p. 964.

[24] Wrobel, p. 179, 6. The same parameter is also given in the Pañca-Siddhāntikā (Neugebauer-Pingree I, p. 157, II, p. 111, Table 23 and p. 114) and by al-Farghānī (Diff. sci. astr., Chap. 17; ed. Carmody, p. 31).

[25] Cf. above note 8.

[26] Cf. above note 11 and p. 687 (1), respectively.

[27] Migne PL, Vol. 83, col. 985.

[28] Cf. above note 8.

[29] Cf., e.g., Lydus, De mens., p. 56, 13f., ed. Wuensch.

Looking back at the material assembled here one can say that beyond the Babylonian parameters only the synodic periods listed in Cleomedes II, 7 are astronomically meaningful. The vast majority of the remaining data – with the exception of a few isolated numbers – are either based on valueless astrological speculations or nonsensical from any point of view. This confirms our impression that during the period between Hipparchus and Ptolemy very little astronomical progress had been made. The outstanding achievements of Ptolemy become ever more evident as we learn more about his predecessors.

2. Planetary Tables

A. Arrangement and Contents

The planetary tables as we know them from Greek and demotic texts of the Roman imperial period differ greatly in their structure from Babylonian ephemerides. The latter progress, at least in principle, in equal intervals of time and tabulate the corresponding longitudes of certain planetary phases while the texts from Egypt register the dates which are associated with the entries of the planet (in direct or retrograde motion) into consecutive signs. In other words the principle of arrangement and the choice of independent variables are basically different in Babylonian and Greek planetary tables.[1] The transformation of one type into the other is, of course, in principle possible but by no means trivial. We shall discuss later some vestiges of this transformation found in the astrological literature.[2]

The new type of arrangement was undoubtedly introduced because of its usefulness for the computation of horoscopes since it makes it possible to know at a glance in what sign a planet was located at a given moment, which is all the information required by the majority of horoscopes.[3] The usual order of the planets, from Saturn to Mercury is indicative of Greek origin, i.e. it is based on a geometric picture for the structure of the cosmos; neither the Babylonian[4] nor the Egyptian[5] sequence is found in any of these tables.[6]

Two major types of arrangement can be distinguished among the planetary tables. Either the number of entries for a given planet in a given year depends on the number of events registered, e.g. for Jupiter[7]

1	day 1	♌
11	16	♍

[1] This does not mean that we have no evidence at all of ephemeris-type tables in Egypt: cf., e.g., at the end of the first century A.D. the lunar table P. Lund Inv. 35a (below V A 2, 1 B) or the "Almanac" P. Mich. Inv. 1454 (below p. 1058) of the 5th century.

[2] Cf. below V A 1, 3 C.

[3] Cf. Neugebauer-Van Hoesen, Greek Horosc., passim.

[4] The Babylonian order in the hellenistic period would be ♃ ♀ ☿ ♄ ♂; cf. above p. 690.

[5] The usual Egyptian order is ♃ ♄ ♂ ☿ ♀; cf. Neugebauer-Parker, EAT III, p. 175.

[6] In contrast to India where the order of the planets is taken from the order of the days of the week, thus ultimately based on a Greek astrological doctrine.

[7] From the Stobart Tables, for the year Hadrian 2 (EAT III, Pl. 77).

This means that the planet was in Leo at the beginning of the year and entered Virgo in month 11 day 16. On the other hand a fast moving planet may require several entries per month, e.g. Mercury[8]

5		10	♑
	day	28	♒
6		13	♓

Or: a second type of tables has always 12 entries for each planet in each year, from Thoth to Mesore. For an outer planet the same sign may be mentioned for several months, e.g. for Saturn from I to VIII always ♓, then IX 4 ♈, finally ♈ for the remaining months.[9] For an inner planet several days can be mentioned in the same month, e.g. 1 ♈, 17 ♉ in month VIII for Mercury.[10]

There is no rigid scheme for the topics listed in these tables. The only thing they have in common is the reference to the dates of entry (or reentry) into consecutive signs. Beyond that it may be mentioned that the planet remains in a sign, or that it reached its midpoint[11]. The date of a planetary phase might be given as well. For example the large demotic papyrus Berlin 8279 (of the Augustan period) gives for Saturn and Jupiter the dates of the characteristic phases Φ, Θ, Ψ, Ω, Γ[12] leaving it to the user to find out the meaning of these dates which are listed without any explanation. Explicit references to the phases of Mercury (rising and setting in the morning or evening) are found in P. Flor. Inv. 75 D.[13]

Horoscopes occasionally distinguish explicitly between the Egyptian and the Alexandrian calendar ("according to the Egyptians" or "Greeks", respectively, or similar expressions[14]) but the tables never indicate which calendar has been used. For us it is usually not difficult to determine the underlying calendar on astronomical grounds. Our material shows clearly that the Alexandrian calendar was widely used both in demotic and in Greek texts at a very early time. On the other hand the greater convenience for astronomical computations of the Egyptian calendar is the reason for its use through many centuries after Augustus. This does not exclude, however, that tables, e.g., of the first and second century A.D. operate with the Alexandrian calendar.[15]

Longitudes are usually to be understood as sidereal longitudes λ^* such that at the beginning of our era $\lambda^* \approx \lambda + 5°$, λ being the modernly determined tropical longitude. Probably this norm has its origin in the Babylonian definition of the vernal point as 8° or 10° of Aries.[16]

On the whole the tables agree reasonably well with the actual facts as established by modern computations. Fig. 2 illustrates the motion of Mercury as computed for the year B.C. 3 (continuous curve) and the dates of entry into the signs as given

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[8] Same year as before.

[9] From P. Tebt. 274, for the year Trajan 15 (Neugebauer [1942, 1], p. 241).

[10] Trajan 10.

[11] E.g. P. Flor. Inv. 75 D, Manfredi-Neugebauer [1973].

[12] Cf. EAT III, p. 229f.

[13] Cf. Manfredi-Neugebauer [1973].

[14] For details cf. Neugebauer-Van Hoesen, Greek Hor., p. 166.

[15] E.g. the demotic Stobart Tables and the Greek P. Tebt. 274, P. Lund Inv. 35b, etc.

[16] Cf. above IV A 4, 2.

by the text[17] (black dots). In general, however, the agreement is not consistently good, in part because of the imperfections of the underlying computational methods, and in part because of the carelessness of computers and scribes.[18]

In the following we give a synopsis of the planetary tables from the time before Ptolemy. For later material cf. below V C 2, 2 and V C 5, 3.

Synopsis of Tables

P. dem. Berlin 8279 (demotic)

Publication: Neugebauer-Parker EAT III, p. 225–240, Pl. 66–73.

Time: original beginning probably −17 Sept. (preserved: −16); ending A.D. 12; written after A.D. 42.

Calendar: Egyptian; regnal years of Augustus.

Contents and Arrangement: year after year, dates of entry of the planets into the signs; for Saturn and Jupiter: phases. Months, occasionally also zodiacal signs, numbered (♍ = 1).

P. Flor. Inv. 75 D (Greek)

Publication: Manfredi-Neugebauer [1973].

Time: A.D. 48 to 52.

Calendar: Egyptian; regnal years of Claudius.

Contents and Arrangement: year by year, month by month; dates of entry of the planets into the signs; midpoints, phases.

P. Berlin Inv. 21226

Publication: Brashear-Neugebauer [1974].

Time: A.D. 50 to 56.

Calendar: Alexandrian; era Augustus 80 to 85.

Contents and Arrangement: Jupiter, month by month (denoted only by α) with additional dates for stations and oppositions, et al.

P. Nelson (Greek)

Publication: Nelson [1970]; Neugebauer [1971].

Time: assuming Alexandrian calendar: A.D. 39 to 41 or 86 to 88; for Egyptian or Alexandrian calendar: −8 to −6.

Contents and Arrangement: two years in one column, month by month, dates of entry of Mars into signs (fragmentary).

Bodleian MS Gr. Class. F 7 (P) (Greek, plaster on wooden tablet)

Publication: Neugebauer [1957, 2] and [1972, 3].

Time: A.D. 101 (± some years).

Calendar: Egyptian; regnal years of Trajan and era Titus.

Contents and Arrangement: year by year, dates of entry of the planets into the signs; months and zodiacal signs numbered (♍ = 1).

P. Tebtunis 274 (Greek)

Publication: Neugebauer [1942, 1], p. 241–242; cf. also EAT III, p. 233 f.

[17] P. dem. Berlin 8279.

[18] Cf. for examples the graphs in Neugebauer [1942, 1] Pl. 1–15.

Time: A.D. 107 to 118.
Calendar: Alexandrian; regnal years of Trajan and era Titus.
Contents and Arrangement: two years in each column, month by month; dates of entry into the signs for the planets and the moon.

P. Lund Inv. 35 b (Greek)
Publication: Knudtzon-Neugebauer [1947].
Time: A.D. 119 and 120.
Calendar: Alexandrian.
Contents and Arrangement: individual columns for each planet, month by month; Mars to Mercury (fragmentary; perhaps part of P. Tebt. 274).

Stobart Tablets (demotic, plaster on wooden tablets; from Thebes)
Publication: Neugebauer-Parker, EAT III, p. 225–228, p. 232–240; Pl. 74–78.
Time: original coverage probably A.D. 63 to 140; extant: 70 to 78, 104 to 119, 125 to 133.
Calendar: Alexandrian. Regnal years of Vespasian, Trajan, Hadrian.
Contents and Arrangement: year after year, dates of entry of the planets into the signs. Months numbered.

O. Bodl. 2177
Publication: Tait-Préaux, Greek Ostraca in the Bodleian Library II (London 1955), p. 389 (incomplete). Neugebauer [1974] (correcting [1962, 1], p. 384f., No. 6).
Time: A.D. 120 to 125.
Calendar: Alexandrian.
Contents and Arrangement: longitudes of Jupiter on Khoiak 15 in 6 consecutive years.

P. Strasb. Inv. 1097 (Greek)
Small unpublished fragment, 2nd cent. A.D.
Contents: dates of entry of Venus into the signs.

B. Notation

A few demotic texts use hieroglyphs related to the Greek names, for the zodiacal signs, e.g. the sitting woman for Virgo or an arrow for Sagittarius.[1] It is not impossible that such writings developed into the later symbols though there is a gap of many centuries in our documentation. The majority of the demotic texts, however, as well as all Greek papyri, do not use symbols but write the names for the zodiacal signs in ordinary orthography.

An important variant of referring to the signs of the zodiac is the simple numbering from 1 to 12. This procedure is found in the demotic papyrus Berlin 8279, particularly in its later sections. The sign of Virgo is counted as No. 1, probably because of the solar longitude in month I.[2] The same counting is, however, still applied in the Greek tablet of the Bodleian Library,[3] a century later.

[1] Cf. EAT III, p. 218f. (Fig. 33), or Neugebauer [1942, 1], p. 246, Fig. 1.
[2] Cf. EAT III, p. 228.
[3] Cf. above p. 787. The same norm is still found in planetary tables for the years 217 to 225 (to be published by J. Rea in a forthcoming volume of the Oxyrhynchus Papyri; Inv. 6 1 B. 8/B (d)).

Demotic tables never use names for the calendaric months but count them consecutively from 1 (i.e. Thoth) to 12.[4] The Greek texts, with the exception of the Bodleian tablet, write the names out (in their "Coptic" form).

No symbols for the planets ever occur in the Greek text; the names are always written out, of course occasionally misspelled or abbreviated. Also the demotic texts usually write in full the Egyptian names for the planets, though the earliest planetary table, P. Berlin 8279, has symbols, e.g. □ for Saturn.[5] There is no evidence, however, for any connection with the later planetary symbols which first appear in the Byzantine codices of the late Middle Ages.

C. Historical Questions

Ptolemy, in Alm. IX, 2, speaks contemptuously about attempts of earlier astronomers to demonstrate "by means of the so-called perpetual tables"[1] that circular motions of eccenters and epicycles can explain the planetary phenomena. This seems to imply that these tables provided only the empirical material for a theory of circular motions without being themselves computed on the basis of a cinematic model. This agrees well with the extant tables which surely were not computed from such models since this would have produced much better results for the retrograde sections than found in our tables. Thus it seems quite plausible that these tables are of the "perpetual" type.

That the term is not Ptolemy's invention is proved by its occurrence in the preamble to a horoscope cast for A.D. 81.[2] It speaks, however, against the interpretation of our tables as "perpetual tables" that in this very horoscope the statement is made that the tables provided degrees and minutes for the positions of the planets (as indeed given in the text). About a century later Vettius Valens also refers to an accuracy of "perpetual tables" down to degrees.[3] Furthermore the previously mentioned horoscope gives the position of the perigee of Mercury,[4] an element entirely foreign to the framework of our tables.

Hence we are not in a position to say for sure what perpetual tables looked like.[5] But at any rate it is interesting to note that Ptolemy knew of astronomers who tried to derive cinematic models on the basis of computed tables rather than of observations. I think this remark is revealing for much of the methodology of ancient astronomy.

We are able to compute Babylonian planetary ephemerides exactly as in the original texts and the same holds for the Ptolemaic theory. For the planetary

[4] For the epagomenal days cf. EAT III, p. 227.

[5] This peculiar symbol is also found in a demotic astrological text. Cf. for details EAT III, Pl. 59, 60, 62–64; also Neugebauer [1942, 1], p. 247 Fig. 2.

[1] *διὰ τῆς καλουμένης αἰωνίου κανονοποιίας* (Heiberg II, p. 211, 5).

[2] Neugebauer-Van Hoesen, Greek Hor., p. 21 (I, 12f.: *κανόνων αἰωνίων*).

[3] Vettius Valens, Anthol. VI, ed. Kroll, p. 243, 7f. (*μοιρικῶς*).

[4] Greek Horosc., p. 22, VII, 157–161.

[5] The term "perpetual tables" is still found in Byzantium, e.g. in the Greek translation (made around A.D. 1000) of Abū Ma'shar's "De revolutionibus nativitatum" (written around 850) and in the Greek version of "De secretis Albumasaris per Sadan." Cf. for these texts: Albumasaris, De Revol. Nat., ed. D. Pingree (Leipzig. Teubner, 1968); for the *αἰώνια κανόνια*, p. 11, 7, Latin: "*universales canones.*" Furthermore: Thorndike, Albumasar in Sadan, Isis 45 (1954), p. 22–32, p. 27 for the English translation of the Latin version. The Greek version (Cod. Angel. 29, fol. 46' lines 2f.) is unpublished.

tables under discussion we are far from such a situation. All we can do is to check a given text against modern tables and thus detect crude scribal errors or determine the meaning of the entries. It is obvious that Babylonian methods cannot be applied directly to the construction of these tables. The transformation of the Babylonian lunar calendar to the Egyptian or Alexandrian norm would in itself constitute a major obstacle. Furthermore the Babylonian ephemerides produce the longitudes of the planetary phases, not the dates of entries into the signs. And in one case where we have the opportunity of comparing Babylonian results with a demotic papyrus one finds rather pronounced discrepancies.[6] Nevertheless our texts have a much greater affinity to a Babylonian approach than to Greek cinematic theories. Hence one may perhaps conjecture that in the early hellenistic period (say in the second century B.C.), when Babylonian astronomy became known to the Greeks in some detail, methods were developed in Alexandria that adapted Babylonian procedures to the requirements of the new hellenistic astrology. But our material does not suffice to uncover the details of such a process.

The general relationship to Babylonian astronomy is also apparent in a small demotic papyrus fragment of a table in two columns of four-digit sexagesimal numbers, the first digit representing degrees.[7] The first column gives the multiples of 0;5,27,17° from 1 to 20; the second column gives the totals of the first, thus beginning with 0;5,27,17 0;16,21,51 and ending with 19;5,29,30. We have here an arithmetical sequence of the second order that fits the day by day progress well of Mercury as morning star between first (Γ) and last (Σ) visibility; similar patterns for the motion of Mercury are also known from Babylonian texts[8] (cf. Fig. 3). Assuming the dates and longitudes of planetary phases known (here Γ and Σ) texts of this type can be used for bridging the gaps and thus to establish the date of crossing from one sign into the next.

Another "auxiliary table," i.e. a table which does not refer to a specific chronological situation, is preserved in a Greek papyrus, now in Florence.[9] Though we have only one column one can reconstruct a significant portion of the text which originally covered 378 days, the synodic period of Saturn, attested also in Cleomedes II, 7 and in the Pañca-Siddhāntikā.[10] The corresponding progress in longitude, the "synodic arc," amounts to 12;31,30°.

The text is arranged in triple columns (cf. Table 1), the middle one (b) giving in every fifth line the day number in the synodic period. To the left and to the right of it are numerical columns, again to four sexagesimal digits, such that the first column represents the differences for the third one. Between the lines 346 and 347 is written Δυcιc, meaning heliacal setting (Ω). The remaining 31 days are traversed with the constant velocity 0;8,15,18$^{\circ/d}$, thus resulting in a progress of 4;24,9,36°. Both the time interval and the longitudinal increment agree well with the period of invisibility of Saturn. Thus our table began (and ended) with heliacal risings (Γ).

[6] Cf. EAT III, p. 238, Fig. 40.
[7] P. Carlsberg 32, published EAT III, p. 240f., Pl. 79 B (not A).
[8] Cf. above II A 5, 3 B and ACT III Pl. 169 Rev. I, −4 to 21.
[9] To be published as PSI 1492.
[10] Cf. below p. 965.

Table 1

	(a)	(b)	(c)	n
I, 1.	0,0, 0, 0		0, 0,17,42	0 = Ψ
2.	0,0, 4,14	230	0, 0,21,56	1
3.	0,0, 8,28		0, 0,30,24	2
4.	0,0,12,42		0, 0,43, 6	3
etc.				
38.	0,2,36,38		0,49,53,44	37
II, 1.	0,2,40,52		0,52,34,36	38
2.	0,2,45, 6		0,55,19,42	39
3.	0,2,49,20		0,58, 9, 2	40
4.	0,2,53,34	270	1, 1, 2,36	41
etc.				
38.	0,5,17,30		3,21,22,42	75
III, 1.	0,5,21,44	305	3,26,44,26	76
2.	0,5,25,58		3,32,10,24	77
3	0,5,30,12		3,37,40,36	78
4.	0,5,34,26		3,43,15, 2	79
etc.				
			7,26,46,14	112
38.	0,7,58,22		7,34,44,36	113
IV, 1.	0,8, 2,36		7,42,47,12	114
2.	0,8, 6,50		7,50,54, 2	115
3.	0,8,11, 4	345	7,59, 5, 6	116
4.	0,8,15,18		8, 7,20,24	117
5. 6.		Δυcιc		Ω
7.	0,8,15,18		8,15,35,42	1
8.	0,8,15,18		8,23,51, 0	2
etc.				
37.	0,8,15,18		12,23,14,42	31
38.	0,8,15,18	(378)	12,31,30, 0	32 = Γ

The whole table was originally arranged in 10 triple columns of 38 lines each. The last 148 lines can be restored on the basis of the last column. This section describes the motion of Saturn from the second station (Ψ) to Ω in 117 days, with velocities increasing from 0° to 0;8,15,18$^{\circ/d}$, resulting in 8;7,2,42° of longitudinal progress. Then follows the above described interval of invisibility from Ω to Γ. Table 1 shows the whole pattern for the last four triple columns (of which only the last column is actually preserved).

We shall see in the next section [11] that tables of a similar type must also have existed for Venus and Mercury. At least it is possible to bring sense into rules given by Vettius Valens (late second century A.D.), obviously without real understanding, for the determination of longitudes of Venus at a given date. The combination of cyclically computed phases with templates for daily motions indeed constitutes an effective procedure for determining dates for given longitudes or

[11] Below p. 798.

longitudes for given dates. This, perhaps, answers the problem of transition from Babylonian ephemerides to Greek and demotic planetary tables.

Mainly in astrological contexts one can find references to a special "phase" of Mars, called "90-day anomaly" or similar, variously related to the stations of the planet or to its elongation with respect to the sun.[12] What all this means becomes intelligible thanks to a passage in Pliny[13] in which he says that Mars is sensitive to the rays of the sun at 90° elongation, called "*primus et secundus nonagenarius*," and that the planet remains six months in the same sign when "stationary" *(stationalis)*.

The latter remark contains the key to all the passages considered here. If Θ represents the exact opposition of the planet then there always exist two neighbouring points, Θ' and Θ'', of equal longitude (cf. Fig. 4). The time interval between Θ' and Θ'' is very nearly six months[14] and for this interval the longitudinal variation of the planet remains well within $\pm 15°$, i.e. within "the same sign." Since the elongation of the planet from the sun is at Θ per definitionem 180° the elongation near Θ' and Θ'', 90 days earlier and later, is $\pm 90°$ since the sun moves 90° in 90 days. Hence the "stationary" interval of Mars is limited by the quadratures, a situation of obvious appeal to astrological speculation. This aspect, however, does not exhaust, I think, the interest in the points Θ' and Θ''. If one uses linear interpolation between Θ'', 90 days after Θ, and Θ', 90 days before the next Θ, one will obtain a fair estimate for the motion of the planet during some 18 or 19 months (cf. Fig. 4). In other words we would have here a simple method of obtaining planetary positions from a sequence of consecutive oppositions. Hence it could be a computational procedure which originated the interest in the "*nonagenarii*."

In later sources[15] we find Pliny's quadratures replaced by elongations of 82°. In effect this means introducing as boundaries of the "stationary" intervals points before Θ' and after Θ'', hence to shorten somewhat the section of linear motion.[16] The origin of the parameter $8° = 90 - 82°$ is probably to be related to the elongation required for first and last visibility of Mars.[17] We know that "opposition" commonly means not exactly 180° elongation but acronychal rising which must take visibility limits into account. Perhaps it was only a formal parallelism which suggested a similar modification of the "quadratures."

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[12] I know of the following cases: P. Mich. 149 (≈A.D. 150) XI, 18-21 (Mich. Pap. III, p. 76): *ενενηκονθημερους ανωμαλιας*; Porphyry (≈300), Introd. Tetrab., CCAG 5, 4, p. 194, 6-11: *ἀνωμαλίαν ποιεῖται* at 82° or 90° elongation from the sun; Martianus Capella (≈400) VIII, p. 467, 9-11, ed. Dick: *quadratura;* Rhetorius (≈500), CCAG 7, p. 217, 24-218, 10: *ἡμέρας ἐνενήκοντα* in quadrature; cf. also below note 15.

[13] NH II, 60 (Budé, p. 26/27).

[14] Our Fig. 4 represents as an example the positions of Mars between A.D. 51 Aug. 13 and 54 May 29. The intervals between Θ and Θ' or Θ'' are about ± 84 days.

[15] To be precise: Porphyry, CCAG 5, 4, p. 194, 9; Paulus Alexandrinus, ed. Boer, p. 32, 3 and 5; the scholion No. 32 (Boer, p. 113, 5) says "82 or 84"; "Heliodorus" (recte Olympiodorus) has 84, corrected to 82 (ed. Boer, p. 20, 24 and p. 22, 9).

[16] In the example shown in Fig. 4 the "stationary" intervals would thus be lengthened to about 7 months. The intervals between Θ and the quadratures (elongation $\pm 90°$) are about ± 95 days.

[17] Again from Porphyry: CCAG 5, 4, p. 228, 18.

3. Planetary Theory in Vettius Valens

In Book I, Chap. 20 of his "Anthology" Vettius Valens[1] gives rules for finding the longitudes of the sun and of the planets by means of "handy" methods, fortunately adding some numerical examples which help to make the general rules a little easier to understand. These examples belong to the years A.D. 109 to 120 but all computations are based on the era Augustus (year 1, egyptian Thoth 1 = −29 August 31).

The intervening less than 150 years succeeded not only in introducing several numerical errors into the basic parameters but also in obscuring almost completely the meaning of the prescribed operations. The formulations in the text give the impression that the obtained results represent true positions. In fact only mean positions of the outer planets can be obtained by the given rules. For the inner planets the situation is more complicated since mean positions are of no interest, being the same as for the sun. We shall see, however, that for the inner planets also the "handy" rules represent only a preliminary step on the way to the true longitudes.

Table 2

No.	Planet	Text	Equivalent Date	A Egypt. Cal.	B Alex. Cal.
1	♄	Trajan 13 VII 18, y. from Augustus 138	Augustus 139 VII 18	110 Febr. 9	110 March 14
2	♃	same	Augustus 139 VII 18	110 Febr. 9	110 March 14
3	♂	from Caesar 139 y.	Augustus 139 VII 18	110 Febr. 9	110 March 14
4	♀ (a)	139 y. VII 18	Augustus 139 VII 18	110 Febr. 9	110 March 14
5	♀ (b)	Hadrian 4 III 30 from Augustus 148 y.	Augustus 149 III 30	119 Oct. 22	119 Nov. 27
6	♀ (c)	Hadrian 4 VI 13 from Augustus 149 y.	Augustus 149 VI 13	120 Jan. 3	120 Febr. 8
7	☿	same	Augustus 149 VI 13	120 Jan. 3	120 Febr. 8

The text itself is in an abominable condition. Before discussing details it is useful to realize that the examples[2] are full of mistakes. Table 2 shows that four of the seven examples concern the same date, Trajan 13 VII 18 = Augustus 139 VII 18. The first example correctly implies that the number of completed years of the era Augustus is 138. But the examples No. 3 and 4 use 139 for the completed years. Similarly Hadrian 4 = Augustus 149 requires 148 completed years as is said in No. 5; but No. 6 takes 149. The same can be said about the months which in all cases are treated as if completed instead of current.

Most likely the rules in question were set up for egyptian years but the computations we have are based on Alexandrian years. This is suggested not only by the better agreement with modern data[3] but we also have in Book III, Chap. 6 a horoscope which can be dated to 110 March 15, i.e. to one day later[4] than Augustus 139 Alexandrian VII 18. The one day difference is due to a better agree-

[1] Ed. Kroll, p. 33–36.

[2] Kroll, p. 35, 15–36, 27.

[3] Cf. below p. 796, Table 3.

[4] Cf. Neugebauer-Van Hoesen, Greek Hor., p. 105f. (correct the longitudes for Jupiter, Mars, and Venus to the values given in Table 3).

ment with the position of the moon given in the horoscope; for the examples of planetary positions we can, of course, ignore this small adjustment which reduces a computational discrepancy for the moon's longitude to a minimum.

A. Solar Longitudes

Since the sun has only one anomaly, the "equation of center," it is easy to design arithmetical schemes which lead to a fair approximation of the true solar motion. The rules given by Vettius Valens in I, 20 are of this type: the 360° of one annual revolution are distributed in such a fashion over the 365 days of the year that the slower motion belongs to an arc centered about Gemini. The faster motion is assumed to be 30° per month (i.e. $1^{\circ/d}$) between ♍ 8° and ♈ 8°, hence totalling 210° during the seven months I to VII; then follow only 29° per month (i.e. $0;58^{\circ/d}$) on the remaining arc, hence a progress of $149;50 \approx 150°$ during 155 days.

This scheme is so simple that one need not assume a direct influence of the Babylonian "System A." The parameter ♈ 8° could be taken as the Babylonian vernal point (in "System B") but Valens' text is not reliable at this point. In a similar rule the text of Paulus Alexandrinus has ♍ 6° as epoch[1] and the same parameter is given in the commentary by "Heliodorus."[2] Actually the (tropical) longitude of the sun at Thoth 1 of Augustus 1 was about ♍ 5°.

B. The Outer Planets

Saturn. In modern terminology the rule for determining the longitude of Saturn is as follows: let t be the number of completed years in the era Augustus, m the number of (completed) months, d of days, in the year Augustus $t+1$. Then determine integers α and β such that

$$t = \alpha \cdot 30 + \beta \qquad 0 \leqq \beta < 30 \tag{1}$$

and compute the arc

$$\gamma = 5 \cdot \alpha + 12 \cdot \beta + m + d/30 \,^{1} \tag{2}$$

reckoned in degrees. Then

$$\lambda = ♋\, 0° + \gamma \tag{3}$$

is the longitude of the planet.

Example (cf. below p. 796, Table 3): for Trajan 13 = Augustus 139 VII 18 we have $t = 138$, $m = 7$,[2] thus (ignoring d) $\alpha = 4$, $\beta = 18$, and therefore

$$\gamma = 5 \cdot 4 + 12 \cdot 18 + 7 = 243.$$

Hence Saturn is in ♓ (because ♋ 0° + 243 = ♓ 3°).

It is easy to explain these rules. The number α counts the number of 30-year periods contained in t. Eq. (2) indicates that the planet gains 5° of mean longitude in each such period, as is reasonably correct.[3] By accepting 30 years as sidereal

[1] Ed. Boer, p. 79, apparatus to line 15.
[2] Ed. Boer, p. 90, 11.
[1] The text has $d/32$ which is certainly a mistake.
[2] One should expect $m = 6$. The shift from Egyptian to Alexandrian dates would amount to about one month (cf. Table 2) but this seems to me to assume too high a sophistication for our author.
[3] Alm. IX, 4 gives an excess of 6;42° in 30 eg. years.

period each single year contributes 12°, thus each month 1° and each day 1°/30. Rather crude is the epoch value ♋ 0° for Augustus 1 I 1; the mean longitude, according to the Almagest, would be ≈♊ 13°, the actual true longitude was about ♊ 21;30°.

Jupiter. Using the same notation as before we are told to find the longitude of Jupiter from

$$t=\alpha\cdot 12+\beta \qquad 0\leqq\beta<12 \tag{4}$$

with

$$\gamma=\alpha+12\cdot\beta+m+0;2\,d;^{4} \tag{5}$$

then the longitude of the planet is

$$\lambda=♉\,0^\circ+(\gamma) \tag{6}$$

where

$$(\gamma)=2;30^\circ\cdot\gamma. \tag{7}$$

Definition (7) shows that artificial new units were used in (5), not degrees, perhaps in order to give (5) the same appearance as (2) for Saturn. The text expresses this change by saying "counting each sign as 12".[5] Indeed $12\cdot 2;30^\circ=30^\circ$.

Example (cf. No. 2 in Table 3): Again $t=138$, $m=7$, thus $\alpha=11$, $\beta=6$ and therefore $\gamma=11+12\cdot 6+7=90$. Hence $(\gamma)=2;30\cdot 90=225^\circ$ and ♉$0^\circ+225=$♐15° which is very nearly the mean longitude of Jupiter for Augustus 139 Alex. VII 18, computed with Alm. IX, 4.

The rationale of this procedure is obvious: α counts the number of 12-year periods of Jupiter, β the remaining single years, each of which contributes 30°, i.e. $12\cdot 2;30^\circ$. The coefficient 1 of α in (5) indicates that the planet supposedly exceeds in each 12-year period complete rotations by 2;30°. The correct value would be about 4;5°. Also the epoch position, ♉ 0°, at Augustus 1 I 1 is rather inaccurate; the Almagest would give ♈ 8;35 as mean longitude. The true longitude was about ♈ 20;15°.

Mars. It is clear from the preceding analysis that the longitudes of Saturn and Jupiter, applying the rules of the text, are only mean longitudes, estimated on the basis of the simple round periods of 30 and 12 years, respectively. Following the same idea for Mars with its very crude period of 2 years can, of course, lead only to results with rapidly increasing errors, even for the mean positions.

Besides the 2-year period also a 30-year period (otherwise unattested) seems to be assumed since we are directed to form

$$t=\alpha\cdot 30+\beta, \qquad 0\leqq\beta<30 \tag{8}$$

where α does not influence the subsequent steps. The text then goes on to form

$$\gamma\equiv 2\cdot\beta+2;30\cdot m \qquad \text{mod. } 60 \tag{9}$$

which shows that units of 6° are used. Thus (9) would be for degrees

$$(\gamma)\equiv 12\cdot\beta+15\cdot m \qquad \text{mod. } 360^\circ. \tag{10}$$

[4] The coefficient 0;2 is the consequence of the coefficient 1 of m. The text is corrupt and seems to have meaningless numbers 1/5 1/9 or similar.

[5] Cf. Kroll, p. 35, 24 (ἀνὰ ιβ′) and p. 35, 31 f. (διδοὺς ἑκάστῳ ζῳδίῳ ἀνὰ ε′).

The coefficient 15 of m shows that 24 months produce one complete rotation, consistent with the 2-year period; the coefficient 12 of the number β of single years is a consequence of the 30-year period.[6]

The 2-year period appears once more in the epoch dates. For odd values of β in (8) the angle (γ) has to be added to ♎ 0°, for even values to ♈ 0°. Indeed, according to Alm. IX, 4 one has for the beginning of

Augustus 1	♎ 3;52	Augustus 0	♓ 22;35
		Augustus 2	♈ 15;9

for mean longitudes of Mars.[7]

The corresponding example is badly garbled. First t is taken to be 139 instead of 138. Secondly $t=139$ is taken to be $4 \cdot 30+11$ $(=131)$, hence $2\beta=22$ and with $m=7$

$$\gamma = 22 + 2;30 \cdot 7 \approx 39$$

thus $(\gamma)=39 \cdot 6=234°$. Since β is odd the planet is said to be in ♉ (since ♎ 0° + 234 = ♉ 24°).

Table 3 shows that this result, based on two essential errors, agrees nevertheless with the true longitude given in the horoscope. Following the rules of the text with $t=138$ one finds ♈ 0° + 318 = ♒ 18° which shows an error of more than a quadrant for the mean longitude. It is clear that the rules for Mars have no practical value whatsoever.

Table 3

No.	Planet	Text I, 20	$\bar{\lambda}$ Alm. IX, 4 110 March 14	Computed A Egypt. Cal.	Computed B Alex. Cal.	Horoscope Text III, 6
1	♄	♓	♒ 27;35	♒ 20;32	♒ 24;32	♓ 1;25
2	♃	♐	♐ 14;35	♐ 17;21	♐ 21;27	♐ 24;18
3	♂	♉	♊ 1;17	♈ 21;12	♉ 12;18	♉ 21; 8
4	♀ (a)	♒	♓ 19;20	♑ 4;22	♒ 7; 7	♒ 9
5	♀ (b)	♐ 20		♍ 13;14	♎ 25;43	
6	♀ (c)	♑ 16		♐ 11; 4	♑ 25;32	
7	☿	♒ 25		♑ 26;37	♑ 29;24	

C. *Venus and Mercury*

The procedure for Venus is based on the synodic period of 8 years during which time the planet comes in conjunction with the sun 5 times. Assuming exact validity of this relation one obtains for the mean synodic arc

$$\Delta\bar{\lambda} = 8 \cdot 360°/5 = 576° \equiv 216° \quad \text{mod. } 360° \tag{1}$$

and for the synodic time[1]

$$\Delta t = 8 \cdot 365^d/5 = 584^d. \tag{2}$$

[6] According to Alm. IX, 4 a 2-years mean motion produces $\approx +22;34°$, a 30-years motion $\approx -21;33°$ mod. 360°.

[7] The true longitudes are about ♍ 17° and ♉ 11°, ♊ 8°, respectively.

[1] This interval is mentioned in Cleomedes II, 7; cf. above p. 784. Note that "years" are taken here to be egyptian years. In fact, however, rigid standards are not a characteristic feature of such treatises and it is not surprising to find Alexandrian years in our computations.

The author of the extant text (Vettius Valens or his source) apparently did not understand that these parameters were underlying his rules. One starts in the same fashion as with the outer planets. Let t denote the number of completed years in the era Augustus and

$$t=\alpha\cdot 8+\beta, \quad 0\leqq\beta<8. \tag{3}$$

The fact that α does not appear in the subsequent calculations implies the assumption of full accuracy of the 8-year cycle. Five values of β are then selected and associated with dates (months and days) and zodiacal signs as follows

β	dates	signs
$\beta=1$	VII 10	♉
3	II 10	♐
4	X 22	♌
6	V 8	♓
7	XII 14	♎

(4)

Obviously we have here a slightly garbled set of dates and longitudes which should be equidistantly spaced with (1) and (2) as intervals, β giving the years of the era Augustus. It requires only modest emendations to replace (4) by a correct, strictly periodic, pattern, e.g., as follows:

year	date	longit.
year 1	VII 18	♉ 20
3	II 22	♐ 26
4	X 1	♌ 2
6	V 5	♓ 8
7	XII 14	♎ 14
9	VII 18	♉ 20

(5)

There can be no doubt that a scheme of this type existed instead of (4); nevertheless the examples in our text operate with the inaccurate scheme (4), at least for the years 3 and 4. For the results the small differences between (4) and (5) are irrelevant.

Since the intervals (1) and (2) underlie the patterns (4) and (5) we can also say that these schemes must always represent dates and longitudes for the same phase of the planet. Which phase this is can be found by computing for the given dates in the era Augustus the longitudinal differences between sun and Venus. Table 4 p. 798 shows the result: Venus is in all cases about 45° to the east of the sun, i.e. at maximum elongation as evening star.[2]

It is clear that we have now found the basic elements for the "handy" determination of the longitudes of Venus. By finding in (3) the residue modulo 8 for a given year $(t+1)$ of the era Augustus we know to which one of the five cycles of the first eight years of Augustus the given year belongs. Thus a given date is now associated with a cycle of 584 days of known initial date and longitude. It is also clear how one must procede from here: for a given date which is d days later than the starting point of its cycle $(d<584)$ we must find the longitudinal increment

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[2] Table 4 also shows that this result is practically independent of the changes from (4) to (5). It is interesting to see how nearly the interval $\Delta\bar{\lambda}=216^\circ$ is valid also for the true longitudes of Venus when $\Delta t=584^d$ as in (5).

Table 4

Augustus	julian	$\lambda_{\odot}$	$\lambda_{♀}$	Text	$\Delta\lambda = \lambda_{♀} - \lambda_{\odot}$
1 VII 10	−28 March 7	♓ 15; 9	♈ 28	♉	43°
3 II 10	−27 Oct. 8	♎ 13; 4	♏ 27;16	♐	44
4 X 22	−25 June 17	♊ 22;29	♌ 8; 5	♌	46
6 V 8	−23 Jan. 3	♑ 11;58	♒ 27;12	♓	45
7 XII 14	−22 Aug. 7	♌ 11;43	♍ 26;49	♎	45

Emended Dates ($\Delta t = 584^d$)				
Augustus	julian	$\lambda_{♀}$	$\Delta\lambda_{♀}$	$\Delta\lambda$
1 VII 18	−28 March 15	♉ 6;46		44°
3 II 22	−27 Oct. 20	♐ 10;58	214;12°	46
4 X 1	−25 May 27	♋ 16;25	215;27	44
6 V 5	−24 Dec. 31	♒ 23;49	216;35	45
7 XII 14	−22 Aug. 7	♍ 26;49	213;49	45
9 VII 18	−20 March 13	♉ 4;56	218; 7	44

from a day-by-day template that is valid for one synodic cycle. We do not have such a text but its existence is secured by corresponding auxiliary tables for Mercury and for Saturn.[3] Vettius Valens, however, does not give such tables and what follows in the three examples for Venus is some makeshift procedure which arrives at a result that has been obtained by some complete and much more accurate computation, eventually incorporated into the data for the horoscope in III, 6 (cf. Table 3, p. 796).

In discussing the three examples given in I, 20 we accept the values of t used by Vettius Valens though the first and the third should be one year lower.[4] Using the epoch dates listed in (4) the following steps are taken:

(a)	$t = 139 = 17 \cdot 8 + 3$	hence from epoch	3 II 10	to 3 VII 18:	158 days	
(b)	$t = 148 = 18 \cdot 8 + 4$		3 II 10	4 III 30:	415	(6)
(c)	$t = 149 = 18 \cdot 8 + 5$		4 X 22	5 VI 13:	236	

According to the rules in the text[5] one now subtracts 120 "degrees" from each of these intervals. Obviously one can only subtract days from days and this is what is done in all three examples.

The meaning of this subtraction must be a change from the epoch dates in (4) or (5) to dates 120 days later. The significance of these new epoch dates is not difficult to determine. For example at the day Augustus 1 VII 10 + 120 = −28 July 5 we find for the sun the longitude ♋ 10, for Venus ♊ 15 (hence being morning star) and we also see that Venus was stationary. Hence we see that the maximum elongations of Venus as evening star do not serve as epoch phenomena in the

[3] Cf. above p. 790f.
[4] Cf. above Table 2 p. 793.
[5] Kroll, p. 34, 34.

first 8-year cycle of the era Augustus but the stationary points as morning star (Φ) do. This also explains some cryptic remarks about stationary points,[6] or that the planet is not stationary for $\beta=2$ or 5.

The concluding steps in the examples for Venus consist in counting one zodiacal sign for each 25 days in d. This means assuming a mean velocity of $30/25=1;12^{\circ/d}$ for the motion of the planet starting at Φ and ending at the next maximum elongation as evening star, as is quite correct for the major part of this section. The corresponding time, one synodic period minus 120 days, i.e. about 460 days, would be associated with a little more than 18 signs of progress in longitude whereas the synodic arc (576°) amounts to a little more than 19 signs. Consequently the examples in Valens' text add (without a word of explanation) one sign to the epoch parameters (4) because they refer to maximum elongations and not to Φ. In this way one finds

(a)	$d=$ 38	thus $\lambda=$ ♑ 0°+ 45;36° = ♒ 15;36	Text: ♒	
(b)	295	♑ 0°+354 = ♐ 24	♐ 20	(7)
(c)	116	♍ 0°+139;12 = ♑ 19;12	♑ 16.	

For the actual true longitudes at the correct dates cf. Table 3 p. 796. The use of the corresponding correct values of t would destroy the agreement in the cases (a) and (c). This supports our suspicion that the results were adapted to previously known data, computed by proper procedures based on auxiliary tables.

A short text, closely related to Chap. I, 20 of Vettius Valens has been published (as far as Venus is concerned) by A. Tihon.[7] The examples correspond to the year A.D. 906 though the method is still the same as in Vettius Valens, including the use of the era Augustus. The Alexandrian epoch dates (above p. 797 (4)) are schematically transposed to julian months (I = September), i.e. the day numbers are kept unchanged, a procedure which produces additional irregularities for the synodic intervals. The substitution of December, instead of January, for Tybi (V) is an obvious mistake; the day number 13 instead of 8 is the result of adding here (at the end of the julian year!) the 5 epagomenal days of the Egyptian calendar.[8] As we have seen the correct synodic interval suggests the date VII 18 for the cycle year 1, hence schematically March 18, a date provocatively close to the vernal equinox. At any rate we have the following relation between the epoch parameters:

β Vett. Val.	Vat. gr. 184	
1 VII 10 ♉	March 21 ♊	
3 II 10 ♐	Oct. 10 ♑	(8)
4 X 22 ♌	June 22 ♍	
6 V 8 ♓	Dec.(!) 13 ♈	
7 XII 14 ♎	Aug. 14 ♏	

[6] Kroll, p. 34, 28; 34, 34; 35, 1–6.

[7] Tihon [1968], from three MSS of the 13th to 15th cent.: Vat. gr. 184, Par. gr. suppl. 464, Rossianus 897 gr. 37.

[8] I owe this neat explanation to my colleague D. Pingree. This corresponds to a rule found in CCAG 11, 2, p. 112, 18–20, for converting julian years and months to Egyptian ones.

The zodiacal signs listed in Vettius Valens correspond to the position of Venus at the given dates (as shown in Table 4, p. 798); i.e. to Venus at maximum elongation as evening star. The positions in Vat. gr. 184, however, are the positions of Venus 120 days later, i.e. at its second station (Φ). This is seen from the following tabulation:

Vett. Val. Augustus	120 days later: Φ	$\lambda_♀$	Vat. gr. 184
1 VII 10	−28 July 5	♊ 15	♊
3 II 10	−26 Feb. 5	♋ 28	♋
4 X 22	−25 Oct. 15	♍ 5	♍
6 V 8	−23 May 3	♈ 6	♈
7 XII 14	−22 Dec. 5	♏ 9	♏

(9)

The combination in (8) of the dates of maximum elongations with the longitudes of Φ, 120 days later, supports our conjecture that special patterns did exist to bridge the gaps.

Mercury. No mention is made of a reduction modulo a synodic period.[9] All we are told is to find for a given date the number d of days elapsed since the preceding Thoth 1. This means either that our text is only fragmentarily preserved, or that it is assumed that one year is an integer multiple (then three) of synodic periods. To d should be added 162 and the total, reckoned as degrees, counted from ♈ 0°. The result supposedly is the longitude of the planet at the given date.

If we apply these rules to the Thoth 1 of Augustus 1 we see that the epoch which precedes this date by 162 days is −29 March 22, the vernal equinox preceding the beginning of the era Augustus. The planet itself was on this day in ♈ 18°, i.e. practically at maximum elongation as evening star, i.e. in the same phase we had found for Venus at the epoch dates (4), p. 797. Again a cryptic remark[10] finds its explanation in this configuration: "if the sun is at the beginning of a sign, you will find the planet behind it (in the sense of the daily rotation, i.e. to the west of the sun), if (the sun is) at the end of a sign (the planet is) in the next sign" (in the order of the signs, i.e. again to the west of the sun).

To reckon days as degrees means proceeding with the mean velocity of the sun. Adding the time interval of the given day in the year from I 1 $-162^d=$ VII 24 to ♈ 0° as degrees gives us the mean longitude of the sun at the given day. Again it would require a template to obtain the corresponding longitude of the planet.

Since this whole procedure only makes sense under the assumption of a fixed relationship of Mercury to the solar longitude, i.e. by taking years as integer multiples of synodic periods deviations must rapidly produce meaningless results. It is sheer luck when the example in our text gives a tolerable approximation. Starting with VI 13 of the year Augustus 149 (cf. Table 2, p. 793) gives $d=$ VI 13 − I 1 = 163[11] thus $d+162=325$ and ♈ 0° + 325 = ♒ 25, interpreted as

[9] One could expect, e.g., intervals of 116^d, mentioned in Cleomedes II, 7.

[10] Kroll, p. 35, 12–14.

[11] We would prefer to count this interval as 162 days but one never knows how dates should be interpreted, as current or completed days.

Mercury's position. In fact the planet is near its station as morning star (Φ) in Capricorn, and more or less the same situation prevails a year before or after.

4. Incorrect Epicyclic Theory

We have remarked before[1] that continued observation of an outer planet could readily suggest an epicyclic model to explain why the planet is seen periodically ahead and behind its mean position. It also would be natural to conclude from the relative narrowness of the loops traversed by the planet that the observer sees the plane of the epicycle more or less edge on (cf. Fig. 5). It is by no means obvious, however, in what sense the planet rotates about its mean position C because whatever sense of rotation is assumed the planet would be seen oscillating between two extrema A and B with respect to C.

Ptolemy, in Alm. IX, 5, gives some arguments which lead to the correct determination of the sense of rotation.[2] In fact the proper answer was already known to Apollonius whose theory of stationary points[3] may well have to do with the solution of this very problem.[4] Nevertheless the opposite assumption found followers. The earliest evidence for it (in combination with sidereal coordinates for the anomaly) seems to be the inscription of Keskinto[5] (about 100 B.C.?). A not very clear description is found in Pliny (NH II, 68 to 71) and once more, as late as in the second century A.D., in the astrological treatise P. Mich. 149. Apollonius probably remained unknown to all of these sources.[6]

Before discussing these erroneous attempts at describing planetary motions it will be useful to note that the distinction between the two directions of rotation is indeed difficult to make when the epicycle is small, as is the case for Jupiter and Saturn. Suppose that r/R is so small that the direction OPQ (Fig. 6) is practically parallel to OC. Then the same phenomena (e.g. stations) will occur in P and Q since one motion is simply the mirror image of the other. Only with increasing ratio r/R will it be become easier to locate the stations correctly in relation to the opposition (cf. Fig. 7).

For the inner planets (in particular for Venus) the situation is greatly simplified by the fact that maximum elongations as evening star (A) and as morning star (B) are directly observable (cf. Fig. 8). The marked difference in time for the motion from A to B and from B to A determines the proper sense of rotation. This result can be confirmed by the great difference in the duration of invisibility at inferior ($\Omega - \Gamma$) and superior ($\Sigma - \Xi$) conjunction.

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1 Cf. Figs. 155 and 156.

2 Cf. above p. 149.

3 Cf. above I D 3, 1.

4 Cf. the discussion in Aaboe [1963] and Neugebauer [1959, 3].

5 Cf. above IV C 3, 5.

6 The same still holds for an astronomer of the 20th century: Zinner, Entst. u. Ausbr., p. 13 depicted (1943) planetary motions as being retrograde at the apogee.

A. Pliny

We are well informed about the life of Pliny, in particular thanks to the letters by his nephew, the "Younger Pliny," whose description of the death of his uncle at the eruption of Vesuvius (A.D. 79) is justly famous.[1] Of Pliny's numerous writings only the 37 books of his "Naturalis Historia" have survived, an inexhaustible source of information on the scientific knowledge and social conditions of the early Roman imperial period.

Astronomical material is mainly concentrated in Book II. It is obvious that Pliny had no real understanding of this topic; his uncompromising collector's attitude effectively prevented him from studying technical problems seriously. Nevertheless his honesty and human warmth shines through his piles of slips[2] and shorthand notes time and again. The fact that he was entrusted by Vespasian and Titus with high public office is a testimony to his practical ability, while occasional remarks, e.g. on the adverse effects of prosperity on scientific inquiry,[3] show him as a keen observer of conditions in contemporary Roman society.

It is only in vague outlines that we can determine the planetary theory which provided the basis for Pliny's discussion in Book II, 63 to 79. He probably did not have a very clear picture and his sources were probably not much better. At any rate he never gives accurate numerical or geometrical data. His terminology is ambiguous, perhaps in part of his own making in an attempt to replace Greek technical terms by common Latin words.

For example "*altitudo*," height, is used both in the general sense of geocentric distance or specifically for the variation between epicyclic apogee and perigee.[4] The Greek term *apsis* is retained but used not only for a specific point on the orbit but also for the whole deferent, e.g. in declaring (II, 63) that "the innermost apsides are shortest." At the same time the following list of "*apsides altissimae*" is given (II, 64):

♄: ♏	☉: ♊	
♃: ♍	♀: ♐	(1)
♂: ♌	☿: ♑	

obviously representing the apogees of the eccenters.[5] In the next section (II, 65) the astrological "exaltations" (but again called *apsides altissimae*) are declared

[1] Pliny the Younger, Letters III, 5 on writing and working habits of Pliny (Loeb I (1969), p. 172–179); VI, 16 and 20 on the eruption of Vesuvius (Loeb I, p. 424–435, 438–447). A short biographical summary is found, e.g., in Donald J. Campbell, C. Plini Secundi Naturalis Historiae Liber Secundus (Aberdeen 1936), p. 1–4.

[2] Using this modern term does not mean that I have a clear conception of how one assembles notes on some 20000 items (NH I, 17) written on tablets or on rolls.

[3] E.g. NH II, 117.

[4] This variable is usually called βάθος, "depth," in Greek astronomy, but P. Mich. 149 (II, 16; II, 25) uses the exactly equivalent term ὕψος.

[5] It is interesting to note that we have here clear evidence of eccentric deferents. The same apogees for the sun and the outer planets are found in the Almagest (cf. above p. 147), excepting Mars whose apogee is at the end of ♋, but rounded to ♌ 0° in the computation of latitudes (cf. above p. 208). The naive idea that all apogees lie in the middle of their signs can perhaps be attributed to Pliny himself. Pliny's apogees for the outer planets are also found in Martianus Capella VIII, 884–886 (Dick, p. 467f.) and in Maass, Comm. Ar. rel., p. 273 and p. 601, 43 (here also Mercury). I do not know where the apogees for the inner planets come from; they are still found in crude medieval diagrams (e.g. Zinner, Entst. u. Ausbr., p. 55).

to influence the planetary phenomena.[6] As a third cause for the planetary phases, Pliny seems to consider optical effects connected with the variation of geocentric distances. On this vague note he ends the general description of his planetary theory.

A few details can be abstracted from the discussion of the outer planets (II, 68–71). Here it is stated explicitly that the motion of the planet is direct at the perigee (we must add: of the epicycle), retrograde at the apogee, similar to the motion of the moon (cf. Fig. 9).

This arrangement has consequences for the latitudes. Since factually the plane of the epicycle is nearly parallel to the ecliptic the point Θ (opposition) will generally appear at greater absolute latitude than Γ and Ω (cf. Fig. 10).[7] Consequently a model in which Θ is located at the apogee of the epicycle must tilt it in the wrong direction (Fig. 11). This explains Pliny's remark (II, 68) that at Γ the latitude as well as the geocentric distance ("*altitudo*") are at their beginning (i.e. minimum); the stations, conventionally put at elongations of $\pm 120°$, are then near the nodes of the epicycle.[8] Where the motion begins to be retrograde the latitude still increases toward Θ which is at the greatest distance from the earth.

What follows (in II, 70 and 71) is a long description of how this cycle of (epicyclic) motion is caused by the rays of the sun, depending on the angles under which they strike the planet. It is, of course, impossible to connect some concrete meaning with these fantasies for which Pliny takes personal credit (end of II, 71). Yet, similar "explanations" are also given by Vitruvius[9] and later by Martianus Capella.[10]

Concerning the inner planets we are told (II, 72) that their "apsides" naturally behave contrary to the outer planets (perhaps postulating a pattern of motion as shown in Fig. 12). Consequently a whole list of contradictory features is compiled (II, 74):

	outer planets:	inner planets:	
at apogee:	slowest	fastest	
at Ω:	fast	slow	(1)
at Γ:	slowing down	accelerating	
retrograde:	between Γ and Ω	between Ω and Γ.	

All this is clearly based on the assumption of opposite directions of rotation for the two groups of planets. Consequently the enumeration of the phases of Venus (II, 75) is correct.

Apparently no satisfactory theory for the latitudes of the inner planets had been found. At least there is no real basis for the rule (II, 75) that the latitude of Venus should increase after Γ, decrease after Ξ.

There follow (in II, 76) some cryptic remarks about "ascent" and "descent" of the planets.[11] The statements that Mercury and the moon take the same time

[6] Similar in Martianus Capella VIII, 884–886 (ed. Dick, p. 467 f.).
[7] Cf. also p. 1276, Fig. 214 and below Fig. 115 (p. 1413).
[8] Perhaps the plane of the deferent is not distinguished from the ecliptic.
[9] Archit. IX, 1, 9 (Budé, p. 12).
[10] VIII, 887 (Dick, p. 469).
[11] *subit/descendit.*

for ascent and descent, while Venus remains about 15 times longer in ascent than in descent, suggest that this remark concerns the ratio of the time of motion faster than mean motion to the time less than it. If one assumes that the rapidly changing velocities of the moon and of Mercury can be represented by a linear zigzag function the above statement would be correct. Venus moves much longer with more than solar velocity than near the retrograde arc, but the ratio should be about 5:1 and not 15:1. Jupiter and Saturn supposedly have a "descent" twice the "ascent," Mars even 4:1.[12]

For the elongations of the inner planets Pliny gives the values

$$46° \text{ for Venus}, \quad 22° \text{ for Mercury} \tag{2}$$

quoting Timaeus[13] and Cidenas with Sosigenes[14], respectively, as his authorities. The same parameters are also found in commentaries and scholia to Aratus[15] and in Martianus Capella.[16] The limits

$$47° \text{ for Venus}, \quad 23° \text{ for Mercury} \tag{3}$$

given by Porphyry[17] are probably only a different form of expression for the boundaries (2). The astrological treatise P. Mich. 149[18] and Paulus Alexandrinus[19] combine

$$48° \text{ for Venus and } 22° \text{ for Mercury} \tag{4}$$

(mistakenly referred to the stations by Paulus[20] while the commentary by "Heliodorus" (recte Olympiodorus) has 48° and 23°, respectively).[21]

A different tradition gives the round values

$$50° \text{ for Venus}, \quad 20° \text{ for Mercury}, \tag{5}$$

attested in Theon of Smyrna,[22] in Chalcidius,[23] in Cleomedes,[24] and again in Martianus Capella[25] who mention the two sets (2) and (5) side by side.[26] In the Latin versions the term for degrees is *partes* in (2), *momenta* in (5); Martianus, excerpting from both classes of sources, does not hesitate to combine them in one single sentence: "50 *momentis* ... *licet* ... 46 *partibus*" (VIII, 882).

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[12] I do not know how one arrived at these figures.

[13] NH II, 38 (Jan-Mayhoff I, p. 138, 17); cf. also below note 26.

[14] NH II, 39 (Jan-Mayhoff I, p. 139, 1).

[15] Maass, Comm. Ar. rel., p. 273, 13; 16 and p. 601, No. 43.

[16] VIII, 882 and VIII, 880 (Dick, p. 466, 3 and p. 464, 10); cf. below note 25.

[17] CCAG 5, 4, p. 194, 22–195, 1.

[18] P. Mich. 149 X, 34 and X, 31 (Mich. Pap. III, p. 75 and p. 113).

[19] Boer, p. 32, 28 and p. 33, 5.

[20] Correctly referred to maximum elongations in Scholion No. 24 (Boer, p. 111).

[21] Boer, p. 15, 12f.; again 48° for the station of Venus, p. 22, 13.

[22] XIII, ed. Dupuis, p. 224, 14–17; Hiller, p. 137, 2–5.

[23] CX to CXII, ed. Wrobel, p. 117f. (Venus only). As will be shown presently (p. 959) the value of 50° is actually based on the more accurate parameter 48°.

[24] II, 7, ed. Ziegler, p. 226, 16f.

[25] VIII, 882 and VIII, 881 (Dick, p. 466, 2 and p. 465, 10); cf. above note 16.

[26] The elongation of 20° for Mercury (in combination with 46° for Venus) is also mentioned in a group of manuscripts of Pliny NH II, 72 and II, 73 (Jan-Mayhoff I, p. 150, 1; 9f.) with the variant 23°.

Isolated values for the maximum elongations of Mercury are 25°[27] and even 28°, the latter in combination with 48° for Venus.[28]

Pliny knows (II, 73) that the inner planets do not reach the extremal elongations given in (2) at every synodic revolution. He explains this fact by postulating an oscillation of the centers of the "*apsides*" (here meaning the epicycles) about the sun's position, a solution which had escaped the specialists (*canonicos*), as he proudly remarks.

At the end of his discussion of the planets Pliny collected all kinds of curiosa (II, 77, 78), e.g. about common aspects and phases of Mars and Jupiter. Some of these statements are obviously wrong (at least as formulated by Pliny), e.g. of the exclusions of retrogradations of Mercury[29] in Taurus, Gemini, and up to Cancer 25°.

A peculiar phase for Mars is mentioned in NH II, 60 at $\pm 90°$ (of anomaly), called first and second "*nonagenarius*", respectively. This definition is of interest because it has parallel in P. Mich. 149, in Martianus Capella, and in Porphyry,[30] treatises which are also otherwise related to Pliny.[31] For its astronomical significance cf. above p. 792.

B. Pap. Mich. 149

This treatise (consisting of fragments from about 22 columns) is oriented toward astrology, in particular toward doctrines which concern the influences of the planets on the parts of the human body. Written in the second century A.D.[1] it is undoubtedly based on older material, much of which is not known to us from other sources. Within the sections which deal with astronomy proper, unmistakable parallels with Pliny's planetary theory suggest some common source (or sources) of the late hellenistic period.

Unfortunately it is not uncommon to find a lack of precision in Pliny's words. Nevertheless it is at least clear that we are dealing with epicyclic models for which even concrete numerical data are given; in column I, 11–25 the radii are listed in increasing order[2]:

☉: 2;30 = 150′	♃: 13;45 = 825′	(1)
☾: 6 = 360′ (error for otherwise 6;30 = 390′)	☿: 21;15 = 1275′	
	♂: 42 = 2520′	
♄: 10 = 600′	♀: 48 = 2880′.	

The parameter for the sun is identical with the solar eccentricity according to Hipparchus and Ptolemy.[3] The remaining numbers are almost always greater

[27] CCAG 4, p. 115, 19; p. 117, 17.
[28] In a scholion to the Handy Tables; cf. Tihon [1973] XI.
[29] Perhaps confused with the omitted phases of Mercury in Taurus? (cf. above p. 241).
[30] Martianus Capella VIII, 884 (Dick, p. 467, 10f.); Porphyry CCAG 5, 4, p. 194, 6–11; P. Mich. 149 XI, 18–20 (Mich. Pap. III, p. 76, p. 113). This papyrus speaks (XI, 19) about "90-day anomalies"; the same in CCAG 7, p. 217, 24–26; p. 218, 9.
[31] Cf., e.g., on the elongations of the inner planets (above p. 804) or the theory of epicyclic rotation (below p. 807).
[1] Edited (with translation and commentary) by Frank E. Robbins in Mich. Pap. III, p. 62–117. The date is determined by palaeographical criteria; the provenance is unknown.
[2] The units are called μοῖραι and λεπτά, to be interpreted as "parts" and their minutes in case they represent lengths for which the radius of the deferent is $R = 60^p$.
[3] Cf. above p. 58.

than the epicyclic radii in the Almagest[4]:

☾: 5;15^{p}	☿: 22;30^{p}
♄: 6;30	♂: 39;30
♃: 11;30	♀: 43;10.

(2)

One will, of course, not expect accurate agreement with results obtained by the sophisticated methods of the Almagest, but it is clear that it is permissible to interpret the numbers in (1) as epicycle radii such that $R=60^{p}$. On the other hand the papyrus gives for Venus and Mercury 48° and 22° as maximum elongations[5] which would require, respectively, the radii

$$\text{Sin } 48^{\circ}=44;35,19^{p}, \qquad \text{Sin } 22^{\circ}=22;28,35^{p} \tag{3}$$

contradictory to (1). Clearly our text is not consistent.

Most serious doubt is cast on any rational interpretation of the parameters (1) by the fact that these numbers are fitted into an extensive numerological pattern based on the following arithmetical relations: let S_7 be the total of all seven radii in (1),[6] then one finds that

$$1/3\,S_7=1/3\;144^{p}=1/3\;8640'=48^{p}=2880'=r_{♀} \tag{4}$$

and therefore for the total S_6 of the first six radii

$$1/2\,S_6=r_{♀}=1/3\,S_7. \tag{5}$$

It seems extremely unlikely that seven astronomically determined radii satisfy (5) by mere accident. Hence one can hardly hope to detect a rational procedure resulting in the given data.

On the other hand daily mean motions that make good sense are mentioned in II, 19–22:

☉: 0;59,8,16$^{\circ/d}$	♃: 0;5$^{\circ/d}$
☾: 13;10[7]	♂: 0;32
♄: 0;2	♀ and ☿: same as sun.

(6)

Finally, Venus and Mercury are said (II, 29–31) to exceed the solar motion by 0;5(?) and 3;6° = 186′, respectively. Since the second value obviously represents the mean motion in anomaly for Mercury[8] one may perhaps emend[9] the first parameter to 0;35$^{\circ/d}$.

A section concerning apogees is unfortunately badly damaged (XIII, 38–42). Only the following is preserved:

☉: A = ♊, ♀: P = ♓, A = ♍, ♃: A = ♋, P = ♑. (7)

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[4] Cf. above p. 76 and p. 146.

[5] Cf. above p. 804 (2) and (4).

[6] Including the correct number 390′ for the moon.

[7] Text: *στερεαι* $\overline{\iota\gamma}$ $\overline{\iota}$ (in contrast to *λεπτα* in the following) meaning "full (degrees)"; the same terminology in P. Lond. 130 (VII, 159/160), a horoscope for A.D. 81 (Neugebauer-Van Hoesen, Gr. Hor., p. 22, p. 24).

[8] Almagest IX, 4: 3;6,24$^{\circ/d}$.

[9] For Venus one would expect about 0;37$^{\circ/d}$. This suggests the reading *λεπτα* $\overline{\lambda\varepsilon}$ *ωστε* where the text has *λεπτα* $\overline{\varepsilon}$ *εωστε* (sic).

This list probably represents a mixture of apogees and exaltations[10] since Venus has its exaltation in ♍ (not the apogee!), Jupiter in ♋.[11]

No parameters are preserved for latitudes. We are only told (in I, 9–11) that the sun has the smallest variation in latitude,[12] whereas the size of the "sphere" (i.e. of the epicycle) of Venus is made responsible (I, 21–23) for a latitude which exceeds the width of the zodiac[13] (presumably $\pm 6^\circ$).

The synodic cycle of the planets is described twice in our text: first in II, 15–17 for the outer planets and in II, 22–31 for the inner planets, then again in X, 30–XI, 4 for the inner planets and in XI, 5–27 for the outer ones. In spite of the fragmentary condition of our text and its inherent lack of clarity of presentation there can be no doubt that the wrong sense of rotation is assumed for the outer planets: at the apogee, near opposition, the velocity is smallest (II, 7), largest at the perigee (II, 14). One can also determine the meaning of other terms connected with the motion from apogee to perigee, independent of the sense of rotation. During the motion after opposition toward heliacal setting the planet is always to the east of the sun. Since the text describes this situation as "the planet is left behind by the sun" (II 12, 13) we see that the terms "behind" and "ahead" (and similarly "overtake," etc.) must be understood in relation to the daily rotation[14] – in agreement with the commonly used notation "leading" (*προηγούμενος*) and "following" (*ἑπόμενος*).[15] Consequently, when we read in II 28, 29 that an inner planet at the perigee[16] "runs ahead of the sun" we see that the sense of rotation is correct, as one should expect for an inner planet[17], and again in agreement with Pliny.[18] This distinction between outer and inner planets is confirmed by all relevant passages throughout the text.[19]

As remarked at the beginning one can intuitively expect nearly the same phenomena, regardless of the sense of rotation, as long as the epicycle is small and its progress on the deferent slow in comparison with its own rotation. A large epicycle, however, which is traversed by a planet such that its motion in anomaly is added at the perigee to the mean motion in longitude excludes retrogradation not only at the perigee but may well result in direct motion also at the far distant apogee. A. Aaboe has in general investigated these phenomena[20] and has shown that a planet moving in the wrong direction on an epicycle of radius $r<R$ will never become retrograde when its mean velocity in longitude $\bar{v}_\lambda$ is greater than half its angular velocity v_α in anomaly:

$$\bar{v}_\lambda > 1/2\, v_\alpha. \tag{8}$$

[10] Reminiscent of Pliny NH II 64, 65 (above p. 802f.).

[11] Exaltations and depressions are listed in column XI, though under the unusual terms "thrones" and "prisons."

[12] As in Pliny, NH II 66, 67; cf. above p. 782.

[13] Again as in Pliny, NH II 66.

[14] The misinterpretation of these terms vitiated translation and commentary in the edition of our text.

[15] Hence we still speak about the "precession" of the equinoxes.

[16] Being "lowest" (*ταπεινούμενος*).

[17] Cf. above p. 801.

[18] Cf. above p. 803.

[19] In X 38, 39 one has to translate *ἑσπέριοι στηρίζουσιν* as "western," not as "evening" stations (the planet being morning star).

[20] Aaboe [1963]; cf. also p. 191.

Since for Mars: $\bar{v}_\lambda \approx 0;32^{\circ/d}$ $1/2\, v_a \approx 0;14^{\circ/d}$
for Venus: $\bar{v}_\lambda \approx 1^{\circ/d}$ $1/2\, v_a \approx 0;18^{\circ/d}$ (9)

no epicyclic model with incorrect sense of rotation can account for the phenomenon of retrogradation of these two planets. Aaboe has also shown that the criterium (8) is an immediate consequence of Apollonius' theorem on stationary points.[21] Hence it was fully within the reach of ancient astronomy to demonstrate rigorously the exclusion of one sense of rotation for these two planets.

It follows from these considerations that the qualitative descriptions of planetary motions as we find them in Pliny or in P. Mich. 149 can never have been associated (at least for Mars) with numerical tables or with specific predictions about stationary points and retrogradations. When, on the other hand, P. Mich. 149 gives essentially correct numerical data for the radii of the epicycles and for the velocities $\bar{v}_\lambda$ and v_a [22] then we know that these data cannot come from a theory which operates with the incorrect sense of rotation of the epicycle. Obviously our text is the result of excerpting, without understanding, sources of very different level. Medieval attitudes have their roots deep in classical antiquity.

§ 2. Lunar Theory

1. P. Ryl. 27 and Related Texts

P. Ryl. 27 belongs to a small group of Greek and demotic papyri which have contributed in an exceptional way to our understanding of Greek astronomy under the influence of Babylonian methods and their transmission to India. At the same time we have a good example of the haphazard way in which our insight into complex historical processes develops, depending on the accidental preservation or publication and irrational combinations of luck, ignorance, and persistence that are the characteristic attributes of working with source material which extends over many centuries and the most diverse areas of civilizations.

P. Ryl. 27 was written shortly after A.D. 250 as is evident from a short chronological table at the end of the text ("Sect. 7"[1]) which gives the regnal years of the Roman emperors from Commodus to Gallus. Our text was written on the verso of the papyrus; the recto,[2] inscribed with Book I of the Iliad, shows, squeezed between two columns, the title of the astronomical text "Treatise of Ptolemy." This is not only chronologically incorrect but a misrepresentation as well of the methods described in the main part of the treatise. It is not surprising, however, that no correct title could be given, since only the first four of the seven sections form a unit belonging to a much earlier methodological level than the remaining sections.

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[21] Aaboe [1963], p. 7; for Apollonius cf. above I D 3, 1. The determination of the right order of magnitude of the parameters in (9) requires nothing beyond the commonly known relations between synodic and sidereal periods.
[22] Cf. above p. 806.
[1] The division into sections (following Neugebauer [1949, 3]) is marked only in part in the text, e.g. by headings for our Sect. 3 and 5.
[2] Published as P. Ryl. 43.

The text was edited in Vol. I of the "Catalogue" of the John Rylands Library at Manchester (1911) with ample notes by Hunt and Smyly. Obviously the main purpose of the treatise, headed by the sign of a crescent, ☾, was the teaching of a procedure to find the moon's position in longitude and latitude for any given moment. But the details only became clear through related texts discovered later which established the connection with Babylonian methods, known since Kugler's "Babylonische Mondrechnung," published in 1900, and with Tamil astronomy, described in Warren's "Kala Sankalita" of 1825.

The procedures of P. Ryl. 27 (= R) can be called a typical product of hellenistic syncretism. Babylonian numerical methods are adapted to the Egyptian calender and its 25-year cycle whose structure was revealed through the discovery of the demotic P. Carlsberg 9.[3] An application of these combined methods came to light in a Greek table of lunar longitudes in P. Lund Inv. 35a[4] (= L) concerning, as far as preserved, the years from Nero 6 to Domitian 3 (i.e. from A.D. 60 to 84). The close relationship of the fragment in Lund to the papyrus in the Rylands collection was not immediately realized but ultimately provided the key for a complete understanding of the theory concerning lunar longitudes in P. Ryl. 27.[5] The right interpretation of the numbers for "latitudes" remained a desideratum until van der Waerden noticed[6] that the argument of latitude was not counted in degrees but in units of 15°, that is to say in "steps" (*βαϑμοί*).[7]

The first evidence for the transmission of this type of hellenistic-Babylonian lunar theory to India was noticed much earlier by Schnabel.[8] With the help of our understanding of the procedures in the Greek papyri it became possible to explain the computation of lunar eclipses in Tamil tradition, still alive in the 19th century.[9] We know today that this is only part of a much larger influx of early hellenistic material into Indian astronomy.[10]

A. P. Ryl. 27

The rules given in P. Ryl. 27 for the computation of lunar (mean) longitudes and latitudes are based on the following set of parameters

during	$\Delta\lambda$ (mod. 360°)	$\Delta\Omega'$ (mod. 24)	$\Delta\omega'$ (mod. 360°)	
A = 248^d	27;43,24,56°	2;43,28,34, 0	40;52, 8,30°	(1)
B = 303	32;33,44,51	3;14,29,34,15[1]	48;37,23,33,45	
C = 3031	337;31,19, 7	9;12,43,48,15	138;10,57, 3,45	
D = 9093	292;33,57,21	3;38,11,24,45[2]	54;32,51,11,15	

[3] First published: Neugebauer-Volten [1938]; "final" version in Neugebauer-Parker EAT III, p. 220–225.
[4] Published Knudtzon-Neugebauer [1947].
[5] Neugebauer [1949, 3].
[6] Van der Waerden [1958, 2], p. 181 f.
[7] Cf. above IV B 5.
[8] Schnabel [1927], last "Nachtrag" on last page.
[9] Warren, Kala Sankalita (1825); Neugebauer [1952], van der Waerden [1956].
[10] Cf., e.g., Pingree [1959, 1] and [1959, 2].
[1] Neugebauer [1949, 3], p. 11 line 29 should read: for [latitudes ...] 3;14,2[9,34,15].
[2] The first badly preserved number must be read 3, not 6 as in the edition and Neugebauer [1949, 3], p. 11 and 15.

The column $\Delta\lambda$ gives the progress of the mean (sidereal) longitude, $\Delta\Omega'$ concerns the argument of latitude counted in steps of 15°, such that

$$\Delta\omega'^{\circ} = 15^{\circ} \cdot \Delta\Omega' \tag{2}$$

is the increment of the argument of latitude, counted in degrees, always measured from the northern limiting point of the lunar orbit; these numbers $\Delta\omega'$ are mentioned nowhere in the text. The last line in (1) is in all cases exactly three times of the preceding line. Hence we have, in fact, only six independent parameters for three time intervals.

The interval A is the well-known length of 9 anomalistic months

$$248^{d} = 9 \cdot 27;33,20^{d} \tag{3}$$

where C gives the slightly more accurate value for 110 anomalistic months

$$3031^{d} = 110 \cdot 27;33,16,22,\ldots^{d}. \tag{4}$$

The interval B is simply $A + 2 \cdot 27;30^{d}$ corresponding to

$$303^{d} = 11 \cdot 27;32,43,\ldots^{d} \tag{5}$$

and hence of no independent interest.

The increments in λ and ω are based, according to (1), on the following daily mean motions

$$\begin{array}{lll} & \Delta\lambda & \Delta\omega' \\ \text{in A:} & 13;10,34,41,50^{\circ/d} & 13;13,45,31,\ 5^{\circ/d} \quad (6) \\ \text{in C:} & 13;10,34,51,58 & 13;13,45,41,13. \quad (7) \end{array}$$

Thus the nodal motion is in both cases

$$\Delta\lambda - \Delta\omega' = -0;3,10,49,15^{\circ/d}, \tag{8}$$

independent of the difference of approximations of the anomalistic month in (3) and (4).

Parameters for the lunar anomaly, longitude, latitude and nodes point to an ultimate interest in eclipse computation. Therefore it is not surprising to find the 25-year cycle for syzygies set in relation to the intervals in (1):

$$25 \text{ years} = 25 \cdot 365^{d} = 9125^{d} = D + 32^{d} = 309 \text{ syn. m.} \tag{9}$$

The table of regnal years in Sect. 7 counts years in the Era Augustus. If we assume the same norm for the Sect. 1 and 2 we can compute the longitude of the moon for a date used as epoch in Sect. 3 and 4 but now expressly counted in regnal years of Commodus. Since the resulting parameters agree[3] we have confirmed our interpretation of the starting day in Sect. 1 and 2. The computational rules of these sections ask for the addition of 2 years and 61 days to the year for which one wishes to compute. This means that the epoch precedes the beginning of the Augustan Era by 2 years and 61 days. Hence

$$\begin{gathered} \text{August 1 I 1} - 2 \text{ years } 61^{d} = \text{Augustus} - 2 \text{ XI 5} \\ (= \text{Nabonassar 716 XI 5} = -31 \text{ July 1}) \end{gathered} \tag{10}$$

[3] Cf. below p. 814.

is the epoch date for Sect. 1 and 2. The reason for this choice becomes obvious when one computes the anomaly of the moon for this date: the tables in the Almagest (IV, 4) give 354;28°, i.e. a position practically coinciding with the apogee. Consequently the periods in (1) will again lead to apogees. This conclusion is fully confirmed by the details of the subsequent procedures. We do not know, however, why it was just the apogee (10) which had been chosen instead of, e.g., the one nearest to the new year's day.

Sect. 1: find the date of a lunar apogee in the year Augustus $N+1$. Let all letters in these formulae denote non-negative integers. Then we are instructed to form

$$n=N+2=\alpha\cdot 25+\beta, \qquad 0\leqq\beta<25 \text{ years} \tag{11}$$

and

$$a=\alpha\cdot 32+\beta\cdot 365+61=\gamma\cdot 3031+\delta, \quad 0\leqq\delta<3031 \text{ days.} \tag{12}$$

We know already that the 2 years in (11) and the 61 days in (12) represent the interval between the epoch and the beginning of the Era Augustus. Since each 25-year cycle contains 32 more than D days (cf. (9)) we have in (12) the excess of days in n years over an integer number of anomalistic months. Because

$$\mathrm{C}=3031=11\cdot 248+303 \tag{13}$$

we must distinguish between two cases with respect to the residue $\delta<3031$: either δ belongs to the last 303 days in C or to the 11 multiples of 248, i.e. to the first 99 anomalistic months of C.[4] Accordingly, if

$$\left.\begin{array}{llll}\text{case I:} & \delta\geqq 11\cdot 248, & \delta=11\cdot 248+\zeta, & 0\leqq\zeta<303\\ \text{case II:} & \delta<11\cdot 248, & \delta=\varepsilon\cdot 248+\zeta, & 0\leqq\zeta<248\end{array}\right. \tag{14}$$

then the date of an apogee in the year Augustus $N+1$ is given by

$$\text{Augustus } N+1, \quad \text{Thoth } 1+\begin{cases}0 & \text{if } \zeta=0\\ 303-\zeta & \text{in case I}\\ 248-\zeta & \text{in case II.}\end{cases} \tag{15}$$

The essence of this procedure consists in converting the number of years since epoch into days and casting out multiples of anomalistic periods.[5] The result is a date of a lunar apogee in the year Augustus $N+1$.

Sect. 2 concerns the determination of the moon's longitude and latitude for the date found in Sect. 1. Since each 25-year cycle contains D days and the excess in (12) additional γC days we have with (1) the increment

$$\Delta\lambda=\alpha\cdot 292;33,57,21^\circ+\gamma\cdot 337;31,19,7^\circ \tag{16}$$

for the longitude and

$$\Delta\Omega'=\alpha\cdot 3;38,11,24,45+\gamma\cdot 9;12,43,48,15 \tag{17}$$

[4] The terminology is peculiar: the two cases are distinguished as "nodes" and "no nodes", respectively (ἐπὶ μὲν τῶν συνδέσμων — ἐπὶ δὲ τῶν μὴ συνδέσμων). Since the periods in question have no relation to the lunar nodes σύνδεσμος must here be taken in its non-technical sense ("bond," etc.).

[5] I do not understand the meaning of the concluding words of Section 1: "the nodes are 386 and 14,23."

for the argument of latitude. For the residue of δ days the proper multiples for A and B in (1) have to be used as required by (14). These increments must be added to the following epoch parameters given in the text:

$$\lambda_0 = ♌ 0° - 49;57,43,50° (= ♊ 10;2,16,10°) \tag{18}$$

for the longitudes, which are reckoned from the beginning of Leo,[6] and

$$\Omega'_0 = 12;10,39,19,15\ ^7 \tag{19a}$$

for the argument of latitude, reckoned from the northern limiting point of the lunar orbit in "steps" of 15°, hence

$$\omega'_0 = 182;39,49,48,45° \tag{19b}$$

in degrees.

These elements can be compared with the mean positions obtained with the tables of the Almagest (IV 4) and one finds for the epoch date (10)

$$\lambda_0 = ♊ 13;20, \qquad \omega'_0 = 185;51$$

hence for the northern limiting point

$$\lambda_N = \lambda_0 - \omega'_0 = 247;29 = ♐ 7;29°$$

and for the ascending node ♍ 7;29° in agreement with the modern estimate ♍ 8°. From (18) and (19) one obtains

$$\lambda_N = \lambda_0 - \omega'_0 \approx 247;22 = ♐ 7;22°. \tag{20}$$

This, incidentally, fully confirms our chronological assumptions.

Example: $N = 93$.

Consequently, from (11) and (12):

$$n = 95 = 3 \cdot 25 + 20 \qquad \text{i.e.} \quad \alpha = 3, \ \beta = 20,$$

$$a = 3 \cdot 32 + 20 \cdot 365 + 61 = 7457 = 2 \cdot 3031 + 1395 \quad \text{i.e.} \quad \gamma = 2$$

and

$$\delta = 1395 = 5 \cdot 248 + 155 \qquad \text{i.e.} \quad \varepsilon = 5, \ \zeta = 155.$$

Hence we are in case II of (14) and obtain for the date of a lunar apogee from (15)

$$\begin{aligned}&\text{Augustus 94 Thoth } 1 + (248 - 155)^d = \text{Augustus 94 IV 4}\\ &(= \text{Nab. } 812 = \text{Nero 11 IV } 4 = \text{A.D. 64 Nov. 8}).\end{aligned} \tag{21}$$

Longitude: $\alpha = 3$, $\gamma = 2$, $\varepsilon = 5$ gives with (1)

$$\begin{aligned}\Delta\lambda &= 3 \cdot 292;33,57,21 + 2 \cdot 337;31,19,7 + 6 \cdot 27;43,24,56\\ &= 157;41,52,3 + 315;2,38,14 + 166;20,29,36 \equiv 279;4,59,53\end{aligned}$$

hence with (18)

$$\lambda = 279;4,59,53 - 49;57,43,50 = ♌ 229;7,16,3 (= ♓ 19;7,16,3). \tag{22}$$

[6] Cf. for this norm above p. 670.

[7] The text has 12;12,... instead of 12;10,... Cf. below p. 815, n.4 for this emendation.

Argument of latitude:

$$\Delta\Omega' = 3\cdot 3;38,11,24,45 + 2\cdot 9;12,43,48;15 + 6\cdot 2;43,28,34$$
$$\equiv 21;40,53,14,45 \quad (\text{mod. } 24)$$

thus

$$\Delta\omega' = 325;13,18,41,15^\circ$$

and from epoch with (19b)

$$\omega' = 325;13,18,41,15 + 182;39,49,48,45 \equiv 147;53,8,30^\circ. \tag{23}$$

Hence for the northern limiting point

$$\lambda_N = \lambda - \omega' = 210;14,7,33 = \text{♎}\ 21;14,7,33^\circ$$

and ♋ 21;14° for the ascending node.

The Sect. 3 and 4 of R involve a change of epoch and will be discussed presently in connection with L.[8] Sect. 5 (headed "On the nodes") gives trivial parameters for the motion of the nodes, based on 0;3,10° as daily increment.

Sect. 6 has very little relation to the rest of the text. It gives a simple rule for the derivation of dates of solstices and equinoxes, starting with the well-known observations in the year Antoninus 3 (A.D. 139/140) reported in the Almagest, making use of Ptolemy's value $365;15 - 0;0,12^d$ for the length of the tropical year.[9] The last preserved section gives the regnal years from Commodus to Gallus.

B. P. Lund Inv. 35a

This papyrus[1] contains fragments of a table of mean longitudes of the moon for dates arranged exactly in the fashion prescribed by the text R. In fact it was this agreement which made it possible to understand the rules given in the Rylands papyrus. We can now follow the easier way and explain the table in L by the methods described in the preceding section.

For our purpose the short excerpt displayed in Table 5 suffices. In the papyrus each column covered 25 years in 3 groups. Only the major part of the column for the years from Nero 6 to Domitian 3 (A.D. 60 to 84) is preserved. Some traces suggest the existence of a preceding and of a subsequent column, suggesting a coverage of at least from A.D. 35 (Tiberius 21) to 109 (Trajan 12). The text was probably written in the first half of the second century.[2]

A look at the differences between the dates and longitudes in consecutive lines demonstrates the close connection between L and R: 11 times one finds A followed by one B with increments $\Delta\lambda$ as shown in (1), p. 809. Not only the differences but the longitudes themselves agree with the results obtainable with R; this is shown by our example p. 812 (22) which agrees with line 3 in Table 5. There appear, however, two discrepancies. First the dates in R are one day higher than in L (IV 4 in our example p. 812 (21) against IV 3 in L); secondly the rules

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[8] Cf. below p. 814.
[9] Cf. Neugebauer [1949, 3], p. 21.
[1] Published Knudtzon-Neugebauer [1947].
[2] Knudtzon, l.c. p. 77.

Table 5

		Eg.	$\lambda_{\mathbb{C}}$	julian	Δt	Nabon.	Aug.
		XI 27	♑ 23;40,26,11	June 30		810	92
(Nero)	10	VII 30	♒ 21;23,51, 7	64 March 4	A	811	93
	11	IV 3	♓ 19; 7,16, 3	Nov. 7	A	812	94
		XII 11	♈ 16;50,40,59	65 July 13	A		
	12	VIII 14	♉ 14;34, 5,55	66 March 18	A	813	95
	13	IV 17	♊ 12;17,30,51	Nov. 21	A	814	96
		XII 25	♋ 10; 0,55,47	67 July 27	A		
	14	X 23	♌ 12;34,40,38	68 May 25	B	815	97
(Vespas.)	1	VI 26	♍ 10;18, 5,34	69 Jan. 28	A	816	98
	2	II 29	♎ 8; 1,30,30	Oct. 3	A	817	99

in R result in an interchange of the intervals B and A in lines 8 and 9 of Table 5. The text L always leaves a line blank at the end of a group of 12 lines where the interval B is applied.[3] The rules in R would place the separation between such groups one line farther down, at least when one follows the rules stated in the Sect. 1 and 2 of R. Consequently one would have for the lines 7 to 9 of Table 5

		in L		in R		line	
Nero	13	XII 25	♋ 10; 0,55,47	XII 26	♋ 10; 0,55,47	7.	
	14	X 23	♌ 12;34,40,38	X 29	♌ 7;44,20,43	8.	(1)
Vesp.	1	VI 26	♍ 10;18, 5,34	VI 27	♍ 10;18, 5,34	9.	

Both discrepancies are eliminated, however, by a secondary rule given in R (Sect. 3 and 4) under the heading "another (and) shorter method." This heading is misleading in so far as the new rule involves only a shift to a new epoch and the counting of years beginning with Commodus 1 (=Augustus 190= Nab. 908 Thoth 1=A.D. 160 July 14). The actual computation, however, uses as epoch Nero 14 X 23 (A.D. 68 May 25), i.e. 73 days before Vespasian 1 I 1. Since this is a date found in L at the beginning of a group (cf. above line 8) all subsequent dates and longitudes agree exactly with L. The reason for this change is unknown. All we can say is that L was computed according to the norm of the "shorter" method which in this way is attested long before Commodus. The rest of the procedure is the same as before.

The parameters for the new epoch can easily be derived from the elements in R. The time interval between the old epoch

$$\text{Augustus} - 2\ \text{XI}\ 5 = -31\ \text{July}\ 1 = \text{jul. day}\ 1709917$$

and the new epoch

$$\text{Nero}\ 14\ (=\text{Nab.}\ 815)\ \text{X}\ 23 = 68\ \text{May}\ 25 = \text{jul. day}\ 1746040$$

amounts to

$$36123^d = 4\cdot 9093 - 248 - 1 = 4\,D - A - 1^d.$$

[3] Cf. the photograph Knudtzon-Neugebauer [1947], Pl. I.

The single day produces the adjustment of the day numbers and can be ignored for the following. From (1), p. 809 one finds for 4 D − A the increments

$$\Delta\lambda = 62;32,24,28°, \quad \Delta\Omega' = 11;49,17,5 \tag{2}$$

hence for the longitude at the new epoch with (18) p. 812

$$\lambda_1 = ♊\, 10;2,16,10 + 62;32,24,28 = ♌\, 12;34,40,38°. \tag{3}$$

For the argument of latitude one should obtain with (19a), p. 812

$$\Omega_1' = 12;10,39,19,15 + 11;49,17,5 = 23;59,56,24,15$$
$$\equiv -0;0,3,35,45 \quad (\text{mod. } 24)$$

or

$$\omega_1' = -0;0,53,56,15°.$$

This would give for the northern limiting point

$$\lambda_N = \lambda_1 - \omega_1' \approx ♌\, 12;36°$$

in good agreement with ♌ 14;5° found with the Almagest. The text, however, committed an error by making

$$\Omega_1' = -0;21,22,14,15 \quad \text{i.e.} \quad \omega_1' = -5;20,33,33,45°$$

which is the result of

$$\Omega_1' = \Delta\Omega' - \Omega_0' = 11;49,17,5 - 12;10,39,19,15 = -0;21,22,14,15.$$ [4]

This would make $\lambda_N \approx ♌\, 17;55°$, not big enough an error to be easily detected.

C. The 25-year Cycle and the Epoch Dates

As we have seen the computational rules of P. Ryl. 27 are based on anomalistic periods of length D = 3 C, 32^d shorter than one 25-year cycle.[1] Table 6 displays the beginning dates of these groups: in R, Sect. 1 and 2, starting with the epoch date Augustus − 2 XI 5, in Sect. 3 and 4 computed back from the new epoch date Nero 14 X 23. The preserved group in L began with Nero 6 VII 2 and from this date the earlier and later column headings can be easily determined.

For the 25-year cycle itself we have initial dates listed in P. Carlsberg 9, ranging from Tiberius 6 to Antonius 7.[2] Extending this pattern back brings us to the new year's day Augustus − 1 I 1 that follows the epoch date of R, Sect. 1, 2. We cannot tell whether this is purely accidental or not.

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[4] The text in Section 2 of R has for Ω_0 the number 12;12, ... which would lead here to 0;23, ... for Ω_1'. Obviously dittography 12,12 is a more plausible scribal error than 21 for 23.

[1] Cf. above p. 810 (9).

[2] The year 2 for Vespasian is restored according to the norm of the "Ptolemaic Canon" (cf. e.g., Kubitschek, Zeitr., p. 62) which makes the 10 years of Vespasian directly following Nero 14 (cf. also here text L, above p. 816, Table 6), ignoring the interregnum of one year; consequently Titus is given 3 years instead of 2. (The same norm is also found in Vettius Valens I, 19 ed. Kroll, p. 32/33.) In our editions of P. Carlsberg 9 (cf. above p. 809, n. 3) we followed the historical count (cf., e.g., Preisigke, Wörterbuch der griechischen Papyrusurkunden III, p. 45) by restoring Vespasian year 1 for A.D. 69/70.

Table 6

P. Ryl. 27

Sect. 1, 2					Sect. 3, 4				
Nabon.	Aug.	Eg.	julian	regnal y.	Nabon.	Aug.	Eg.	julian	regnal y.
716	−2	XI 5	−31 July 1		716	−2	II 26	−32 Oct. 25	
741	+23	X 3	−6 May 24		741	+23	I 24	−7 Sept. 17	
766	48	IX 1	+19 Apr. 16		765	47	XII 27	+18 Aug. 10	
791	73	VII 29	44 March 6		790	72	XI 25	43 July 3	
816	98	VI 27	69 Jan. 29	Vesp. 1	815	97	X 23	68 May 25	Nero 14
841	123	V 25	93 Dec. 22						
866	148	IV 23	118 Nov. 14						
891	173	III 21	143 Oct. 7						
916	198	II 19	168 Aug. 29	Comm. 9					
941	223	I 17	193 July 22	Severus 2					
965	247	XII 20	218 June 14	Elag. 1					
990	272	XI 18	243 May 7	Gord. 6					
1015	297	X 16	268 March 29						

P. Lund Inv. 35a					P. Carlsbg. 9				
Nabon.	Aug.	Eg.	julian	regnal. y.	Nabon.	Aug.	Eg.	julian	regnal y.
707	−11	XI 10	−40 July 8		717	−1	I 1	−31 Aug. 31	
732	+14	X 8	−15 May 31		742	+24		−6 25	
757	39	IX 6	+10 Apr. 23		767	49		+19 19	Tib. 6
782	64	VIII 4	35 March 16	Tib. 21	792	74		44 12	
807	89	VII 2	60 Febr. 6	Nero 6	817	99		69 6	Vesp. [2]
832	114	V 30	84 Dec. 29	Dom. 4	842	124		94 July 31	Dom. 14
857	139	IV 28	109 Nov. 21	Traj. 12	867	149		119 25	Hadr. 3
					892	174		144 18	Ant. 7

Finally we have a Greek papyrus from the early second century B.C., P. Ryl. Inv. 666[3] where it is stated that the first year of a cycle coincides with the first year of Philometor which is the year Nabonassar 568 that begins with Thoth 1 = −180 Oct. 7.[4] If we add 6 cycles, i.e. 150 years, we find Nabonassar 718 = Augustus 0 as the first year of a cycle, i.e. one year later than the cycles of P. Carlsberg 9. Obviously the evidence from our four papyri, Greek and demotic, does not support the hypothesis of the existence of a well established numbering of the 25-year cycles during the Ptolemaic and Roman period.

Since it is a characteristic feature of the 25-year cycle to restore syzygies, because 25 years = 9125 days are very nearly 309 mean synodic months,[5] one may expect that its association with the determination of lunar longitudes and latitudes in the rules given in R is made in such a fashion that either new- or full-

[3] Published Turner-Neugebauer [1949]; now called P. Ryl. 589 (Vol. IV, p. 56–62).

[4] This counting of the years of Philometor from −180/179 is confirmed by the date of the lunar eclipse −173 Apr. 30/May 1 given as Philometor 7 VII 27/28 in the Almagest (VI, 5 Manitius, p. 350, 7).

[5] Cf., e.g., the tables for syzygies in Alm. VI, 3 (above I B 6, 1).

moons are selected at endpoints of the 25-year groups. Comparison of the dates in P. Carlsberg 9 with modern computations shows[6] that indeed new moon dates coincide with the starting points of 25-year periods, always associated also with the Egyptian Thoth 1.

P. Ryl. Inv. 666 states explicitly the interest in new moons[7] but nevertheless its cycles are counted in regnal years of Philometor which begin with the second year of the Carlsberg pattern, such that not Thoth 1 but Thoth 20 (−180 Oct. 26) is its first new moon's date.

In R the first new year's day after epoch is Augustus −111 which is a new moon's day according to the Carlsberg scheme (cf. Table 6). No obvious relation to new moons seems to exist, however, for the second epoch in R or for L. It is perhaps only accidental that the epochs in L (but not in R) are full moon's days (e.g. −40 March 2).

D. India

1. The Tamil Tradition. Indian sources, the classical Sanskrit works and the Tamil oral tradition, supplement our knowledge of arithmetical methods otherwise accessible to us in fragmentary fashion through cuneiform texts and Greek papyri. The latter present us with an adaptation of Babylonian methods to the calendaric system of hellenistic astronomy, i.e. to the use of Egyptian years and the corresponding 25-year cycle. In India the basis of astronomical time reckoning is the number of days since epoch, the ahargaṇa ("sum of days"), i.e. in principle the idea underlying the reckoning with "julian days." Counting by Egyptian years, however, lies outside the framework of Indian astronomy.

From the Greek papyri we know of two anomalistic periods of the moon[1]

$$\begin{aligned} A &= 248^d = 9 \cdot 27;33,20^d, \\ C &= 3031^d = 110 \cdot 27;33,16,22,\ldots^d. \end{aligned} \tag{1}$$

The Indian material[2] provides us with another composite period

$$E = 4C + A = 12372^d = 449 \cdot 27;33,16,26,11,\ldots^d \tag{2}$$

which is very close to the Babylonian parameter

$$1 \text{ anom. m.} = 27;33,16,26,57,\ldots^d \tag{3}$$

obtainable for System B of the lunar theory.[3] It seems likely that this period E is a product of hellenistic astronomy.

In order to determine the longitude of the moon at a given day one has first to find the number n_a of days elapsed since the moon was at an apogee of longitude λ_0 at a certain epoch date. Exactly as in the Greek texts we are given longitudinal

[6] Neugebauer-Volten [1938], p. 401.
[7] Cf. Turner-Neugebauer [1949], p. 86.
[1] Cf. above p. 810.
[2] Neugebauer [1952], p. 261 f. The period A is also attested in the Pañca-Siddhāntikā II, 2–6 and VIII, 5, the period C in the Khandakhādyaka (Chatterjee I, p. 73; Sengupta IX, 5, p. 140); the parameter 27;33,20 in al-Farghānī, Diff. sci. astr., Chap. 17 (ed. Carmody, p. 31).
[3] By multiplication of 29;31,50,8,20^d (cf. above p. 483 (3)) with the ratio 4,11/4,29 (cf. p. 482 (3)).

increments $\Delta\lambda$ which correspond to the three anomalistic periods

	Tamil	Greek
A	27;44, 6°	27;43,24,56°
C	337;31, 1	337;31,19, 7
E	297;48,10	[297;48,41,24]

(4)

If

$$n_a = a\cdot \mathrm{A} + c\cdot \mathrm{C} + e\cdot \mathrm{E} + \delta, \quad 0 \leqq \delta < 248^d \tag{5a}$$

the progress of the moon will be

$$\Delta\lambda = a\,\Delta\lambda_A + c\,\Delta\lambda_C + e\,\Delta\lambda_E \tag{5b}$$

as far as the motion between apogees is concerned. For the remaining δ days a fixed sinusoidal pattern of lunar motion, beginning at minimum velocity for $\delta=0$, is assumed which enables the computer to directly determine the additional motion. Since our Greek sources do not provide us with information for the lunar motion beyond the position of the mean moon at apogee we shall not discuss the details of the Indian procedure any further.[4]

For the latitudes the Indian procedure is in so far different from the Greek one as one directly finds the position of the ascending node for the given moment which falls n_b days after a certain epoch of known longitude of the ascending node. The computation follows a rule which can be expressed in the following formula

$$\Delta\lambda = 360^\circ - \left(n - \frac{9n}{169\,809}\right)\frac{30^\circ}{566} \tag{6a}$$

which is the equivalent of

$$\Delta\lambda = 360^\circ - n\left(1 - \frac{1,0}{5,14,27,40}\right) 0;3,10,48,45,\ldots^\circ. \tag{6b}$$

Here we see that a nodal motion of $-0;3,10,48,45,\ldots^{\circ/d}$ had been assumed, very near the hellenistic parameter[5] $-0;3,10,49,15^{\circ/d}$. If one applies the minute correcting factor in the parenthesis the motion becomes $-0;3,10,48,9,\ldots^{\circ/d}$.

Both for the longitudes of the moon and for the nodes epoch dates were chosen near to what we must consider the time of the last codification of these ancient methods. Instead of counting the ahargaṇa from the traditional beginning of the Kaliyuga (−3101 Febr. 18) one added $1\,600\,984^d$ in order to reach a lunar apogee[6] and $1\,600\,066^d$ to obtain a lunar position at the ascending node. Hence in julian days[7]:

$$\begin{aligned} 588\,466 + 1\,600\,984 &= 2\,189\,450 = 1281 \text{ May } 22 \\ 588\,466 + 1\,600\,066 &= 2\,188\,532 = 1279 \text{ Nov. } 16. \end{aligned} \tag{7}$$

[4] Details can be found in Neugebauer [1952] and van der Waerden [1956] (who eventually retracted (p. 222, p. 230) his unfortunate assumption of an equant model).

[5] Cf. above p. 810 (8).

[6] This has been realized first by van der Waerden [1956].

[7] Cf. below p. 1066.

The corresponding positions are given as

$$\lambda_a = 212;0,7^\circ, \quad \lambda_b = 0;40^\circ. \tag{8}$$

Modern computation gives

$$\begin{aligned} &\lambda_a \approx 224;30^\circ \quad \text{anomaly } \alpha = 0^\circ \\ &\lambda_b \approx \ 14;50^\circ \quad \text{node} \approx 12;40^\circ \text{ thus } \omega \approx 0. \end{aligned} \tag{9}$$

The difference of about 12° between modern longitudes and the given epoch parameters can be explained by precession,[8] since sidereal coordinates are used in Indian astronomy.

Table 7

ω	Sin ω	$\beta = 4;30^\circ \sin\omega$	ω	$4;30^\circ \sin\omega$	β Tamil	ω	$4;30^\circ \sin\omega$	β Tamil
3;45°	3,45	0;17,40,12	1°	0; 4,42,43	0; 4,43°	10°	0;46,53, 5	0;46,53°
7;30	7,29	0;35,15,42	2	0; 9,25,26	0; 9,29(!)	11	0;51,32, 3	0;51,32
11;15	11,11	0;52,41,47	3	0;14, 8,10	0;14, 8	12	0;56, 8,10	0;56, 8
15; 0	14,50	1; 9,53,44	4	0;18,50,34	0;18,51	13	1; 0,43,21	1; 0,43
18;45	18,25	1;26,46,48	5	0;23,32, 2	0;23,32	14	1; 5,18,32	1; 5,19
			6	0;28,13,30	0;28,14	15	1; 9,53,43	1; 9,54
			7	0;32,54,58	0;32,55	16	1;14,23,52	1;14,24
			8	0;37,35,11	0;37,40(!)	17	1;18,54, 2	1;18,54
			9	0;42,14, 9	0;42,19(!)	18	1;23,24,11	1;23,24

In order to compute the latitude of the moon one must first determine the distance of the moon from the node, a problem completely solvable by means of the preceding rules. A fixed pattern leads finally to the corresponding latitude. Accidentally the sources at our disposal concern only the computation of eclipses and for this reason we know the pattern in question only for nodal distances for integer degrees from 1° to 18°.[9] Our Table 7 reveals the arithmetical basis on which this pattern was built: first were computed the values for 4;30 sin α with the Indian table of sines[10] which gives the values for $\alpha = 3;45^\circ$, 7;30°, 11;15°, 15°, and 18;45°. The required values for integer arguments are then obtained simply by linear interpolation. The basic values are again known to us from hellenistic astronomy as the "steps" of 15°, modified here to intervals for sines from the original chords.[11]

2. Appendix: Modern Exploration. The rapid development of Kepler-Newtonian astronomy greatly stimulated the search for accurate methods for the determination of distances within our planetary system. It was Edmund Halley who, in 1691, first pointed out[1] that transits of Venus could be utilized for a much

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[8] Cf. van der Waerden [1956], p. 225.

[9] Reproduced from Warren (Kala Sankalita, p. 342) and Le Gentil (Mém. 1772 II, p. 238) by van der Waerden [1956], p. 228.

[10] The values of Sin $\omega = R \sin\omega$ are based on $R = 3438' = 57;18^\circ$ such that $2R\pi = 6,0^\circ$ (cf. below p. 1129). Cf. Āryabhaṭīya I, 10 ($\approx$ A.D. 500) or the "modern" Sūrya Siddhānta II, 15–17 (12th cent.); $i = 4;30^\circ$ is commonly assumed in Indian astronomy for the inclination of the lunar orbit.

[11] Cf. above p. 671.

[1] Halley [1691].

more accurate measurement of the solar parallax than hitherto known. The first transits following Halley's suggestions were those of 1761 and 1769.[2] Astronomers the world over prepared for the event. India was chosen by the French as the basis for their observations and Le Gentil, Cassini's assistant at the newly founded observatory of Paris, was appointed to the arduous task. He left France in 1760 for the French possessions on the Coromandel Coast. After a dangerous circumnavigation of Africa and being plagued by severe illness he learned upon his arrival at Mahe that the British had taken and burned Pondicherry (1761). Thus he had to return to Mauritius and Madagascar, though determined to wait for the transit of 1769. He therefore moved on to the East Indies and the Philippines. Here he spent the next years, returning only in March 1768 to Pondicherry after the Seven Years War had ended (1763). The British of Madras put their best telescope at Le Gentil's disposal and everything seemed promising for the observation of the transit on June 3 of 1769 when a sudden cloudiness obscured the sun for the duration of the transit. After 11 years of absence Le Gentil returned in 1771 to Paris.[3]

While waiting in Pondicherry for the transit of 1769 Le Gentil tried to gather information about native astronomy. There seemed to have existed some rivalry between the Hindu Brahmins and the native Tamil population, the Brahmins trying to conceal the existence of a Tamil tradition of astronomy beside their own. Nevertheless Le Gentil eventually contacted a Tamil who was versed in the astronomical methods of his people. With the help of an interpreter he succeeded in having computed for him the circumstances of the lunar eclipse[4] of 1765 August 30 which he himself had observed and checked against the best tables of his time, the tables of Tobias Mayer.[5] The Tamil method gave the duration of the eclipse 41 seconds too short, the tables of Mayer 1 minute 8 seconds too long; for the totality the Tamil was 7 minutes 48 seconds too short, Mayer 25 seconds too long.[6] These results of the Tamil astronomer were even more amazing as they were obtained by computing with shells on the basis of memorized tables and without any aid of theory. Le Gentil says about these computations[7]: "Ils font leur calculs astronomiques avec une vitesse et une facilité singulière, sans plume et sans crayon; ils y supléent par des cauris (espèce de coquilles) qu'ils rangent sur une table, comme nos jetons, et le plus souvent par terre. Cette méthode de calculer m'a paru avoir son avantage, en ce qu'elle est bien plus prompte et plus expéditive que la nôtre ..." It is indeed a plëasant idea to see the members of the Academie Royale compute on the floor with cauri shells.

[2] For the irregular sequences of Venus transits cf. above p. 228f.

[3] Map and vista of Pondicherry in Le Gentil, Voyage I Pl. 7 and 5.

[4] The existence of methods to predict lunar eclipses had to be known long before Le Gentil. In a letter from Pondicherry, written in Jan. 1709 by "pere de la Lane, missionaire de la Compagnie de Jésus" [1669–1746] we are told "les Brames ont les tables des anciens astronomes pour calculer les éclipses, et il savent même s'en servir. ... Il y a toujours quelque Brame qui s'applique à comprendre l'usage de ces tables; il l'enseigne ensuite à ces enfans, et ainsi par une espèce de tradition, ces tables ont été transmises des pères aux enfans, et on a conservé l'usage qu'il en falloit faire." [Lettres édifiantes et curieuses, X Recueil, Paris 1713, p. 36/37; nouvelle edition t. 6, Paris 1819, p. 378/379.]

[5] Novae tabulae motuum Solis et Lunae. Commentarii Societatis Scientiarum Göttingensis, Vol. 2 (1752).

[6] Le Gentil, Mém. 1772, II, p. 248.

[7] Le Gentil, Voyage I, p. 215 (= Mém. 1772, II, p. 174).

About half a century after Le Gentil another contact with Tamil astronomy was made, this time under British auspices. In 1825 Colonel John Warren[8] published a book in Madras of over 500 quarto pages on Indian time reckoning, called Kala Sankalita. Warren had heard[9] about the remarkable achievements of the Tamil "calendar makers" and succeeded in finding one of them in Pondicherry. Le Gentil's experiences were repeated almost identically. The lunar eclipse of 1825 June 1 was the test case and the results were as good as one could expect,[10] the error for its middle being only 23 minutes.[11] For us the main interest lies in an independent check of the methods, Warren's report being, in a few instances, more accurate than Le Gentil's who condensed a few minor steps into one.[12] There appear also some small deviations in the numerical values quoted by the natives,[13] but on the whole excellent agreement exists.

The information gathered by Le Gentil and Warren about astronomy in India was neither the only information nor the earliest. The successes of first the Genoese and then of the Dutch and the British in their attempts to establish themselves in India caused Louis XIV to try the same. India itself being no longer inviting because of too manifest a competition, the French efforts were concentrated upon Siam (the modern Thailand).[14] Splendid embassies were exchanged which contributed to the glory of the court. It was from these embassies that M. de La Loubère, the emissary of Louis XIV in 1687 to Siam, brought back the first records of Hindu astronomy, dealing with the computation of solar and lunar positions. The authenticity of these records is secured by the fact that their meaning was unintelligible to the Jesuit fathers who wrote them down while in Siam. It was first Cassini[15] who penetrated the purpose of these procedures and explained them in La Loubère's book and again in the Mémoires of the French academy.[16] Though these texts are much less complete than the Tamil records

[8] Warren himself was French [1769–1830]; cf. the biography in R. H. Phillimore, Historical Records of the Survey of India, Vol. II (1950), p. 449–453. I owe the reference to this publication to Mr. D.S. Jonah, Librarian of Brown University.

[9] He once mentions Le Gentil (preface p. II).

[10] The eclipse in question is a very small partial one; Oppolzer, Canon No. 4690, magnitude 0.3.

[11] Warren, p. 347; also Neugebauer [1952], p. 253.

[12] E.g. in the determination of the position of the ascending node.

[13] Warren, Tables p. 33; Le Gentil [1772, II], p. 226. Warren Tables p. 62 (XLVII); Le Gentil [1772, II], p. 232.

[14] From observations listed in Souciet, Observations ... II, p. 287 one can deduce that "Siam" has the coordinates of about 100;50° east of Greenwich and $\varphi \approx 14;20°$. For early maps see La Loubère, I, p. 4/5.

[15] Giovanni Domenico Cassini (1625–1712) had become famous by his discovery of four satellites of Saturn, of the axial rotation of the sun and of Jupiter, etc. (cf. Arago, Œuvres, Vol. 3, p. 315–318 and Tables p. 41–43). On the other hand he stubbornly tried to replace Kepler's elliptic orbits by curves now called "Cassini ovals" (cf., e.g., G. Loria, Spezielle algebraische und transcendente Kurven, (2) Leipzig 1910, p. 208).

[16] Cassini, Giovanni Domenico: Règles de l'astronomie Siamoise, pour calculer les mouvemens du Soleil et de la Lune, traduites du Siamois, et depuis examinées et expliquées par M. Cassini de l'Académie Royale des Sciences. [Published in: La Loubère, Du Royaume de Siam ... II, p. 141–294. Summary in Acta Eruditorum anno 1692 publicata, p. 488–491. Independent version, published three times:]

Règles de l'astronomie Indienne pour calculer les mouvemens du soleil et de la lune, expliquées et examinées par Monsieur Cassini de l'Académie Royale des Sciences. [Published as 7th treatise (64 pp.) in] Recueil d'observations faites en plusieurs voyages par ordre de sa majesté, pour perfectionner

(Continuation of footnote on p. 822)

one can easily see that they belong to a different school; it suffices to mention the distinction between mean motion and anomaly in a form typical for the "trigonometric" methods. We have here the first contact with the school of thought for which the Sūrya Siddhānta is the main representative; its prototype is again a hellenistic theory, this time of eccenters and epicycles.

E. PSI 1493

Closely related to the methods described in the preceding sections is a papyrus of the second century (judging from the writing), now in the collection of the University of Florence (to be published eventually as PSI 1493). This text is, however, in so far different from the material discussed here as it does not concern any specific date but constitutes a template for the anomalistic lunar motion, obviously based on the 248 day period known from R and L.[1]

The extant text is a fragment of a table of which only the end of one column and four subsequent columns are preserved. I reproduce here in Table 8 only one group of three related columns (III to V of the preserved fragment) and the corresponding differences. Δ IV shows that we are dealing with the daily motion of the moon, varying between about $11;41^{\circ/d}$ and $14;40^{\circ\, d}$ in essentially linear fashion. Computing accurately this would lead to a constant second difference of about 0;12,59. Our column $\Delta\Delta$ IV shows indeed in most cases the difference 0;13 with some irregularities of 0;12 or 0;14. If one computes Δ V one finds again what was meant to be a linear zigzag function exactly in phase with Δ IV. The units in column IV are obviously degrees, in column V hours since the limits are 0 and 24. If one converts degrees to hours one finds from the extrema in Δ IV the extrema of Δ V:

$$11;41^\circ = 0;46,44^h \approx 0;47^h$$
$$14;40^\circ = 0;58,40^h \approx 0;59^h.$$

Column III counts integer days. Hence it is easy to determine, at least approximately from a minimum value, e.g. in day 111, the values in day 1. One finds that both columns IV and V must have started at day 1 with zero and with minimum velocity.

This result explains the structure of the table: in each group of three connected columns the first counts the days, beginning with 1 and ending with 248. The next column gives the longitudes, counted from 0° at day 1, being the result of a velocity function that is a linear zigzag function — in good Babylonian tradition.[2]

(Footnote 16 — continued)
l'astronomie et la geographie. Avec divers Traitez Astronomiques. Par Messieurs de l'Academie Royale des Sciences. A Paris, l'imprimerie royale, 1693.

Regles de l'astronomie Indienne, pour calculer les mouvemens du soleil et de la lune, Expliquées et examinées Par Monsieur Cassini, de l'academie royale des sciences. Memoires de l'academie royale des sciences. Depuis 1666. jusqu'a 1699. Tome 8. A Paris ... 1730, p. 211–299.

Règles de l'astronomie Indienne pour calculer les mouvemens du soleil et de la lune, expliquées et examinées Par M. Cassini. Memoires de l'academie royale des sciences, contenant les ouvrages adoptez par cette academie Avant son Renouvellement en 1699. Tome 5. Divers ouvrages d'astronomie. Par M. Cassini. A La Haye, ..., 1731, p. 279–362.

[1] Cf. above p. 810 (3). The text probably contained 10 triple columns, each of about 25 lines (the lines are not very carefully spaced).

[2] Cf., e.g., ACT, p. 58.

Table 8

					IV		V
	III	IV	V		Δ	ΔΔ	Δ
1.	105	307;41	20;53	1.			
		320;31	21;44		12;50	0;12	0;51
		333; 7	22;35		12;36	0;14	0;51
		345;30	23;24		12;23	0;13	0;49
5.		357;41	0;13	5.	12;11	0;12	0;49
	110	9;39	1; 1		11;58	0;13	0;48
		21;2[4]	1;49		11;45	0;13	0;48
		33;16	2;36		11;52		0;47
		45;21	3;25		12; 5	0;13	0;49
10.		57;3[9]	4;14	10.	12;18	0;13	0;49
	115	70;1[0]	5; 5		12;31	0;13	0;51
		82;53	5;56		12;43	0;12	0;51
		95;50	6;48		12;57	0;14	0;52
		1[08;]59	7;40		13; 9	0;12	0;52
15.		1[22;]21	8;34	15.	13;22	0;13	0;54
	120	1[35;]56	9;29		13;35	0;13	0;55
		149;43	10;24		13;47	0;12	0;55
		163;44	11;20		14; 1	0;14	0;56
		177;57	12;17		14;13	0;12	0;57
20.		192;23	13;15	20.	14;26	0;13	0;58
	125	207;[3]	14;14		14;40	0;14	0;59
		221;[29]	15;12		14;26	0;14	0;58
		23[5;]42	[16;]9		14;13	0;13	0;57
		249;42	17; 5		14; 0	0;13	0;56
25.		263;30	18; 1	25.	13;48	0;12	0;56

PSI 1493

The third column, however, is not attested elsewhere. Its numbers express in hours and minutes the delay of the moon's return to the sidereal position the moon had at day 1 at 0^h, the daily amount being proportional to the moon's velocity at this day. By introducing the solar motion this delay could be easily transformed into the delay of the moon's culmination or into the delay of moon rise by considering the length of daylight. As the text stands, however, it is simply a template for the anomalistic motion of the moon and of the delay of its sidereal returns.

2. Vettius Valens

It will be obvious to anyone who tries to intelligently read the astronomical chapters of Book I of the "Anthology" that whole sections are out of place, or misunderstood and badly formulated. Among the worst of these sections is Chap. 19 that pretends to reproduce a Hipparchian method for the determination of the lunar longitude.[1] That the text as we have it could be representative of Hipparchus' astronomy is, of course, out of the question. All we can say with

[1] Cf. below p. 824 f.

some confidence is that the rules concerning the longitudes of the moon and the planetary methods described above[2] are basically of the same type,[3] operating mainly with mean motions and rather crude periodicities. Also a certain tendency to present supposedly "handy" procedures in an unnecessarily devious fashion is common to several sections. It would be interesting to know the milieu in which these texts originated.

A. Lunar Longitudes and Phases

Book I Chap. 15 concerns only the moon. Suppose we know the lunar longitude on a certain day (e.g. $\lambda_{☾} = ♏\, 7^1$); then we are told that the moon on the 3rd day is one sign ahead, a quadrant on the 7th day, and 180° on the 14th day.[2] If we take the "3rd day" to mean two complete days motion, i.e. $\approx 26;20°$ motion in longitude, and $7 \cdot 13;10 \approx 92°$, $14 \cdot 13;10 \approx 184°$ then the text is sufficiently correct in view of the low requirements in a section of mainly astrological purpose.

The moon's elongation from the sun is brought into play in the Chap. I, 13, I, 9, and I, 19, making the simple assumption that the elongation reaches 180° in 15 days, i.e. that its daily increment is 12°. Chap. I, 13 gives a direct consequence of this assumption: the daily delay of the moon-set with respect to sunset must be $12^h/15^d = 0;48^{h/d}$.[3]

Chap. I, 9 contains another obvious application of the norm of $12^{\circ/d}$ for the increment of the elongation. If for a given date the longitudes of both sun and moon are known, hence also the elongation, one can immediately find the date of the nearest conjunction or opposition. Since the intervals to be bridged can only amount to a few days one finds quite correct dates in the examples given in the text.[4]

Chap. I, 19 takes a more sophisticated attitude; the longitude of the moon can be found from the longitude of the sun if the dates of the straddling syzygies can be found independently. The latter is indeed possible, at least in principle, by starting from any known conjunction and progressing in steps of 19 years, i.e. in steps that contain very nearly an integer number of lunations. Having found the 19-year interval into which the date of the given solar position falls one can proceed from the conjunction at the beginning of this interval to the conjunction nearest to the given date; hence one also knows the time interval that provides us with the amount of the corresponding elongation, hence with $\lambda_{☾}$.

The numerical execution of this simple idea (which is ascribed to "Hipparchus") is needlessly complicated. We are only interested in the number δ of days between the given civil date and the preceding conjunction. All one had to do would be to add up the number of days which correspond to the years, months, and days elapsed within the present 19-year cycle and to reduce the total modulo $s = 29;30^d$,

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[2] Cf. V A 1, 3 B and 3 C.

[3] A remark in the section on Venus explicitly refers to the procedure for the moon (Kroll, p. 34, 31).

[1] This is the position which appears in several examples from the year Hadrian 4; cf. above Table 2 (p. 793) and below p. 827, n. 5.

[2] The text has instead a meaningless "40th day."

[3] Cf. also below p. 830.

[4] One example which takes its departure from 114 July 26 (as we know from a parallel in Book VII, 5) finds correctly for the preceding conjunction July 19 and for the subsequent opposition August 4/5. The second example concerns again 120 Feb. 8 (cf. above note 1); the preceding opposition falls on Feb. 2 and the following conjunction on Feb. 16/17.

assumed for the length of a synodic month. Instead the text treats each component of the civil date separately, a procedure which causes all the complications.

It would be out of place to discuss the errors and variants in the extant manuscripts and to compare them in detail with the simple mathematical consequences of the previously mentioned basic idea. I shall therefore develop first the proper formulae and then compare the result with the rules given in the text.

Let t be the number of completed egyptian years within the 19-year cycle into which the given date falls. Let

$$e=365^d, \quad c=30^d, \quad s=29;30^d \tag{1}$$

be the basic time intervals in question and determine two integers α and β such that

$$t=\alpha\cdot 3+\beta, \quad 0<\beta\leqq 3 \quad (0\leqq t<19). \tag{2}$$

Since $e=12s+11$ we have

$$t\cdot e=12ts+10t+t\equiv\alpha c+10\beta+t \quad \text{mod. } s.$$

Furthermore, because $c=s+1/2$, the number m of completed months beyond the t years contributes $m/2$ days mod. s. The number δ of days since the last conjunction is therefore

$$\text{for } \beta=\begin{cases}1\\2\\3\end{cases} \quad t+\begin{cases}10\\20+1/2(m+\\0\end{cases}\begin{cases}\alpha)\\\alpha)\\\alpha+1)\end{cases} \quad +d\equiv\delta \quad \text{mod. } s. \tag{3}$$

During these δ days the moon has supposedly obtained an elongation of 12δ from the sun. Thus, if $\lambda_\odot$ is the sun's longitude at the given date the longitude of the moon will be

$$\lambda_{☾}=\lambda_\odot+12\delta. \tag{4}$$

This solves our problem.

The text gives essentially the rule (3) with the exception that the contribution of α to the number of 30-day intervals is ignored. This is a modest error since $\alpha/2$ is at the most 3. Furthermore the elongation is only expressed in zodiacal signs, not in degrees.[5] Since the elongation increases 30° in 2;30 days the last term in (4) would lead to a contribution of $\delta/2;30$ signs. The text mistakenly multiplies by 2;30 and subtracts the product from $\lambda_\odot$.

Total chaos is produced by the reduction modulo 19 of the years of the era Augustus. At the end of Chap. 19 one finds a royal canon which gives the regnal years of each ruler from Augustus to Gordianus and Philip (i.e. to almost A.D. 250, a century beyond Vettius Valens). Instead of giving the totals in the era Augustus and reducing them modulo 19 each individual interval is reduced modulo 19 and some of the longer ones modulo 30 which makes no sense. Hence the values of t obtained from this canon are meaningless. Nevertheless an example computed for Hadrian 3 (=Augustus 148) III(a) 28 (=A.D. 118 Nov. 24) gives $\lambda_\odot=$ ♐, $\lambda_{☾}=$ ♍, $\delta=24\ 1/2^d$ while in fact $\lambda_\odot=$ ♐ 2;10, $\lambda_{☾}\approx$ ♍ 10, preceding new moon = 118 Nov. 1 (in ♏ 7;50) thus $\delta=23^d$.

The text also discusses the inverse problem of finding the day number d in the current month from the elongation, i.e. from the corresponding δ in (4).

[5] This is due to the predominantly astrological interest of our text. The emphasis on zodiacal signs is again a parallel to the planetary sections (cf. above V A 1, 3).

Obviously one then can find d by means of (3). Probably it is this inverse problem which introduced multiplication by 2;30 instead of division.

A short fragment from a collection of astrological treatises[6] seems to be related to the procedures described here. We are told to reduce modulo 19 the years of the era Augustus increased by 2; in other words Augustus -1 is introduced as epoch date. The reason becomes obvious if one compares the following

Augustus	1 I(e) 1 =	-29 August 31	New moon:	August 9
	0	-30 August 30		August 20
	-1	-31 August 31	=	August 31
	-2	-32 August 31		Sept. 11

The corresponding solar or lunar longitude is ♍ 5;17.

In the following, obviously corrupt, passages a factor (?) 3 is mentioned, perhaps in connection with the β of our formula (2), and subtraction "from ♋" then supposedly leads to the longitude of an "ecliptic node," i.e. to a situation when syzygies and nodes coincide. The reference to Cancer may have to do with the rules for finding the position of the ascending node discussed in the following pages.

B. Lunar Latitude

The Chap. 16 to 18 of the "Anthology" concern the determination of the position of the moon's ascending node. This problem is solved in two ways: either directly by multiplying the time since epoch by the nodal mean motion and subtracting the result from the position of the ascending node at epoch, or by using the argument of latitude in relation to the longitude of the moon at the given moment. This second procedure applies the archaic terminology of "steps" for angles and uses the northernmost point of the lunar orbit instead of the ascending node.

The direct method[1] is easy to understand. The following (retrograde) nodal motions are assumed

1 egypt. year	19;20°	
30 days	1;35	(1)
1 day	0;3.	

Similar parameters are found in P. Ryl. 27[2]: 0;3,10° for the daily motion and 1;35° for each month but only 19° for one year (perhaps by mistake). The values (1) are basically correct.[3]

As the nodal position at epoch, Augustus 1 I(e) 1 (= -29 August 31) is assumed ♋ 30 = ♌ 0°. The Canobic Inscription[4] gives for this day ♋ 25;31°, modern computation ♋ 26;0°.

[6] CCAG 8, 4, p. 239, 14–21.
[1] First section of Chap. 16, Kroll, p. 29, 25 to 30, 6.
[2] Section 5 (Neugebauer [1949, 3], p. 20f.); cf. above p. 813.
[3] Alm. IV, 4 would give 19;20,1°, 1;35,20°, 0;3,11°, respectively.
[4] Ptolemy, Opera II, p. 152, 12.

An example is computed for Hadrian 4 = Augustus 149 Phamenoth (VII) 19. In the Alexandrian calendar the equivalent julian date would be 120 March 18, in the Egyptian calendar February 8. A glance at Table 2 (p. 793) shows that we must use the second interpretation since it concerns the same date that had been used for Venus and Mercury (Nos. 6 and 7); this conclusion is confirmed in the next section[5] by the lunar longitude ♏ 7 that agrees only with Feb. 8. But no word in the text alerts the reader to a change in the calendaric norm amounting to 36 days.

The numerical example given in the text assumes for the time since epoch 148 years 6 months 19 days, hence the motions

$$148 \cdot 19;20 = 2862° \qquad \text{(accurately: } 2861;20°)$$
$$6 \cdot 1;35 + 19 \cdot 0;3 \approx 10° \qquad \text{(accurately: } 10;27°)$$

thus a total of

$$2872° \equiv 352° \text{ mod. } 360° \quad \text{(accurately: } 351;47°).$$

Therefore the longitude of the ascending node

$$\lambda_a = \text{♋}\,30 - 352° = \text{♌}\,8°. \tag{2}$$

Modern computation gives about ♌ 4;45°. The discrepancy is, of course, due to the use of sidereal coordinates.

The second method is not described in general terms but only exemplified numerically for the date of the preceding example, i.e. 120 Feb. 8. The extant text is badly garbled[6] but its mathematical contents can be restored as follows: let arcs be measured in "steps" (*βαθμοί*), i.e. such that[7]

$$1^{st} = 15°. \tag{3}$$

Then two intervals are introduced, one of

$$\Lambda_0 = 12;18^{st} = 184;30° \tag{4a}$$

and a second, 204 days "from epoch," of

$$\Lambda_1 = 11;37^{st} = 174;15°. \tag{4b}$$

Next their total is formed

$$\Delta\lambda = \Lambda_0 + \Lambda_1 = 23;55^{st} = 358;45° \tag{5}$$

and with it

$$\text{♌}\,0° + \Delta\lambda = \text{♋}\,28;45° \tag{6a}$$

and

$$\text{♋}\,28;45 - \text{♉}\,0° \approx 89°. \tag{6b}$$

Now it is supposedly known that the longitude of the moon at the given moment was

$$\lambda_{☾} = \text{♏}\,7° \tag{7}$$

[5] Cf. below (7). Other examples using the same data 120 Feb. 8 are found in I, 4 and III, 13, I, 15, I, 20. A very specific date is given in I, 10: Hadrian 4 = Augustus 149 VI(a) 13 1[h] night (Kroll, p. 26, 21 f.); similar in I, 4.

[6] Kroll, p. 30, 11–34; the chapter division is at the wrong place and a whole section is duplicated.

[7] Cf. above IV B 5.

as in the preceding examples.[8] This, then, gives the longitudes of the ascending node

$$\lambda_{\Omega} = \lambda_{☾} - 89° = ♌\ 8° \tag{8}$$

in agreement with the previously obtained result (2).[9]

The purpose of this procedure becomes clear when one condenses the preceding steps into a single formula. From (8) and (6 b) one obtains

$$\lambda_{\Omega} = \lambda_{☾} - ♋\ 28;45 + ♉\ 0°$$

hence with (6 a)

$$\lambda_{\Omega} = \lambda_{☾} - \Delta\lambda - (♌\ 0° - ♉\ 0°) = \lambda_{☾} - \Delta\lambda - 90°. \tag{9}$$

Now

$$\lambda_{\Omega} + 90° = \lambda_N \tag{10}$$

is the longitude of the northernmost point N of the lunar orbit and therefore, with (9),

$$\Delta\lambda = \lambda_{☾} - \lambda_N = \omega' \tag{11}$$

the "argument of latitude" with respect to N, well known not only from the Almagest[10] but also, e.g., from P. Ryl. 27.[11] Hence the second method tells us no more than that the ascending node can be found from the argument of latitude, assuming the moon's longitude known.

Our text is obviously incomplete because it does not give us any information about the mean motions of ω' during years, months, and days. All we have is the epoch value (4 a)

$$A_0 = \omega'_0 = 184;30°$$

and the increment (4 b) since epoch

$$A_1 = \omega'_1 = 174;15°.$$

To check these parameters we may, of course, assume that the mean motion of ω' is nearly the same as given in the Almagest (IV, 4).[12] Thus we can expect as epoch value for Augustus 1 I (a) 1 (= −29 Aug. 30)

$$\omega'_0 \approx 192;45°,$$

a value only 8;15° greater than in our text, corresponding to about 16^h motion. This difference could correspond to a shift from noon epoch (Almagest) to midnight epoch and to a minute change in the basic parameters.

The increment due to a time interval of 148 years, 6 months, 19 days = 54219 days would amount, according to the Almagest, to

$$\omega'_1 \approx 175;24°$$

hence only insignificantly different from the textual value 174;15°. Unexplained remains only the remark that this increment should have been obtained from a

[8] Cf. above p. 827, n. 5.

[9] The text gives also a "variant" of the relations (4 a) to (6 a), consisting, however, only of a trivial rearrangement of terms.

[10] Above p. 80.

[11] Above p. 810.

[12] From the parameters in P. Ryl. 27 (above p. 809 f. (1) and (8)) one finds a daily increment of about 13;13,45,41,15° as compared with 13;13,45,39,49° in Alm. IV, 4.

time interval of 204 days between epoch and the given date.[13] Indeed, according to Alm. IV, 4 the increment for 204 days is about 178;45°. Apparently the author of this section had nothing but the data (4a) and (4b) at his disposal and simply motivated his ω_1' by an interval Δt that would more or less lead to the desired result.[14]

§ 3. Visibility Problems

It is not surprising that the phenomena of rising and setting of the moon, of planets and stars, play a dominant role in early astronomy. They provide secure marks for the return of the seasons and thus acquire major calendaric significance – the rising of Sirius is the classical example. On the other end of this development we have the Greek attempts to relate meteorological data to the risings and settings of the fixed stars, theories handed down in the treatises on "phenomena,"[1] Ptolemy's "Phaseis" being the latest one preserved in Greek.[2]

It is an obvious empirical fact that nearness to the sun is the cause of invisibility of other celestial bodies. Numerical parameters were assigned at a relatively early date to the limits of invisibility in relation to the sun. The neat norm that a constant length of one sign of the zodiac (i.e. the interval $\lambda_{\odot} \pm 15°$) is removed from visibility is attested in Babylonian astronomy[3]; it appears also in the earliest extant Greek treatise on spherical astronomy "On the Risings and Settings" by Autolycus (around 300 B.C.) and still occupies a central position in Theodosius' "De Habitationibus"[4] (about 100 B.C.).

The 15°-limit remained for centuries the generally accepted estimate. We find it in the Eudoxus papyrus,[5] in P. Mich. 149,[6] as well as in the strictly astrological literature, e.g., in Serapion,[7] in Antiochus (or Rhetorius),[8] in Porphyry,[9] and Paulus Alexandrinus[10] – examples which could be readily multiplied.

1. Moon

The theory of first and last visibility of the moon represents the most refined section of Babylonian mathematical astronomy. That no trace of such a theory has been found in Greek astronomy (in spite of the Greek adherence to lunar calendars) is a strong argument in favor of the opinion that not much more than some basic parameters were transmitted from Mesopotamia to the West.

[13] Kroll, p. 30, 14.
[14] The existence of tables is alluded to in the text, Kroll, p. 30, 31.
[1] "Many wrote Phainomena": cf. Maass, Comm. Ar. rel., p. 324, No. 5.
[2] Cf. below V B 8, 1.
[3] For Ω and Γ of Mars; cf. above p. 411 (22b).
[4] Cf. above IV D 3, 4 and IV D 3, 3 A.
[5] Sect. 37 to 39 and 44 (Tannery, HAA, p. 290f.).
[6] X 25f., 40f.; XI 6f. (Mich. Pap. III, p. 75f.; p. 113).
[7] CCAG 8, 4, p. 227, 25f.
[8] CCAG 1, p. 145, 4–6.
[9] CCAG 5, 4, p. 193, 25; 194, 18f.; p. 228, 15f.
[10] Chap. 14 (ed. Boer, p. 29).

Of course, the general idea of arithmetical methods is undoubtedly of Babylonian origin and found wide application in early Greek astronomy. But nowhere do we meet any attempt of analyzing the influence of the different components of the lunar motion on lunar visibility.

In some sense the most advanced Greek attempt to cope with the variable conditions of the lunar visibility is a rule in Vettius Valens.[1] There it is stated that, approaching conjunction, the moon becomes invisible at the longitude

$$\lambda_{☾} = \lambda_{\odot} - 1/2\,\rho_i \qquad (1)$$

where ρ_i is the rising time of the i-th zodiacal sign to which $\lambda_{\odot}$ belongs. In the examples for this rule, given by Vettius Valens, the ρ_i are the rising times of System A for Babylon.[2] Since the mean value of all rising times is 30° we see that (1) is a certain refinement of the crude rule, mentioned before, that celestial objects become invisible at $\lambda_{\odot} \pm 15°$. That ecliptic arcs can be related to equator arcs and that visibility conditions depend on the variable inclination of the ecliptic to the horizon — these are insights underlying the Babylonian theory[3]; but the rule (1) is no more than a faint reflection of these basic theoretical facts.

Pliny says[4] that the visibility of the moon increases each day by $3/4 + 1/24$ of one hour (i.e. $19/24 = 0;47,30^h$). This is a simple estimate based on a schematic month of 30 days since a moonrise 12^h after sunrise would correspond to 15;9,... days after conjunction. Vettius Valens assumes[5] similarly an increment of $1/2 + 1/4 + 1/20$ hour per day (i.e. $4/5 = 0;48^h$) which leads to exactly 15 days for 12 hours delay. One obtains the same result by postulating a daily elongation of 12° which is also the equivalent of $0;48^h$ in time.[6] There is not the slightest evidence for a Babylonian origin of these estimates.

In N.H. XVIII, 324[7] Pliny gives the increment $10/12 + 1/48$ (i.e. $0;51,15^{h/d}$) that would lead to 12 hours delay in about 14;3 days. This makes no sense; it is probably a mistaken consequence of assuming 28 days for the duration of the moon's visibility.

2. Planets

The study of more or less random criteria for first and last visibility of the planets provides an excellent background for the evaluation of the progress made by Ptolemy by introducing the general concept of arcus visionis,[1] a concept that made it possible to establish individual conditions by means of geometric considerations which relate the planetary models to the horizon and the sun's position. In contrast most of the data found in the popular literature or in the

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[1] Anthology I, 14 (ed. Kroll, p. 28).

[2] Cf. above p. 368.

[3] Cf. above II B 10, 2.

[4] N.H. II, 58 (Budé II, p. 26).

[5] Anthol. I, 13 (ed. Kroll, p. 28, 6–18); cf. above p. 824.

[6] A. Rome in Bidez-Cumont, Mages II, p. 176–178 (from it, including misprints, van der Waerden [1951]; p. 27). It is probably the same limit of $12° = 0;48^h$ that underlies Pliny's remark N.H. XVIII, 219 that stars are visible only at least 3/4 of one hour (i.e. $0;45^h$) before sunrise or after sunset.

[7] Loeb V, p. 292/293.

[1] Cf. above I C 8, 2 and I C 8, 5.

astrological treatises have no theoretical background whatsoever and probably often enough not even an observational basis. It is, of course, not surprising to find the visibility limit of $\pm 15°$ for the fixed stars also applied to the planets[1a].

An exception with respect to empirical data is perhaps a remark by Pliny[2] concerning Mercury: evening risings (Ξ) are said to be rare in Pisces, frequent in Virgo; morning risings (Γ) are very rare in Leo, common in Aquarius.[3] Indeed the elongations of Ξ are near a minimum in Pisces, near a maximum in Virgo, whereas the elongations of Γ are low in Leo, high in Aquarius.[4]

Otherwise Pliny reports only crude estimates.[5] The moon should be at least 14° from the sun to be visible, whereas the planets are sometimes seen at only 7° elongation — which is taken as proof that the planets are larger than the moon. On the other hand the outer planets make their morning rising (Γ) at a distance of at least 11° from the sun.[6] The astrological papyrus P. Mich. 149 (of the second century A.D.) considers 15° elongation the limit of visibility common for all planets.[7]

Porphyry in his Introduction to the Tetrabiblos says[8] that the rays of the sun "reach" 15° forward and backward. Though he uses exactly the same expression also for the moon and the planets its meaning must be different: the $\pm 12°$ for the moon obviously refer to the elongation from the sun[9] and similarly for the planets:

$$\text{Saturn and Jupiter: } \pm 9°, \quad \text{Mars: } \pm 8°, \quad \text{Venus and Mercury: } \pm 7°. \tag{1}$$

The symmetry of this pattern makes one suspect numerological speculations. The same numbers are also interpreted as "sizes of the stars"[10] — gratuitously adding a size of 3° for the lunar nodes.[11]

Quite different parameters are found in Firmicus Maternus[12]:

$$\text{Saturn: } 15°, \quad \text{Jupiter: } 12°, \quad \text{Mars and Venus: } 8°, \quad \text{Mercury: } 18°. \tag{2}$$

The limits of 12° for Jupiter and 8° for Venus appear in astrological disguise also in the Tetrabiblos.[13] A scholion to Paulus[14] gives

$$\begin{array}{l} \text{outer planets: } \Omega \text{ at } 15°,\ \Gamma \text{ at } 15° \text{ or } 10° \\ \text{inner planets: } \Xi \text{ and } \Gamma \text{ at } 15°. \end{array} \tag{3}$$

1a Cf., e.g., P. Mich. 149 col. X, 25; Paulus Alex., p. 29, 12, ed. Boer; scholion quoted Neugebauer [1958, 2], p. 238, n. 1.

2 NH II, 77 (Budé II, p. 33).

3 For the text of the passage in question cf. Campbell (above p. 802, note 1) p. 97 ad § 77.

4 Cf. Fig. 250c (p. 1293).

5 NH II, 58 (Budé II, p. 26).

6 NH II, 59; cf. also Maass, Comm. Ar. re., p. 274, 1–3. The numbers 14° and 11° are probably only another formulation for the well-known elongations of 15° and 12° (e.g. CCAG 5, 4, p. 228, 15f.).

7 Cf. above p. 829, n. 6.

8 In a short section "On the rays of the planets" (CCAG 5, 4, p. 228, 14–19).

9 Cf., e.g., Cramer, Anecd. gr. Par. I, p. 372, 30f.: "one day from conjunction."

10 CCAG 5, 3, p. 130, 11–16. The latest evidence for the parameters (1) is the Quadrivium of Pachymeres (late 13th cent.) with the only difference that Jupiter is given 8° (ed. Tannery, p. 398).

11 Perhaps this is an oblique reference to the eclipse limit of 12° (cf., e.g., above p. 125f.) which is 3° less than the visibility limit of 15° mentioned at the beginning.

12 Math. II, 9 (ed. Kroll-Skutsch I, p. 51, 22–52, 8).

13 Boll-Boer: III, 11, p. 134, 7–9 and 20; Robbins: III, 10, p. 284/5.

14 No. 22, ed. Boer, p. 111.

Martianus Capella assigns 12° elongation to the visibility of Saturn and of the inner planets.[15]

A passage, supposedly from Rhetorius,[16] assumes for the outer planets a limit of 10°, both at Ω and Γ. For the inner planets he gives

$$\text{at}\quad \begin{array}{l}\Xi:\\ \Omega:\\ \Gamma:\\ \Sigma:\end{array}\quad \text{Venus:}\ \begin{array}{c}20^\circ\\ 6\\ 5\\ 2\end{array}\qquad \text{Mercury:}\ \begin{array}{c}3^\circ\\ 2\\ 3\\ 2.\end{array} \tag{4}$$

The elongations for Mercury and for Σ of Venus are obviously wrong.

The determination of the elongations that are associated with first and last visibility of a planet could have offered more than accidental interest because other important parameters of an epicyclic planetary model are readily associated with these elongations. Let τ be the limiting angle for visibility and σ the maximum elongation of an inner planet (cf. Fig. 13[17]), such that the radius r of the epicycle is given by $r = \operatorname{Sin}\sigma$. Consequently, if α_1 and α_2 are the epicyclic anomalies which measure the half-duration of the arcs of invisibility, then $\alpha_1 - \tau$ is the angle at Ξ and $180 - \alpha_2 - \tau$ the angle at Ω. Hence one finds for the triangles ΞCO and ΩCO

$$\operatorname{Sin}(\alpha_1 - \tau) = R\frac{\operatorname{Sin}\tau}{\operatorname{Sin}\sigma} = \operatorname{Sin}(\alpha_2 + \tau). \tag{5}$$

The mean motions in anomaly are known[18] thanks to the relations between sidereal and synodic revolutions; hence (5) provides us with the duration of invisibility at superior and inferior conjunctions.

For an outer planet we know that the radius $C\Gamma$ of the epicycle must be parallel to the direction $O\odot$ (cf. Fig. 14). Therefore we have in the triangle ΓCO

$$r = R\frac{\operatorname{Sin}(\alpha - \tau)}{\operatorname{Sin}\tau}. \tag{6}$$

The duration Δt of invisibility can be observed easily and quite accurately and since the mean daily motion v_p in anomaly is given one also knows $\alpha = 1/2\, v_p\, \Delta t$. Hence one can find the radius r of the epicycle as soon as one accepts an estimate for the elongation τ necessary for visibility, e.g. the traditionally assumed $\tau = 15°$.

Unfortunately we have only very few data for invisibility intervals from a pre-Almagest level. Pliny NH II, 78[19] mentions

Saturn and Mars:	160 to 170 days		
Jupiter:	26 to 36 days		(7)
Venus:	52 to 69 days (Maass: 52 or 168)		
Mercury:	13 to 17 days (Maass: 13 to 18).		

[15] VIII, 886, ed. Dick, p. 468, 6f.

[16] CCAG 7, p. 214–224.

[17] Fig. 13 is drawn to scale for $\sigma = 22°$, $\tau = 15°$, i.e. for the parameters of Mercury according to P. Mich. 149 (cf. above p. 804 (4) and above p. 831).

[18] Cf., e.g., above p. 806.

[19] To be supplemented by Maass, Comm. Ar. rel., p. 273 f.

Martianus Capella gives Venus more than 4 months of invisibility at superior conjunction and less than 20 days at inferior conjunction[20]; only the second parameter makes sense (for the longest invisibility between Ω and Γ). Pliny's values for Jupiter and for Venus (at superior conjunction) are practically correct and so are the 13 days for Mercury as minimum near inferior conjunction. The remaining numbers in (7) are badly garbled; there is, of course, no similarity possible between Saturn and Mars.

A duration of 13 days of invisibility of Mercury at inferior conjunction corresponds to $2\alpha_2 \approx 40;23°$. If we assume $\sigma = 22°$ as maximum elongation and $\alpha_2 \approx 20°$ one obtains from (5) $\tau \approx 11°$ as visibility limit.[21] Similarly one finds for Venus at superior conjunction limits τ between 7° and 9° from invisibilities between 52 and 69 days (with $\sigma = 48°$).

The invisibility of Jupiter during 26 or 36 days corresponds to an angle $\alpha \approx 11;45°$ and 16;15°, respectively for the anomaly at Γ. If one assumes for τ in the first case an angle of about 9;40°, in the second case $\tau = 13;15°$, one obtains values of r between 13 and 14, i.e. an epicycle of the size assumed in P. Mich. 149.[22] In other words, elongations ranging between about 10° and 13° would lead for Jupiter to correct radii for its epicycle.

From the foregoing it is clear that angles of elongations at first and last visibility as recorded in our sources could have provided, on the basis of (6), reasonably correct estimates for the radii of the epicycles of the outer planets. It should be noted, however, that the correct sense of rotation of the planet on the epicycle is an essential condition for obtaining the relation (6). Again, the model with incorrect epicyclic rotation is seen unfit for providing numerical data concerning planetary motion.[23]

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[20] VIII, 883 (ed. Dick, p. 466, 15). In the text the conjunctions are interchanged by mistake.

[21] The same relation (5) leads to $2\alpha_1 \approx 84°$, i.e. to an invisibility of about 27^d at superior conjunction, as is quite correct as lowest value.

[22] Cf. above p. 805 (1).

[23] Cf. above p. 808.

B. Ptolemy's Minor Works and Related Topics

§ 1. Biographical and Bibliographical Data

We are in the fortunate situation that almost the only source for a biography of Ptolemy is Ptolemy's work itself.

The Almagest is probably the earliest work of Ptolemy. It contains data for his own observations ranging from A.D. 124 to 141 (cf. Fig. 15). The "Canobic Inscription"[1] is dated to Antoninus 14[2], i.e. A.D. 146/7. These are the only secure data at our disposal.

Claudius indicates that Ptolemy was a Roman citizen, belonging to a family of some distinction. A misreading of the Arabic rendering of Claudius made him a native of Pelusium, an error correctly explained by Reiske in 1787 and by Buttmann in 1810[3] but still flourishing, e.g., with Duhem who speaks about "l'astronome de Péluse."[4] Some Greek and Arabic biographical remarks are of practically no value.[5] He is said to have been born in Ptolemais Hermeiou[6] and died at the age of 78 years, statements which can neither be confirmed nor disproved. Boll, weighing all available evidence, came to the conclusion ([1894], p. 64) that Ptolemy must have lived from about A.D. 100 to 178.

In late antiquity and in the Middle Ages he is often called "godlike" (θεῖος[7]) or made a king of Egypt.[8]

The chronological arrangement of Ptolemy's writings can be established within reasonable limits. The fundamental work is the "Almagest," followed soon after by the closely related "Canobic Inscription." The "Handy Tables" are a systematic continuation of the Almagest while the "Planetary Hypotheses,"

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1 Cf. below p. 913.
2 Opera II, p. 155, 3.
3 Reiske [1787]; Buttmann [1810], p. 483 f. Cf. Boll [1894], p. 58; Kunitzsch, Alm. p. 125–128.
4 SM I, p. 478 et passim.
5 Collected and carefully discussed by Boll [1894], p. 53 to 63; summary by van der Waerden RE 23, 2 col. 1788 to 1791.
6 In Upper Egypt near Akhmīm, now a "schmutziges Araberdorf" according to the Baedecker of 1913, p. 205. Cf. Boll, Kl. Schr., p. 143, n. 1.
7 The "Anonymous of 379" speaks about the *θειότατος Πτολεμαῖος* (CCAG 5, 1, p. 198, 4 f.; p. 204, 9). Not much later, about A.D. 400, are passages in Hephaistio (ed. Pingree I, p. 32, 10; p. 42, 24, etc.) and in the "Periplous" of Marcianus of Heracleia Pontica (Geogr. gr. min. I, p. 516, 16). Cf. also "Palchus" (CCAG 1, p. 80, 1) and Boll [1916], p. 77, 2.
8 Examples: Malalas (6th cent.) [Dindorf, p. 196, 13 f.; Spinka-Downey, p. 11, p. 34]; Isidorus Hisp. (7th cent.) [Etym. III, 26]; Gerard of Cremona (12th cent.) [Haskins SMS, p. 105]; Albertus Magnus (13th cent.) [CCAG 5, 1, p. 89, 11 f.]. Cf. also Buttmann [1810], p. 459–475.

which quote the Almagest,[10] have many details in common with the Handy Tables, e.g. the use of the Era Philip.[11]

All these works (excepting, of course, the Canobic Inscription) are dedicated to Syrus, a person otherwise unknown to us. To him are also dedicated the "Analemma" and the "Planisphaerium,"[12] as well as the "Tetrabiblos" which refers to the Almagest.[13] It seems plausible to consider all these writings the fruit of Ptolemy's earlier years, comprising all his strictly astronomical works if we add the "Phaseis" of which only Book II is preserved.[14]

The "Geography" certainly belongs to the later period of Ptolemy's life, although it appears to have been planned from the time he wrote the Almagest[15]. Schnabel [1950] investigated the sequence of Ptolemy's geographical studies and showed that the "*Ἔκθεσις*," which later became the core of the last book (VIII) of the Geography, was written before the list of the "Important Cities" from the Handy Tables,[16] all of which were later than the Tetrabiblos.

Besides these two major groups there remain both the "Optics" and the "Harmonics," works of highest level with lasting influence. They do not seem to contain definite clues to their dating but at least the Harmonics has been plausibly placed after the Tetrabiblos.[17] For a philosophical work of Ptolemy cf. Boll [1894] and Lammert in R.E. 23, 2 cols. 1854 to 1858; for fragments: Opera II, p. 263 to 270.[18]

From the Almagest it is certain that his observations were made in Alexandria; it seems (cf. Fig. 15) that he collaborated until A.D. 132 with a certain Theon who may have been his teacher.[19] Theon's four observations cited in the Almagest all concern Venus and Ptolemy's early observations also deal with planetary theory. What earlier material is explicitly utilized in the Almagest can be seen from Fig. 16.

At least from the third century onwards the manuscripts of the Almagest included an epigram after the table of contents to Book I[20] which was finally incorporated in the Greek Anthology[21]:

> Well do I know that I am mortal, a creature of one day.
> But if my mind follows the winding paths of the stars
> Then my feet no longer rest on earth, but standing by
> Zeus himself I take my fill of ambrosia, the divine dish.

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[10] Ptolemy Opera II, p. 72, 7.
[11] Cf. below V B 7, 5 and p. 971.
[12] Below V B 7, 2 and 3. Reference to the Almagest in the Planisphaerium: Heiberg, p. 234, 16.
[13] Tetrab. I, 1, ed. Boll-Boer, p. 3, 4.
[14] Cf. below V B 8, 1.
[15] Alm. II, 13 (Manitius I, p. 129). Conversely: Geogr. VIII 2, 3 (Nobbe, p. 195, 25) refers to the Almagest.
[16] Cf. p. 937 and p. 971: p. 1025.
[17] Düring, Harm. I, p. LXX.
[18] Also R.E. 23, 2 col. 1839/40.
[19] There is no evidence in favor of an identification with Theon of Smyrna. Cf. below p. 949 f.
[20] Cf. Boll, Kl. Schr., p. 152; also [1894], p. 74.
[21] Cf. the Loeb edition by W. R. Paton, Vol. III (1958), p. 320/321, No. 577 [but ignoring the text-critical results of Boll (1921), Kl. Schr., p. 143 to 155].

In the Anthology Ptolemy is considered as the author [22] and modern scholars tend to agree.[23] Tycho Brahe and Kepler composed Latin versions [24] while in the 12th century Abbot Suger of Saint Denis was inspired by it to a praise of the new Gothic architecture.[25]

Tycho Brahe in his Progymnasmata [26] assumed without any discussion that Ptolemy's catalogue of stars was simply taken over from Hipparchus, similar to mediaeval practice, still followed by Copernicus.[27] Brahe also knew Pliny's story about the Nova which induced Hipparchus to compile a fixed star catalogue [28] and obviously took it for granted that Hipparchus had defined the positions of the stars in ecliptic coordinates.

As we have seen these assumptions are by no means confirmed by a more detailed investigation. H. Vogt has shown that the Ptolemaic coordinates cannot be derived from known Hipparchian data,[29] nor is it at all likely that there ever existed a Hipparchian catalogue with stellar positions given in ecliptic coordinates.[30]

Nevertheless Brahe's casual remarks seem to be the ultimate origin of the often repeated statement that Ptolemy made scarcely any observations and that most of the Almagest is basically Hipparchus' work. Such statements were already common in the 18th century (e.g. with Lemonnier [31] and Lalande [32]) and culminated in Delambre's hostile attitude and distrust toward all of Ptolemy's work, a position which has greatly influenced the later (mostly secondary) literature. Among the few who tried to restore a proper estimate is L. Ideler [33] (1806) and in recent decades, beside H. Vogt,[34] students of specific Ptolemaic works.[35] It is perhaps only Bīrūnī who can be considered Ptolemy's peer in the whole period before Kepler and Newton.

1. The "Almagest"

There can be no doubt that the name "Almagest" is derived from an Arabic "al-majastī." The form *μεγίστη σύνταξις* – instead of the original *μαθηματική σύνταξις* – actually occurs in Greek, although I know it only from a late author

[22] Synesius, however, in a letter to Herculian (trsl. Fitzgerald, Synesius, No. 143, p. 238) says only that it was older than his own epigram (cf. below p. 875) and that it came from "an ancient fragment." Obviously neither Herculian nor Synesius connected it with Ptolemy.
[23] Boll, Kl. Schr., p. 151 to 153; Cumont, Eg. astr., p. 206.
[24] Opera I, p. 151; Werke I, p. 3 (title page of the *Mysterium cosmographicum*); cf. Boll, Kl. Schr., p. 154.
[25] Cf. E. Panofsky, Suger, p. 21 (reprinted: p. 129).
[26] Brahe, Opera II, p. 151, etc.; cf. above p. 280, n. 1).
[27] De Revol. II 14.
[28] Cf. above p. 284.
[29] Cf. above p. 281 ff.
[30] Cf. above p. 283.
[31] Lemonnier, Inst. astron., p. XXVIII (1746).
[32] Lalande [1757], p. 420/421; also Lalande, Astronomie[3] I, p. 119, No. 344 (1792).
[33] Ideler, Astron. Beob., p. 301-306.
[34] Vogt [1920], [1925].
[35] So Luckey [1927]; Düring, Harm. (1930-1934); Rome, C.A. (1931-1943); Curtis-Robbins [1935] (p. 92, n. 1); Mžik, Ptol. Erdkunde (1938); Lejeune, Opt. (1948-1957); Neugebauer [1956].

(Symeon Seth, about A.D. 1080).[1] Eutocius (about A.D. 500) says μεγάλη σύνταξις.[2] M. Koppe suggested in 1893[3] that a contracted title "μεγα. συντ." was transliterated into Arabic. This implausible hypothesis was unfortunately accepted by J. Ruska in his thesis on the Quadrivium of Severus Bar Shakkū.[4] From here Brockelmann took it over into his Geschichte der arabischen Litteratur[5] but abandoned it later on,[6] influenced by Suter's skepticism.[7] Nallino again followed Koppe[8] for the sole – and as we know incorrect – reason that μεγίστη does not occur in Greek sources. Finally Plessner followed Nallino in the new edition (1960) of the Encyclopedia of Islam.[9]

The manuscript tradition of the Almagest is unusually good. Heiberg for his edition had at his disposal two codices of the 9th century (his A and B), two of the 10th century (C and D[10]), in a total of 36 codices. Because (or in spite) of this ample documentation the Almagest is one of the best preserved literary works of Antiquity.

The only modern edition which is based on a systematic investigation of all available manuscripts is Heiberg's edition in the Teubner series (1898/1903) from which Manitius' excellent German translation (1912/1913) was made.[11] A century earlier the Abbé Halma had published an edition of the Greek text from some manuscripts in Paris, faced by a French translation (1813/1816).[12] Halma's edition was repeatedly reprinted; its pages are also given on the margin of Manitius' translation. An English translation, based on Heiberg's text, appeared in 1952.[13] A monograph by Olaf Pedersen on the Almagest has just been published (Odense 1974).

The story of the mediaeval transmission of the Almagest through Arabic translations and Byzantine manuscripts and of the European revival in the Renaissance, of Latin translations and printing, has been told often and need not to be repeated here.[14] Little known but interesting details about the numbering of figures or the counting of "theorems" in the ancient commentaries to the Alma-

[1] *De util. corp. caelest.*, ed. Delatte, AA II, p. 110, 3/4. Much earlier (3rd cent.?) is the occurrence of the name "the Roman μεγίστη" in Pahlavi (cf. Bailey, Zor. Probl., p. 86).
[2] Archimedes, Opera III, p. 232, 16/17, ed. Heiberg. Cf. also Cassiodorus (6th cent.) who ascribes to Ptolemy "*duos codices ... quorum unum minorem, alterum maiorem vocavit astronomum*" (Institutiones, ed. Mynors, p. 156, 1-3).
[3] Reviewed in Z. f. Math. u. Phys. 39 (1894), Hist.-lit. Abt., p. 18f.
[4] Heidelberg 1896.
[5] GAL I, p. 203 (1896).
[6] GAL Suppl. I (1937), p. 363. Cf. also Rosenthal [1956], p. 439, note 1.
[7] Enc. of Islam (1911) I, p. 313.
[8] Nallino, Scritti V, p. 262 (1911/12).
[9] Enc. of Islam[(2)] (1960), p. 1100. Cf. now Kunitzsch, Alm. p. 115-125.
[10] Heiberg, Ptol. Opera I, 1, p. V assigns D (=Vat. gr. 180) to the 12th century but the Vatican catalogue (Cod. Vat. gr., p. 206) corrects this date to the 10th century.
[11] Manitius, Vol. II, p. 436-440 gives a list of preferable variants and changes in the text. A reprint edition of 1963 contains some corrections of Manitius' translations and explanatory notes but only to the extent permitted by technical and other limitations.
[12] For a contemporary review (1818) cf. Letronne. Œuvres choisies sér. 2, 1, p. 95-126.
[13] Translated by R. Catesby Taliaferro; Vol. 16 of the "Great Books of the Western World", p. 1-465 (Chicago 1952).
[14] Cf., e.g., the introduction to Manitius' translation or R.E. 23, 2 cols. 1791-1793 (Ziegler). Kunitzsch, Alm. is the most recent monograph devoted to this topic.

gest were revealed by A. Rome in the introduction to his edition of Pappus' commentary.[15]

Two large commentaries on the Almagest were written in late antiquity, one by Pappus, covering at least the first six books, only V and VI of which are preserved,[16] the other to all 13 books by Theon.[17] Pappus wrote his commentary shortly after A.D. 320,[18] Theon about 360.[19]

An elementary commentary to Book I of the Almagest, mainly concerning the arithmetical operations has, in all probability, Eutocius as its author[20] (therefore written around A.D. 500[21]). No doubt many more explanatory treatises of this type existed in Byzantine didactic literature.

2. Later Tradition

All mediaeval astronomy — Byzantine, Islamic, and eventually western — was related to Ptolemy's work, a situation which prevailed until the invention of the telescope and the concepts of Newtonian mechanics opened entirely new possibilities. It is therefore not surprising that we know of commentaries to and translations of almost all works of Ptolemy.[1] Only a few examples will be mentioned.

Next to the Almagest the "Handy Tables" are of the greatest importance for mathematical astronomy.[2] The indefatigable Theon wrote two commentaries on this work, one shorter one[3] and one large one in five books.[4] Another commentary by his daughter Hypatia is lost[5] (thus before A.D. 514) and one by Tribonianus[6] (who died around 542). A Latin version of Theon's commentary on the Handy Tables which contains an example for August 534 is called "*preceptum canonis Ptolomei*".[7] This seems to be the last trace of the Handy

[15] Rome, CA I, p. XVIII-XX.
[16] Available in an excellent edition by A. Rome (CA I, 1931); cf. below p. 966.
[17] Books I to IV edited by A. Rome (CA II and III, 1936 and 1943). For the remaining books cf. below p. 968, n. 30.
[18] Rome CA I, p. XIII; cf. also R. E. 18, 3 col. 1087–1089.
[19] Rome [1939].
[20] Cf. Mogenet [1956]; also Rome [1955].
[21] For the date of Eutocius cf. also Neugebauer-Van Hoesen, Greek Horosc., p. 188 f.
[1] Cf. for western Europe in general: Haskins, SMS; for Byzantium: Pingree [1964]; for Ptolemy in particular: Boll [1899], Chap. I (p. 79–88). Du Bus ([1938], p. 81) reports the existence of some 190 Ptolemaic manuscripts in France alone.
[2] Cf. below V C 4.
[3] Edited, with French translation, by Halma, H.T. I, p. 27–83 (1822).
[4] Much information about these commentaries is found in A. Rome's introductions and notes to his CA I to III. The often repeated hypothesis of a commentary on the Handy Tables by Pappus (e.g. R.E. 18, 3 col. 1089) is only the consequence of Boll's wrong date for the Helios diagram in the Vat. gr. 1291; cf. below p. 978, note 3).
[5] Deduced from an ambiguous passage in Suidas (ed. Adler IV, p. 644, 4); cf. Tannery, Mém. Sci. I, p. 77. All that seems certain is that she cooperated on her father's commentary on the Almagest; cf. Rome CA II, p. LXXXIII ff. and CA III, p. CXVI ff. Cf. also next note.
[6] Suidas, ed. Adler IV, p. 588, 17/18. Here as well as in his note on Hypatia I consider *Κανόνα* to mean the Handy Tables, not the Almagest (i.e. the *Σύνταξις*); cf. Suidas, ed. Adler IV, p. 254, 7/8.
[7] Cf. below p. 970.

Tables in the west. Their continued use is well attested within the area of the Byzantine empire and in Islamic astronomy.[8]

One has a commentary to the Tetrabiblos[9] and to the first part of the Harmonics from Porphyry (about A.D. 230 to 300).[10] The latter work, through Boethius' *De institutione musica*, has greatly influenced the western Middle Ages, through al-Fārābī (10th century) Islamic theory.[11] A paraphrase of the Tetrabiblos, often ascribed to Proclus (who died in 485), is probably another author's work.[12]

Some of Ptolemy's works are in whole or in part preserved only through Arabic translations, the foremost being the Optics which is extant only in a Latin version made from the Arabic in the 12th century by the Emir Eugene of Palermo[13] (who also collaborated on a Latin translation from the Greek of the Almagest). Of the Planetary Hypotheses the end of Book I and the whole of Book II is only known through the Arabic,[14] the Planisphaerium is preserved in a Latin translation of Maslama al Majrītī's Arabic version (around A.D. 1000).[15] The Analemma has survived in part in a Greek palimpsest and in a Latin translation[16] by William of Moerbeke "the Fleming" (about 1250) of whom Roger Bacon said "that he knows no Greek science in the original language ... and therefore he translates everything wrong and inhibits the understanding of the Latins."[17]

§ 2. The "Analemma" and its Prehistory

1. Introduction

A masterful little treatise by Ptolemy is entitled "On the Analemma."[1] Ordinarily this term denotes a supporting structure, e.g. a retaining wall. In astronomical context, however, "analemma" means a very specific method of finding by means of geometric constructions in the plane certain arcs and angles which determine a point on the celestial sphere. Using modern terminology one might call it methods of "descriptive geometry." The main field of application of an "analemma" is the theory of sun dials[2] and we know from a remark by Proclus that works specifically on the analemma had been written in the first century B.C.,[3] although this says nothing about the date of the actual invention of the

[8] E.g. by Severus Sebokht in A.D. 662 (cf. Neugebauer [1959, 2]).
[9] Edited by Boer and Weinstock in CCAG 5, 4, p. 185–228.
[10] Edited by Düring, Harm. II. For an alleged commentary by Pappus cf. Düring, Harm. I, p. LXXIV, note 1. For the influence on Kepler cf. below V B 8, 2.
[11] Düring, Harm. I, p. LXXVI f.; for the Byzantine revival cf. p. LXXVII ff.
[12] Cf. Boll, Sphaera, p. 219, note 1 (or Boll [1916], p. 8), against Boll [1899], p. 86 f.
[13] Cf. the introduction in Lejeune, Opt.; also below V B 5.
[14] Edited by Goldstein [1967]; cf. below V B 7, 1.
[15] Cf. below p. 871. Arabic "Notes" on this treatise by Maslama are extant in the Bibliothèque Nationale in Paris (cf. Vajda [1950], p. 8, No. 10).
[16] Cf. below V B 2, 1.
[17] Setton [1956], p. 62.
[1] Edited by Heiberg [1895] and again in Ptol. Opera II, p. 189 to 223 (cf. also p. XI f. and p. CLXXIX).
[2] "Analemma" does not mean, however, a sun dial itself (as stated in Liddell-Scott on the basis of an unfounded interpretation of CIG 2681).
[3] Cf. below p. 841 f.

method. Vitruvius and Heron obviously consider analemmata as well-known[4] and thus establish for us the continuity of the tradition, however fragmentary the extant remnants are.

The Greek text of Ptolemy's version has come down to us only in one palimpsest,[5] the beginning and the end being lost. Fortunately, however, we have a Latin translation, made by William of Moerbeke[6] (about 1250) which preserved the beginning (with the dedication to Syrus as in the Almagest and the Planisphaerium) and reaches without major gaps to the concluding numerical tables of which, however, only one out of seven has survived. Delambre conjectured[7] that even more is missing, namely a discussion of the applications to the construction of sun dials.[8] I see no basis for this assumption; at any rate no trace of a continuation beyond the tables has been preserved.

No translation into a modern language has ever been published.[9] Commandinus added ample commentaries to his version of the Latin text. An excellent discussion of Ptolemy's nomographic procedures[10] was given by Luckey [1927], who also summarized the modern literature since Commandinus.

2. Diodorus of Alexandria

A. Biographical Data

The lifetime of Diodorus, "the mathematician from Alexandria," is known within narrow limits: he is quoted by Eudoros,[1] a philosopher near the end of the first century B.C.; on the other hand he himself is excerpting Posidonius,[2] who is also of the first century B.C.

Of his writings only a few sentences are preserved. They concern the difference between astronomy ("mathematics") and physics ("physiologia"),[3] the meaning of the terms "cosmos"[4] and "star",[5] the nature of the stars[6] and of the Milky Way.[7]

There are some passages which deal with constellations and might belong to a commentary on Aratus.[8] Marinus, the successor of Proclus, in his commentary

[4] Cf. below V B 2, 3 and V B 2, 4 A.

[5] Discovered by Heiberg; cf. [1890], p. 4 note **).

[6] First published by Commandino (Rome 1562) in a very free rendering of the rather corrupt and barbaric text. Heiberg's Latin text is based on Moerbeke's autograph, preserved in Vat. Ottobon. lat. 1850 (cf. Ptolem. Opera II, p. XI).

[7] Delambre, HAA II, p. 471.

[8] This part of the theory has been reconstructed by Commandinus; cf. Delambre, HAA, p. 471 to 503.

[9] Only one section (No. 10 in Heiberg, p. 206, 17/14 to p. 210, 2) is translated in Luckey [1937], p. 113f.

[10] Below (p. 852f.).

[1] Maass, Comm. Ar. rel., p. 30, 20; cf. also Diels, Dox., p. 19, n. 2.

[2] Maass, l.c. p. 39, 6–9; cf. also Diels, Dox., p. 19/20.

[3] Cf. note 1.

[4] Maass, l.c. p. 35, 29 to 36, 7 (= Diels, Dox., p. 20).

[5] Maass, l.c. p. 41, 17.

[6] Cf. note 2.

[7] Macrobius, Comm. (ed. Eyssenhardt, p. 545, 26; trsl. Stahl, p. 149).

[8] Maass, l.c. p. 377, 21 and p. 387, 14 (= Maass, Aratea, p. 70); also p. 179, 5.

to Euclid's "Data" quotes the opinion of Diodorus, as well as of Apollonius and Ptolemy,[9] about the meaning of the term "given" (τεταγμένον).

These references which range from the first century B.C. to the fifth century A.D., show that Diodorus was a well-known author. His name is even found in the Anthologia Palatina[10]: "O Diodorus, of great renown among the gnomonists, tell me the hour ..." And again in connection with those who designed sun dials he is mentioned in the Hypotyposis of Proclus[11]: the gnomonists and "those who first wrote Analemmata, e.g. Diodorus," assume that one may ignore solar parallax.

That the "Analemma" of Diodorus was an important work is shown by the fact that Pappus wrote a commentary on it, as he tells us in his "Collection".[12] We shall discuss in the following a passage from this commentary.[13] But it was only recently that at least one substantial section from the Analemma of Diodorus was identified[14] thanks to a reference by al-Bīrūnī in his treatise on "Shadows"[15] to a method "based on what is said in the book Analemma of Diodorus" concerning the determination of the local meridian. It is not new that Diodorus was known to muslim scholars; an-Nairīzī (about A.D. 900) mentions him in connection with Euclid[16] and Ibrāhīm b. Sinān (908 to 946, grandson of Thābit b. Qurra) refers to him in connection with the theory of sun dials.[17] But until recently we did not know that a method described by the Roman geodesists Hyginus[18] (about A.D. 100) and also by Abū Saʿīd aḍ-Ḍarīr al-Jurjānī (about A.D. 850)[19] "from the book Analemma" represents the method of Diodorus.[20] We shall discuss this method in the next section; its correct explanation was discovered at least three times: by Mollweide in 1813,[21] by Schoy in 1922, and by Kennedy in 1959.

B. *The Determination of the Meridian Line*

We consider to be given in the plane of the horizon three different shadow lengths, GA, GB, GC, cast by a vertical gnomon GF of length g (cf. Fig. 17). We wish to find the meridian line GS.

The method of solution is based on the following considerations. We assume an observer to be in the northern hemisphere and that the sun has, e.g., a southern

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[9] Euclid, Opera, ed. Heiberg-Menge, Vol. VI, p. 234, 17; translation and discussion: Michaux, Comm. de Marinus, p. 55 and p. 26.
[10] Anthol. Pal. XIV, 139 (Loeb V, p. 100/101).
[11] Proclus, Hypotyp., p. 112, 5-12.
[12] Pappus, Coll. IV, XXVII, p. 246, 1 Hultsch.
[13] Cf. p. 843.
[14] Kennedy [1959].
[15] Boilot [1955] No. 15.
[16] Euclid, Opera, Suppl. ed. Curtze, p. 35, 1 and p. 65, 23 in the Latin version by Gerard of Cremona. Arabic version in Besthorn-Heiberg, Cod. Leid. I, p. 24, 10 and p. 118, 8. In both passages Diodorus is associated with an "Abthiniathus" or "Anth...athus" (the *b* or *n* and the *ini* depends on the choice of the diacritical dots) of unknown Greek equivalence. The discussion concerns the theory of parallels.
[17] Luckey [1948], p. 505.
[18] Cf. Mollweide [1813], followed by Cantor, Agrimensoren, p. 68 f. and Schoy [1915], p. 563-566. Text: Röm. Agrimensoren I, p. 189, 16-191, 11. Fragments also in the Geometry of Gerbert (ed. Olleris, Chap. 94, p. 470; ed. Bubnov, "Geometria incerti auctoris", p. 363/4, No. 61).
[19] Schoy [1922]: cf. Suter MAA, p. 27, No. 48. Aḍ-Ḍarīr = the blind one.
[20] Remarked by Hermelink, Zentralblatt für Mathematik 89 (1961), p. 4.
[21] Summary also in Dilke [1967].

declination. We may consider the tip F of the gnomon as the center of the celestial sphere and the sun rising in the horizon of F at H, setting at Δ. Let FN represent the direction to the north celestial pole for the given locality. Then FN is the axis of a straight circular cone made by the solar rays which meet the horizontal plane through G in the hyperbola ABC. We consider on this cone the circle $H'C\Delta'$ which is parallel to the day-circle $H\Delta$ of the sun and goes through the point C. Since this circle is parallel to the equatorial plane it intersects the horizontal plane through G in a line parallel to the direction east-west. As soon as we can construct this intersection we have only to draw the perpendicular direction GP as meridian line.

Since C is by construction one point of the intersection in question we need only to find one more point of it. To this end one determines the points A′ and B′ where the rays FA and FB meet the circle $H'C\Delta'$. Here a slight difference appears between the procedure followed by Hyginus and by Bīrūnī. Abū Saʿīd describes them both which suggests that they both belonged to Diodorus.[1]

According to the first method one projects A′ perpendicularly to the horizontal plane onto A″, B′ onto B″. Then A″B″ is the projection onto the horizontal plane of the chord A′B′ which belongs to the plane of the circle $H'C\Delta'$. But the plane of the triangle ABF intersects the horizontal plane in the line AB through the given endpoints A and B. Thus A′B′ meets the horizontal plane in the intersection D of A″B″ and AB; hence D is a point of the intersection of the plane $H'C\Delta'$ with the horizontal plane. Thus CD is the East-West direction and PGS the meridian line. Bīrūnī also finds the point D but not by means of the intersection of A″B″ with AB but by operating in the plane of the triangle ABF and thus intersecting simply AB with A′B′.

The corresponding analemma constructions are as follows. First one finds the distances $d_1 = FC$, $d_2 = FA$, $d_3 = FB$ by constructing in Fig. 18a in true size the three right triangles [F]GC, [F]GA, [F]GB where GA, GB, GC are the given shadow lengths and $G[F] = g$ the length of the gnomon. Fig. 18b then represents the horizontal plane in which GA, GB, GC are the given shadows in proper size and direction. From Fig. 18a we know the distance GA″ and GB″. Thus we can find D as intersection of A″B″ and AB.

The other version, which is followed by Bīrūnī, operates only with one figure, located in the horizontal plane (cf. Fig. 20). Again GA, GB, GC are the given shadows. The right triangles $G[F]_1C$, $G[F]_2A$, $G[F]_3B$ provide as hypotenuses the distances d_1, d_2, d_3 as before. If we determine [F] such that $A[F] = d_2$ and $B[F] = d_3$ then AB[F] represents in true size the triangle ABF. Therefore a circle with center [F] and radius $d_1 = [F]_1C = [F]A' = [F]B'$ determines the position of the points [A′] and [B′] in true scale. Thus [A′][B′] intersects AB in the same point D as A′B′ in space the line AB.

Abū Saʿīd gives still another method for the determination of the East-West line, based on the altitude of the sun when crossing the prime vertical.[2] Though the procedure is simple enough and fully within the reach of the Greek analemma methods one cannot be sure that Abū Saʿīd found it in the book by Diodorus.

[1] Abū Saʿīd says explicitly "taken from the book Analemma."

[2] Cf. Schoy [1922].

C. Pappus' Commentary to the "Analemma"

The section from his commentary to the "Analemma" of Diodorus which Pappus repeats in his "Collection"[1] unfortunately does not concern an astronomical problem in the modern sense of the word. It deals, unexpectedly for a modern reader, with the trisection of an angle. Nevertheless the problem is of astronomical origin and, in all likelihood, was as such discussed in the treatise of Diodorus. Otherwise it would not have been included in Pappus' commentary if we may judge from his commentary to the Almagest in which he keeps very close to the primary text. The problem is in fact well motivated in a treatise on the solution of questions of spherical astronomy by means of geometric constructions: from an analemma one can find the length of daylight to given solar longitude,[2] i.e. the arc of a circle of constant declination between sunrise and sunset. One twelfth of this arc gives the length of one seasonal hour. Its accurate construction implies the trisection of an arbitrary angle.

The solution described by Pappus is based on the curve known as cochloid (or conchoid) of Nicomedes[3] which he used for the duplication of the cube. This curve is the locus of all points B such that $AB = \text{const}$ where A lies on a fixed straight line and BA is directed toward a fixed point P.

To show that this curve also serves to trisect a given angle[4] α we construct a right triangle QPA_0 (cf. Fig. 20) with angle α at P and arbitrary, but fixed hypotenuse d. We construct to P as pole and to A_0Q as given straight line the cochloid B_0B such that its characteristic parameter, the fixed distance $A_0B_0 = AB$ is $2d$. Consider a point B on this curve such that $BQ \parallel A_0B_0$. Let R be the midpoint AB and join Q with R. Then the angle β occurs not only at B and P but also at Q. Since $QR = d = QP$ we see that the angle $\gamma = \alpha - \beta$ at P is the same as the angle 2β at R. Thus $\beta = 1/3\,\alpha$.

3. Vitruvius

The first actual discussion of an "analemma" is preserved for us in the work "On Architecture" by Marcus Vitruvius Pollio, dedicated to Augustus. It is evident that Vitruvius was familiar with the fundamental concepts of astronomy from many remarks in his book. Book IX is specially devoted to mathematical and astronomical topics. We mention here planetary periods,[1] the discussion of the lunar phases according to Berosus' opinion,[2] Aristarchus' correct explanation,[3] and the description of the major constellations, supposedly based on Democritus.[4]

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[1] Pappus, Coll. IV, 27, ed. Hultsch, p. 244, 21–246, 14; trsl. Ver Eecke, p. 187f; trsl. Thomas, Greek Math. I, p. 301.
[2] Cf. below p. 844f.
[3] Perhaps second century B.C.
[4] Pappus does not give any detail; the following is a slight modification of a construction suggested by Heath, Greek Math. I, p. 236.

[1] Cf. above p. 782–784.
[2] The moon is assumed to be half luminous; Vitruvius IX, II 1 and 2 (Budé, p. 16f.).
[3] Cf. p. 635.
[4] Cf. p. 577.

In Chap. 6 of Book IX one finds remarks about the "genethlialogia" of the "Chaldeans" and the weather prognostications of Greek "parapegmatists" from Eudoxus to Hipparchus, known to us also directly from the "calendars" of Geminus or Ptolemy.[5] The final chapters are devoted to clocks, in particular Chap. 7 to sun dials, Chap. 8 mainly to water clocks. From the latter stems our information about the drive of an anaphoric clock.[6] The "analemma" is discussed in IX, 7.

At the beginning a short list is given of equinoctial noon shadows (s_0) of a vertical gnomon (g). This information we can, of course, directly transform to geographical latitudes φ by means of $s_0/g = \tan\varphi$. It should be emphasized, however, that the concept "geographical latitude" occurs nowhere in Vitruvius; he only speaks about the "*declinatio caeli*" as defined by shadow lengths.

The analemma, as described by Vitruvius, serves only for the determination of the length of the noon shadow s for a given solar longitude λ at a place of known geographical latitude, characterized by s_0 and g. Fig. 21 shows the procedure. We may consider the top A of a vertical gnomon the center of the celestial sphere, hence a circle of center A (and radius g) as the meridian which intersects the equator in the diameter DE. Vitruvius assumes for the inclination of the ecliptic 1/15 of the circle's circumference, i.e. $\varepsilon = 24°$. Then HA and FA give the directions of the solstitial shadows, while the parallels to the equator FG and HK are the traces of the solstitial circles in the meridian plane. The diameter LO of the horizon intersects the diameters of the solstitial circles at M and N, respectively.

All these constructions take place in the plane of the meridian. It is characteristic for the analemma that the same plane is also used to show circles in planes orthogonal to the meridian by rotating them 90° about their diameters. Thus FPG is one half of the summer solstitial circle, HQK of the winter solstitial circle. The orthogonals PN to FG and QM to HK are the intersections with the horizon,[7] thus FP and HQ are the half day-arcs at the longest and shortest day, respectively. The points which divide these arcs into 6 equal parts represent the positions of the sun at the end of each seasonal hour at the solstices.

Vitruvius does not tell us how one can find length and direction of the shadow for each of these hours but it is not difficult to solve this problem by geometric constructions since we now have the orthogonal coordinates of the sun for each hour (e.g. P_1N_1 and N_1R_1 in Fig. 21). Hence the solstitial shadow curves can be constructed.[8]

For general solar longitudes the lengths of the seasonal hours can be found by means of an auxiliary circle (cf. Fig. 22) with the chord KG as diameter,[9] called "menaeus".[10] Again Vitruvius does not complete the construction but

[5] Cf. IV A 3, 3 and below p. 928f.

[6] Cf. below p. 870. The reader must be warned not to trust the translation in the Loeb Classical Library. May it suffice that the translator (F. Granger) thinks that the stationary points of a planetary orbit are the nodes.

[7] These intersections are given a special name, variously spelled (e.g. *laeotomus* or *locothomus*, according to our printed texts) and variously explained. I think one may assume an original form *λοξοτόμος* i.e. skew (or asymmetric) intersection with the horizon.

[8] For Ptolemy's procedure cf. below p. 855f. Cf. also below note 12.

[9] Vitruvius, incorrectly, assumes E (Fig. 21) as center and EG as radius.

[10] In the MSS spelled *maneus;* also called *circulus menstruus* (obviously from a *μηνιαῖος κύκλος*).

it is not difficult to supply it.[11] Make the arc ST on the menaeus equal to the solar longitude λ and draw TUV parallel to the equator. Then the semicircle of diameter UV is one half of the solar travel on this day and WX divides the day-arc from the night-arc; hence VW represents 6 seasonal hours of daylight.

In order to demonstrate the correctness of this construction we turn the ecliptic about its diameter FK into the meridian plane; hence Y is the vernal point $\lambda=0$. We wish to show that the point Λ has the given longitude λ (i.e. we wish to show that $\theta=\lambda$) if $Z\Lambda$ is perpendicular to FK. Under this condition $Z\Lambda$ is the intersection between the ecliptic and the parallel circle UWV. But $AZ=g\sin\theta$ and therefore $ZZ'=g\sin\theta\sin\varepsilon$. The radius of the menaeus is $g\sin\varepsilon$, hence $TT'=g\sin\varepsilon\sin\lambda$. But $TT'=ZZ'$ thus $\sin\theta=\sin\lambda$ q.e.d.

In this way one can find for every λ and for every hour the orthogonal coordinates of the sun with respect to the planes of the horizon and of the meridian. This suffices for the construction of the hour lines for given λ.[12]

4. Great Circle Distance between Two Cities

A. Heron

Among the voluminous writings of Heron of Alexandria is found a treatise "On the Diopter"[1] which is of great interest for our understanding the development of ancient geography[2] since it describes methods for the determination of terrestrial distances, large and small. We shall concentrate here in particular on Chap. 35[3] because it contains an application of the "analemma" to the determination of the great circle distance Alexandria-Rome, making use of a lunar eclipse supposedly observed in both cities. But before describing the details of Heron's method I shall remark on the background of this chapter and the form in which its problem is formulated and solved.

The treatise takes its name from the description of a sighting instrument of high technical sophistication. The actual sighting device is mounted on two orthogonal turntables, as on a theodolite, moved by worm screws and controlled by a water level, with adjustable sighting slots, etc. The application of such an instrument to the direct measurement of great circle distances between celestial objects is explicitly mentioned by Heron[4] but has left, at least to my knowledge, no trace in ancient or mediaeval astronomical literature (which is completely dominated by the instruments described in the Almagest).

Heron then gives a long sequence of examples of problems of terrestrial triangulation for which his instrument can be used, leading to the determination of areas, a problem which gives rise to the proof of the "Heronic formula" for the area

[11] Drecker, Sonnenuhren, p. 2; cf. also Neugebauer [1938], p. 7 and Fig. 2.
[12] Many examples are worked out in the older literature, e.g., in Vitruvius, ed. Barbari (1567), p. 310ff.
[1] Published by H. Schöne in Heron, Opera III (Teubner 1903), p. 187 to 315 with German translation.
[2] Heron is consistently ignored in the standard works on ancient geography, e.g. Berger, Erdk.; Honigmann, SK; Thomson, HAG; etc.
[3] Heron, Opera III, p. 302 to 307.
[4] Chap. 32; cf. also Chap. 2.

of a triangle from its sides.[5] The conclusion of the work is made by two chapters on mechanical devices to measure distances travelled by a wagon or a ship[6] (by using the rotation of wheels reduced by gears[7]) and on the determination of large distances by astronomical means — our Chap. 35.

This latter problem is in principle simple enough. Geographical latitudes can be easily determined while the difference in longitude of two localities follows from the difference in local time at the simultaneous observation of a lunar eclipse, a fact well-known at least since Hipparchus.[8] Hence one knows the relative position of the two localities and can determine the great circle distance by direct measurement, e.g. by the length of a string stretched on a globe. We shall see that Heron in fact ends up exactly with this type of totally unmathematical procedure. What is of interest to us, however, is the fact that he does not simply localize the two points by their spherical coordinates. The crucial point is, in fact, the very absence of such coordinates. What Heron assumes at his disposal are observational data in a form commonly used at his period: the local times are given in local seasonal hours, hence he has no simple Δt; and the latitudes are defined by gnomon shadows, not by angles φ. It is for this reason that an analemma has to be used in order to find the spherical positions of the two places. Vitruvius as well as Heron[9] have not yet reached the methodological level of Ptolemy's Geography[10] which is based on geographical longitudes and latitudes, i.e. on data directly applicable to globes or maps.

It is from "Dioptra 35" that Heron's date can be securely fixed to the middle of the first century A.D.[11] As we have just mentioned the essential difficulty in his problem consists in operating with local seasonal hours. The example which he uses to illustrate his method is most ill suited for his purpose because he operates with a lunar eclipse which took place only 10 days before the vernal equinox. At that time the difference between seasonal and equinoctial hours is hardly recognizable and hence the analemma constructions, if made to correct scale, would collapse. The only conclusion left from his choosing such an example is obviously that Heron appealed for the eclipse to the recent memory of his readers (or his own). Indeed, the circumstances he gives for the observation in Alexandria — 10 days before the vernal equinox, 5th hour of the night — are only once satisfied between about -200 and $+300$, namely by the eclipse of A.D. 62 March 13, beginning at $20;51^h$ Greenwich time, i.e. for Alexandria $23^h = 5^h$ of night, 10 days before equinox. This eclipse reached almost 9 digits and was fully visible in Alexandria for about 3 hours.[12]

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[5] Chap. 30. We know through al-Bīrūnī that this formula had been already proved by Archimedes; cf. Suter [1910], p. 39.

[6] Chap. 34 and Opera III, p. 312, 23 (of dubious numbering); the Chap. 37 (on lifting heavy weights) is obviously out of place in this work.

[7] Vitruvius, Archit. X, Chap. 9 describes very similar hodometers.

[8] Cf. Dicks, Hipp., p. 64/65; Berger, Geogr. Fr. Hipp., p. 12f.; Berger, Erdk., p. 470.

[9] Heron is nowhere quoted by Vitruvius who consistently refers to his predecessors. These considerations alone would limit Heron to the period between 0 and 150 A.D.

[10] Cf. p. 934.

[11] Neugebauer [1938], p. 21 to 24. Cf. also Drachmann [1950].

[12] Oppolzer, Canon, No. 1960; Ginzel, Kanon, No. 1037. For the discussion of ancient eclipse records it is of interest to notice that there the "time" of the eclipse refers to its beginning.

It is an ironical fact that the hotly contested problem of the date of Heron found its solution at the same time that it lost most of its interest. The decipherment of the mathematical cuneiform texts made it clear that much of the "Heronic" type of Greek mathematics is simply the last phase of the Babylonian mathematical tradition which extends over some 1800 years. Hence neither Heron nor Diophantus can any longer be considered as exceptional figures in Greek mathematics. In fact the astronomical procedures discussed in the following are of much more interest for a specific Greek development than most of Heron's mathematics.

B. "*Dioptra 35*"

We can now turn to Heron's procedures (cf. Fig. 23 a) to determine the spherical coordinates for two cities.[1] The construction takes place inside of a hemisphere with horizontal upper rim, i.e. a rim parallel to the horizon of the city A (Alexandria). Heron does not say so but one may assume that he did not require a hollow hemisphere especially manufactured for the purpose at hand. He probably meant one to use a sundial of this type (known as *σκαφή*[2]). Its vertical radius is the gnomon OZ, the tip O of which represents the center of the celestial sphere. For the drawing of great circles one could have used a thin circular metal template of the same radius as the hemisphere. The accurate drawing of a parallel circle, however, is not so easy; one could use, perhaps, an orthogonal great circle as guide. Heron keeps silent about these practical questions.

Since we are dealing with an observation at night the foot Z of the gnomon represents the nadir of A. Let BZD be the meridian, AHG the equator, ΘKL a parallel circle, the arc ZK being known, of course, from an analemma (cf. Fig. 23 b). This analemma is also used to find the position of the sun 10 days (or λ°) before the equinox, at the 5th seasonal hour of night. We construct on the "menaeus"[3] the arc λ — for the sake of clarity much exaggerated in Fig. 23 b, and thus obtain the intersection K with the meridian of the parallel circle travelled by the sun on the day in question. The point Θ represents sunset, ΘK six seasonal hours, hence MK is the distance of the sun from the meridian at the 5th hour of night. This gives us also in Fig. 23 a the position M of the sun on the parallel circle ΘKL at the moment of the lunar eclipse.

A similar analemma (Fig. 23 c) has to be constructed for the city **B** (Rome) where the moment of the eclipse is supposed to be at the third hour of the night. Hence we know now on the day-circle ΘMKL in Fig. 23 c the length of the arc from the sun M to the point S on the meridian of **B**.

If HE = 90° on the meridian of A then E is the south pole of the equator. Hence the great circle QESP is the meridian of **B**.[4] Let A′ be the intersection of this meridian with the equator; it occurs also in the analemma of Fig. 23 c where we can find the length of the arc A′B′ which we again transfer to the hemisphere. Hence we now have not only the nadir Z of A but also the nadir B′ for **B**. The arc ZB′ is therefore the great circle distance between A and **B**, to be measured directly in the hemisphere.

[1] First explained by Rome [1923], then, independently, by Neugebauer [1938], [1939].
[2] Cf., e.g., Vitruvius, Archit. Book IX, Chap. 8, 1.
[3] Cf. above p. 844.
[4] Obviously the angle ZES is the difference in geographical longitude of A and **B**.

Heron says that if ZB′ had been found to be 20° the distance between Alexandria and Rome would be 14000 stades, because 360° = 252000 stades, hence 1° = 700 stades. This is the relation proposed by Eratosthenes and accepted by Hipparchus.[5] The time difference, given as two hours, is, of course, not the result of an actual simultaneous observation of the eclipse of A.D. 62 March 13 but an estimate, perhaps based on sailing times.

In reducing observations made by Menelaos in Rome to the meridian of Alexandria Ptolemy assumes (Alm. VII, 3[6]) a time difference of $1;20^h$, i.e. 20° in longitude. In Book VIII of the Geography[7] the time difference Rome-Alexandria is explicitly given as $1\ 1/2\ 1/8^h$ $(=1;37,30^h=24;22,30^\circ)$ whereas the corresponding table of local times in the Handy Tables[8] has $\Delta t=1;36^h$ $(=24^\circ)$. The large list of "Important Cities,"[9] however, assigns to Rome the longitude 36;20°, to Alexandria 60;30°, thus $\Delta t=24;10^\circ=1;36,40^h$. Finally the longitude of Rome is changed in the Geography[10] to 36;40° hence Δt to $23;50^\circ=1;35,20^h$. All these estimates are considerably better than Heron's value of about 2^h time difference; the correct value would be about $1;10^h$.

If one determines the actual great circle distance Alexandria-Rome one finds 17;32°, thus only 2 1/2° less than Heron's figure. Computing, however, with the geographical latitudes 30;58° and 41;38° which correspond to the equinoctial noon shadows 5:3 and 9:8, respectively,[11] one finds, with $\Delta t=2^h$, a distance of 26;15° instead of Heron's 20°. This shows again that Heron did not actually carry out his construction.

5. Spherical Coordinates

Ptolemy's "Analemma" deals with the following problem: assume to be known the geographical latitude φ, a specific hour h on a given day, hence (from tables) the solar longitude λ or the corresponding right ascension and declination; one wishes to find certain spherical coordinates which describe the place of the sun at the given moment (in modern terms, e.g., azimuth and altitude) in a way convenient for "gnomonic" constructions, i.e. useful for the design of sun dials and for related problems.

The basic attitude in Ptolemy's approach is clearly formulated in the introduction: there are only three mutually orthogonal directions possible in space.[1] Therefore it is possible to construct with respect to three orthogonal axes three uniquely determined great circles on the celestial sphere that can serve as circles

[5] Cf., e.g., Berger, Erdk., p. 473; above p. 305, n. 27.
[6] Manitius II, p. 26/27 and p. 28.
[7] VIII 8, 3 (Nobbe, p. 205).
[8] Halma H.T. III, p. 34.
[9] Cf. Honigmann SK, p. 197, 94 and p. 198, 188.
[10] III 1, 61 (Nobbe, p. 151). For the chronology of Ptolemy's geographical writings cf. below p. 934.
[11] Vitruvius, Arch. IX, 7, 1 (Loeb II, p. 249, Budé, p. 26), presumably for Alexandria. Cf. also above p. 101, n. 1.
[1] We know from remarks by Simplicius (6th cent.) and Eustratius (12th cent.) that Ptolemy had written a special treatise Περὶ διαστάσεως in which he dealt with the three-dimensionality of space (Heiberg, Ptol. Opera II, p. 265f.).

of reference for spherical coordinates on the sphere. In the choice of a strictly symmetric arrangement lies the progress of Ptolemy over his predecessors.[2]

A. Ptolemy's Coordinate System

We consider three orthogonal coordinate planes (cf. Fig. 24): the plane of the "horizon," the plane of the "meridian," and the plane of the "vertical." Their intersections are called, respectively, "meridian line" (for the sake of convenience here denoted as M-axis), "equinoctial line" (E-axis), and "gnomon" (Z-axis).

In order to define coordinates for an arbitrary point S on the sphere (for the purpose at hand this point will represent the sun), Ptolemy introduces three movable great circles, one for each axis (cf. Fig. 25) or, if one wishes, he makes one plane always movable in each pair of coordinate planes which intersect in one axis. These planes in general position define angles between the axes and S in cyclic arrangement:

the plane rotating about the E-axis contains the "*hektemoros*"
the plane rotating about the M-axis contains the "*horarius*"
the plane rotating about the Z-axis contains the "*descensivus*."

These names as such are not Ptolemy's invention though he uses them in a new way as we shall see presently.[3] The name "hektemoros" is supposed to refer to the six (seasonal) hours during which the sun travels from the horizon to the meridian; the "horarius" also represents the hourly solar motion while the "descensivus" is our zenith distance. No attempt is made to relate these coordinates used by the "gnomonists" to the ecliptic or equatorial coordinates of cinematic astronomy.

Any pair of these three angles can be used to define the position of S. But instead of using the angles in the movable planes one can also take the three angles made by the movable planes by their intersections with the coordinate planes (cf. Fig. 25). These angles are simply denoted by the names of the coordinate planes "*horizontalis*," "*meridionalis*," and "*verticalis*," again in cyclic arrangement.[4] Any pair of angles of this second triple determines the position of S as well.

These two triples constitute the "six angles" of Ptolemy's coordinates. It is his goal to show how these angles can be found by direct geometric construction from given φ, h, and λ.

B. The "Old" System of Coordinates

The elegance of Ptolemy's definitions becomes manifest if one compares them with the corresponding system of the "old ones" (as Ptolemy says with the characteristic lack of nearer chronological specification which we would like to have).

In this older arrangement the coordinate planes are the same as with Ptolemy but of the movable planes only two will always contain S (cf. Fig. 26) whereas

[2] Cf. below Sect. B.

[3] Below p. 850.

[4] Our Figs. 24 and 25 represent the angles in question for a solar position before noon and to the south of the vertical. For a position after noon angles are counted from the western axis of the horizon; similarly the northern axis replaces the southern M-axis if the sun lies to the north of the vertical. Thus all angles are restricted to the interval from zero to 90°.

the equator serves as the third plane, independent of S. Consequently the spherical coordinates which determine the position of S become asymmetric. Only two arcs end at S (called "*descensivus*" and "*horarius*" where only the second term kept its meaning with Ptolemy,[5] the first being the complement of the Ptolemaic descensivus). The "*verticalis*" is the same in both systems but the horizontal angle in the earlier terminology is the complement of Ptolemy's "horizontalis" and is called "*antiskion*," i.e. "countershadow" (as is readily understood from Fig. 27). As "*aequatorialis*" is denoted the angle in the equatorial plane between the E-axis and the intersection of the equator with the horarius. The older terminology also knows a "*hektemoros*" but as an angle in the meridian plane and in fact identical with Ptolemy's "*meridionalis*" (cf. Fig. 28 and Fig. 25,[6] p. 1380). Hence we see that the older system had all three movable planes which contain S but did not use them in a symmetric fashion. Instead, the fixed plane of the equator was added, but related to the variable position of the horarius.

It is obvious that Ptolemy's strictly cyclic arrangement represents a methodological progress which must make itself also felt in the convenience of actual constructions. The ancient way of representing spherical configurations (cf. Fig. 28 and 29[7]) leaves no doubt in the superior elegance of the Ptolemaic definitions.

6. Construction of the Ptolemaic Coordinates

In the discussion of Heron's method for the astronomical determination of geographical distances[8] we have seen that an Analemma can degenerate in the case of the equinoxes, i.e. when the solar circle of the given day is a great circle. In this case there is also no difference between the "old" arrangement of Fig. 26 (or Fig. 28) and Ptolemy's (Figs. 25 or 29) because equator and hektemoros coincide. Consequently Ptolemy distinguishes two cases in his description of the construction of the "six angles": one for the equinoxes, one for the general case.

Except for the construction of the hektemoros in the general case Ptolemy does not give the proofs for his rules, obviously because they are known from the traditional theory. In the case of the non-equinoctial hektemoros, however, Ptolemy's angle is new and he therefore explains its construction in greater detail. We follow this procedure since it facilitates the understanding of the remaining cases.

A. The Hektemoros

Let the circle with center O represent the meridian (Fig. 30), AB the diameter of the day circle of the sun, turned in the familiar fashion into the plane of construction, such that [S]T shows the true distance of the sun from the plane of the meridian. The plane of the hektemoros intersects the plane of the meridian under

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[5] The text (Latin only, Heib., p. 191, 22) states incorrectly that the horarius is the angle between OS and the E-axis instead of the M-axis. The correct definition is given Heib., p. 193, 4.

[6] For the sake of clarity I have not shown this angle in Fig. 26.

[7] These figures in no way pretend to accurately depict the ancient drawings (Heiberg, p. 192 is a little nearer to the originals) but they at least show their principle in contrast to modern axonometric figures (Figs. 24 to 26).

[8] Above p. 847f.

a right angle (cf. Fig. 25) in the line OTN. We now turn the plane of the hektemoros as well into the plane of construction, i.e. into the plane of the meridian, by swinging it around OTN as axis. Then [E] represents the east point of the horizon if O[E] is perpendicular to OTN; hence [E]ON is the quadrant of the hektemoros in true size. The sun in the hektemoros plane has the distance [S]T from the axis of turning, hence [S]' is again the sun if [S]'T = T[S]. Hence [S]'O[E] is the true size of the arc called hektemoros which we wished to find.

B. The Six Angles

We first discuss the case of the equinoxes, thereby considering the circle of center O at the same time as the representation of the meridian and of the equator, turned into the plane of the meridian about its intersection AO (cf. Fig. 31). Then:

(1). The arc MA is the "*meridianus*" $(=90-\varphi)$.

(2). Let [S] be the sun in the equator and [S]T perpendicular to AO. Then angle T[S]O = SOE = "*hektemoros*" (cf. Fig. 32).

(3). The points X, T, W, and S belong to a plane parallel to the horizon (cf. Fig. 32). Therefore ZW = ZS = "*descensivus*".

(4). The points U, T, V, and S belong to a plane parallel to the vertical (cf. Fig. 32). Therefore MV = MS = "*horarius*".

(5). Angle XOR = ZOB = "*verticalis*" (cf. Fig. 32). But ST = XR. Hence we can find ZB in Fig. 31 by making XR = [S]T[9] and using the circle of center O as representation of the vertical.

(6). Angle UPO = POQ = COE = "*horizontalis*" (Fig. 32). But ST = PU. Hence we can find EC in Fig. 31 by making UP = [S]T[10] and using the circle with center O as representation of the horizon.[11]

General case (cf. Fig. 33): S being not in the plane of the equator, we first construct the day circle for the sun and find as before the true distance [S]T of the sun from the meridian plane.[12] For the rest of the construction Fig. 32 remains valid with the provision that the circle ESA no longer represents the day circle of the sun. Hence the point A in Fig. 32 has not the same meaning as A in Fig. 33 (and Fig. 30) and we therefore call the intersection of the plane of the hektemoros with the plane of the meridian OTA'. Then:

(1). The arc MA' is the "*meridianus*."[12a]

(2). For the "*hektemoros*" below p. 1381 (Fig. 30).

(3). ZW = "*descensivus*."

(4). MV = "*horarius*."

(5). Make XR = [S]T; then XOR = ZOB = "*verticalis*."

(6). Make UP = [S]T; then UPO = EOC = "*horizontalis*."

The justification for the rules (3) to (6) is the same as before (Figs. 32 and 31).

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[9] Ptolemy places R on the same side of ZO as T, which is, of course, perfectly correct but makes the construction more crowded than necessary.

[10] Ptolemy places P on the same side of MO as T; cf. note 9.

[11] The text (Latin only; Heiberg, p. 200, 20) is here corrupt and contaminates cases (5) and (6). Read gec = [*verticalis*, get =]*horizontis*.

[12] If T falls to the other side of OZ the "meridianus" would be counted as NA'; cf. p. 849, note 4. Likewise for the other angles.

[12a] Cf. also notes 9 and 10.

Ptolemy also adds the construction of the "*equinoctialis*" of the older coordinate system (cf. Fig. 26, p. 1380). Since this angle (EOF in Fig. 34) appears also in the sun's day circle which is parallel to the equator (HDS in Fig. 34) its true size is shown by the angle D[S]T in Fig. 33.

C. Graphic Solution

The foregoing discussion would make it possible to determine numerically each one of the six (or seven) angles in an analemma simply by measuring the lengths of the arcs in question in drawings of the type of our Figs. 30, 31, and 33. For the determination of whole sequences of data, however, (as needed, e.g., in the design of sun dials) the lines would soon become much too crowded. To avoid this Ptolemy offers two methods of procedure.

First one can follow up the preceding constructions, step by step, with the corresponding trigonometric computations, all based on plane trigonometry since all parts appear in the plane of the drawing. The possibility for this construction in the plane is indicated by Ptolemy in the form: "Since the arc ... is given, also ... will be known ...." If one replaces these sentences by the corresponding trigonometric formulae one can obtain the modern equivalent of the relations thus derived. We do not need to do this here since it was done explicitly by Luckey[13] and since the result, but not the analytic form, is the same as in the ancient procedure.

The second way, however, is of great historical interest in so far as it reveals the use (and invention) by Ptolemy of methods which today are known as "nomographic." The correct understanding of this part of the "Analemma"[14] is entirely due to P. Luckey ([1927]), who has clearly brought out the main ideas as well as the details in Ptolemy's treatise.

Ptolemy sets out to mechanize the constructions provided by the general theory. To this end two classes of lines are distinguished: those which are "permanent" in the sense that they will be needed for all constructions, irrespective of the specific parameters φ, $\lambda_{\odot}$, and t, e.g. the meridian circle; the other group of lines depends on the individual problem, e.g., the inclination of the horizon. All permanent lines are shown on a circular plate (cf. Fig. 35); if this plate is of metal or stone these lines are engraved, or, if it is wooden, painted in black and red.[15] The surface of the plate is then covered with a layer of wax (obviously thin enough to be transparent) in which the temporary lines can be easily drawn and erased.

We first describe the permanent lines to be drawn on the plate (cf. Fig. 35). Let O be the center of the plate and the circle of radius OM represent the meridian. The declination of the sun for $\lambda = 30°$, 60°, and 90° is known to be about 11;40, 20;30, and 23;50, respectively.[16] If the diameter MO$\varLambda$ is the intersection with the meridian of the equator we can draw the "Month Circles" according to these

[13] Luckey [1927] col. 32 (10) to (15) and col. 23. Cf. also Drecker, Sonnenuhren, p. 6ff.
[14] Heiberg, p. 210ff.
[15] Cf. the similar instructions in Alm. VIII, 8 for the making of a celestial globe.
[16] Rounded from Alm. I, 15 (or from the Planisphaerium, below p. 862 (6a)): 11;39,59, 20;30,9, and 23;51,20.

declinations. For the sake of greater clarity the first (♉) and the third (♋) semicircle are turned to the right, the second (♊) to the left. Each semicircle of the meridian is divided in 12 equal parts which are projected onto the equatorial diameter for the counting of equinoctial hours. A quadrant of a circle with the same radius as the meridian is drawn (with the center in Z) and divided in 90 parts. This quadrant is used to read off angles by using a compass opened to the length of the corresponding chord.[17] On the quadrants of a somewhat larger circle with center O (radius OΛ) are marked the geographical latitudes of the seven climata[18] (in Fig. 35 denoted by I to VII):

I 16;25°	IV 36;0	VII 48;30.
II 23;50	V 40;55(?)	
III 30;20	VI 45;0	

The second and third quadrant is again divided in single degrees[19] in order to make the construction adaptable to any value of φ. This completes the permanent inscriptions on the disk.

For the handling of temporary data two instruments are used: a "plate," i.e. a right angle, and a compass. All analemma constructions can be carried out with these instruments and eventually lead to arcs on the meridian circle of radius OM. In order to measure such arcs in degrees the compass is opened to span the distance between the two endpoints and then transferred to the graduated quadrant of center Z and the same radius.

Ptolemy describes in detail the construction of the six angles both in the case of the equinoxes and for the other solar longitudes. We shall not follow him here since the principle of these constructions remains the same as explained before. The practical side of the procedure will become sufficiently clear from the following example.[20]

Example: find the hektemoros for a location in climate III when the sun is at the beginning of Sagittarius, one seasonal hour after sunrise. For constructions in climate III the horizon III O III[21] is drawn (this is a "temporary" line which can be erased if data for another climate are sought). This line meets the intersection with the meridian of the Month Circle for ♐ in a point G. Hence G[H] is the intersection of the Month Circle with the horizon and [H] is the point of sunrise, while A is the point of culmination for that day. Dividing the arc [H]A into six equal sections provides us with the solar positions for each seasonal hour of daytime

[17] As remarked before all spherical coordinates range only between 0° and 90° (above p. 849, n. 4).
[18] The following numbers are rounded from Alm. II, 8, the minutes being expressed in the text as unit fractions. The values for the climata I to IV and VI are correctly transmitted in the Greek version. For climate VII the Greek text has 48 1/2, the Latin text 48 1/2 1/10, whereas the value in the Almagest (or in the Handy Tables) is 48;32. For the 5th climate the Greek text gives 40 1/3 1/4 1/10, i.e. 40;41, against 40;56 in the Almagest. The Latin text has (incorrectly for climate IV) 43 1/4 which is probably a mistake for 40 1/3 1/4 = 40;35. Perhaps one should emend the text to 40 2/3 1/4 = 40;55.
[19] For the sake of greater clarity Fig. 35 shows only 5° divisions.
[20] The details for all six (or seven) angles are given in Luckey [1927].
[21] Cf. Fig. 36. According to construction the arc from Λ to the left point III is $90-\varphi$ which is the elevation of the equator. Hence IIIOIII in Fig. 36 is the horizon. The perpendicular diameter is the gnomon, not shown in Fig. 36, but directly obtainable by drawing the other diameter IIIOIII in Fig. 35.

(from 1 to 6=A); in our example the sun [S] is at point 1. We now repeat the construction from Fig. 30, p. 1381: we put the right angle (shaded in Fig. 36) with its right angle at O such that one side is at point T, the projection of [S] on AG. Then the other side defines a point [E] on the meridian circle (the east point of the horizon). With the help of the compass of opening T[S] a point [S]' on the meridian circle is found. Keeping one point of the compass fixed in [S]' the other is moved to [E] at the edge of the plate. Then the chord [S]' [E] is the chord of the hektemoros. Its corresponding angle is measured at the circular scale by making SE=[S]'[E].

The construction in the preceding example would be equally valid for the first hour of daytime at a solar longitude of ♒ 0°. It is also valid for hours after noon, counted backwards from A toward [H]. Hours of night are obtainable by dividing the arc B[H] in six equal parts.[22] For the solar longitudes ♌ 0° and ♊ 0° day and night arcs have to be interchanged; the above construction would then concern the first (or last) hour of night.[23] Hence one can use the same basic diagram (Fig. 35) for all zodiacal signs and for every seasonal hour. Ptolemy advises constructing the complete sequence of hours by moving compass and right angle in the same pattern and recording the readings step by step. In this way all data for all climata can be systematically assembled.

D. Tables

Ptolemy's treatise concluded in tables which showed the results of the preceding nomographic constructions for each of the seven geographical climata and the four distinct solar longitudes (♋ and ♑, ♊ ♌ and ♐ ♒, ♉ ♍ and ♏ ♓, ♈ and ♎) at each seasonal hour. Since these tables give the arcs in question to 5 minutes[1] one must assume for the instrument divisions which showed halves and quarters of one degree, a rather formidable requirement beyond the division into single degrees mentioned in the text.[2]

Table 9

hour	hekt.	horar.	desc.	mer.	vert.	horiz.
☉-rise	24;55	65; 5	90; 0	0; 0	90; 0	24;55
1 11	25;15	69;15	75;10	35;15	74;50	20
2 10	31;20	73; 0	60;55	59; 5	60; 0	18;50
3 9	46;50	76	46;10	72;10	45; 5	17;15
4 8	60;10	79;10	31	78;30	30;10	18
5 7	75; 0	81;20	17;30	81;30	15;10	27; 0
noon	90; 0	82;35	7;25	82;35	0; 0	90; 0

[22] The need for hours of nighttime is known, e.g., from Heron's analemma (above p. 847).

[23] Ptolemy turns the plate by 180° such that Month Circles to the left of ΛO always represent negative declinations, to the right positive ones.

[1] One entry, 46;6°, does not fit the common pattern; it is probably an error for 46 1/6=46;10.

[2] Ptolemy, in Alm. I, 12 (Man. I, p. 41 and p. 43), speaks merely of "parts" (μέρη) of degrees on his instruments. Proclus, being a philosopher, does not mind describing instruments with divisions into minutes of arc or even seconds (Hypot. III, 10, Man., p. 44, 23, and III, 69, Man., p. 74, 23f.).

Table 10

hour	hekt.	horar.	desc.	mer.	vert.	horiz.
⊙-rise	24;57	65; 3	90; 0	0; 0	90; 0	24;57
1 11	25;19	69;36	75;38	35;26	74;40	21; 5
2 10	34; 1	73;50	60;58	60; 9	59;39	18;34
3 9	46;34	77;28	46; 9	72;36	44;47	17;31
4 8	60;34	80;12	31;20	78;46	29;55	19; 2
5 7	75;10	82; 0	16;58	81;43	14;59	28;32
noon	90; 0	82;35	7;25	82;35	0; 0	90; 0

Unfortunately only one of these 28 tables has survived (for climate I ($M = 13^h$) and $\lambda_{\odot} = ♋ 0°$ [3]) and this only in the Latin translation. I give here in Table 9 the table from Heiberg's edition,[4] in Table 10 the values recomputed by Drecker[5] which I also used for the graphs in Fig. 37.

For the moments of sunrise and noon most of the angles are known without construction. Indeed (cf. Fig. 25, p. 1380)

at noon: hektem. = horiz. = 90°
horar. = merid. = $90 + \varphi - \varepsilon = 90 + 16;25 - 23;50 = 82;35$ [6]
desc. = 90 − merid. = 90 − 82;35 = 7;25
vert. = 0
at sunrise: vert. = desc. = 90°
merid. = 0
hektem. = horiz. = 90 − horar.

The last mentioned identity is not satisfied in the extant table since

hektem. = horiz. = 24;15, horar. = 65;5.

It is, however, easy to see that the first number should be emended to 24;55 because we know from the Almagest[7] that the ortive amplitude of the summer solstitial point in climate I is 24;57. Hence 24;55 is the value to be expected when rounded to 5 minutes; this then confirms the value 65;5 for the horarius.[8]

*E. Application to Sun Dials**

Ptolemy's "Analemma" does not contain[1] any application to the actual construction of sun dials but the elegance of his definitions becomes fully apparent

[3] This was probably the first table of the set.

[4] With two emendations: one according to above note 1, the second (at 0^h for hekt. and hor.) changing 24;15 to 24;55 (cf. below).

[5] Drecker, Sonnenuhren, p. 9, correcting Delambre HAA II, p. 471. Three of Drecker's numbers have to be changed in order to satisfy the identities for 0^h and 6^h.

[6] Since for the first climate $90 - \varphi + \varepsilon > 90$ when $\lambda_{\odot} = ♋ 0°$ the angles are counted from the northern M-axis (cf. above p. 849, n. 4).

[7] Cf. above Table 1 (p. 38).

[8] Cf. also above Table 10.

[1] At least in the preserved version, but there is no compelling reason for the assumption that Ptolemy went beyond the theory and practice of the analemma proper.

when one looks at the consequences for the design of the three simplest types of dials in which a "gnomon" is mounted perpendicular to one of the three principal planes.

The origin O of the three orthogonal axes is now the top of the gnomon, its foot G lies in the plane which receives the shadow, parallel to one of the coordinate planes. We call u, v, w the three coordinate axes (cf. Fig. 38), α the spherical coordinate in the u, v-plane which is parallel to the receiving plane, β the angle with respect to the w-axis, such that α, β are the Ptolemaic coordinates of the sun S. Then the endpoint s of the shadow can be constructed in the plane of the dial (Fig. 39): in the right triangle OG[s] one has the length of the shadow in true size given by G[s] if the angle at O is β, while α gives the direction of the shadow with respect to the (v)-axis. Hence s is the endpoint of the shadow when the sun is at S.

This construction holds for all three coordinate planes in exactly the same fashion:

Plane of dial	α	β	u-axis	v-axis
horizon	horiz.	descen.	M-axis	E-axis
vertical	vert.	horar.	E-axis	Z-axis
meridian	merid.	hektem.	Z-axis	M-axis

As an example one finds in Figs. 40 to 42 the curves for the endpoint of the shadow between sunrise and noon for climate I at the summer solstice,[2] using the data from Table 9 (p. 854).

If $\lambda_{\odot}$ is held constant and the (seasonal) hours are varied between 0 and 6 (and symmetrically to 12) one obtains as the locus of s the "day-curves" (as shown in Figs. 40 to 42). If one varies $\lambda_{\odot}$ and keeps the hour constant one obtains the "hour-curves."

Experience shows that for localities not too northerly the hour-curves are practically straight lines (cf. Fig. 43). This is very useful as a check in the design of a plane dial. The theory, however, is less simple and caused much discussion.

Thābit b. Qurra (d. 901) denied straightness[3]
Abū Isḥāq Ibrāhīm ibn Sinān (d. 946) disproved straightness[4]
Commandinus (1562) assumed straightness
Maurolicus (1575) denied it
Clavius (1581) disproved it
Delambre (1814) at first insisted on straighfness
Settele (1816) disproved it.[5]

The proof applies to all plane sun dials since it depends only on the question whether or not the hour-curves on the celestial sphere are great circles or not. We have, of course, no idea to what extent the theory of sun dials was developed in antiquity beyond the very foundations known to us through the Analemma.

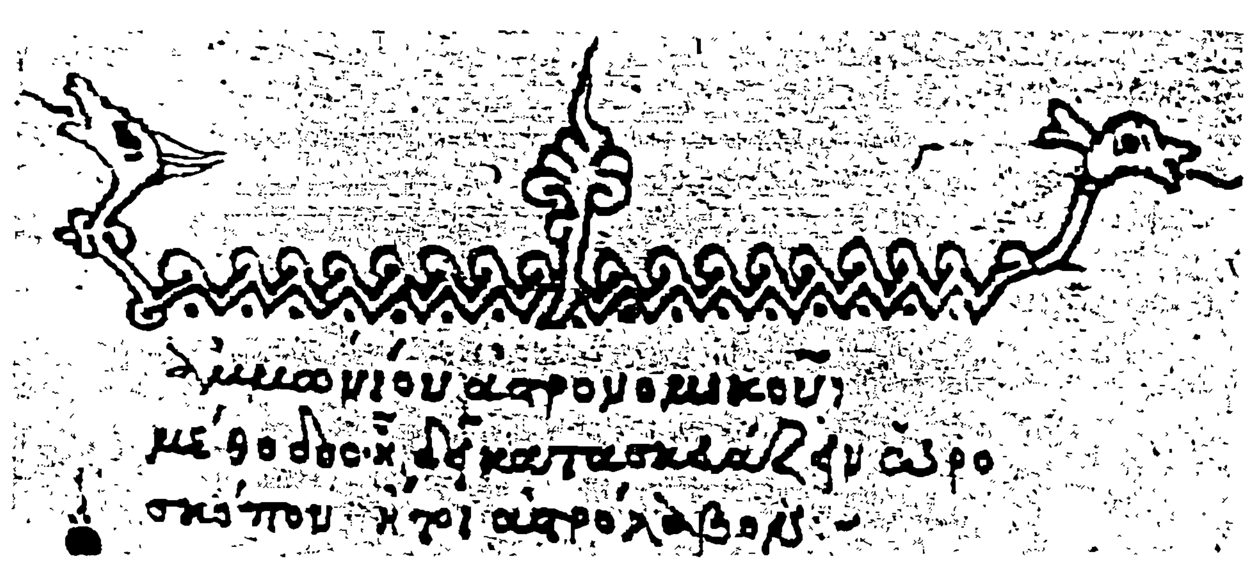
[2] Note that the sun rises to the north of east.
[3] Garbers, Ṯābit p. 7.
[4] Luckey [1948], p. 508/9. Abū Isḥāq was a grandson of Thābit b. Qurra.
[5] Cf. Drecker, Sonnenuhren, p. 12ff. for the history of the problem and for the final proof.

7. The Origin of the Conic Sections

As is well-known[1] the earliest form of definition of "conic sections" in Greek mathematics considers these curves as generated by the intersection of a plane with a circular cone such that one generating line is perpendicular to the plane in question. The three types of curves (by Apollonius[2] called ellipse, parabola, hyperbola) correspond to the three possibilities for the angle at the vertex, less, equal, or greater than 90°, respectively.

I cannot see any other motive for the investigation of this specific class of curves than the theory of the shadow curves for sun-dials.[3] The segment of fixed length of the generating line which is orthogonal to the plane corresponds to the vertical gnomon, the vertex of the cone is the tip of the gnomon at the center of the celestial sphere, and the cone is circular since the shadow-casting rays originate at the day-circle of the sun on the celestial sphere. The endpoint of the shadow indeed describes each day a conic section.

Unfortunately, the condition that the gnomon itself should be part of the conic surface requires, for the vertical gnomon, that the sun reaches the zenith, thus restricting us to geographical latitudes $|\varphi| < \varepsilon$ and even then to only two days in a year. This latter restriction remains, even if one assumes a tilted shadow-plane, as long as one requires that the shadow curve should contain the foot of the gnomon. In other words the hypothesis of an origin of the theory of the conic sections in the design of sun-dials leads to a model without practical usefulness. I do not know how to resolve this contradiction.



§ 3. The "Planisphaerium"

1. Introduction

An important little treatise by Ptolemy is known in the late Middle Ages as the "*Planisphaerium*" because it deals with the representation of the celestial

[1] Cf., e.g., Heath, GM II, p. 111.
[2] Heath, GM II, p. 138.
[3] Cf. Neugebauer [1948, 2].

sphere by means of figures in the plane. This mapping, known today as "stereographic projection," enjoys two important properties: preservation of circles and conformality. As far as we know only the first fact was recognized in antiquity; on it is based the construction of the famous instrument later called the "astrolabe"; to the second quality is due the role played by stereographic projection in the modern theory of complex variables. We shall prove both properties before turning to a description of Ptolemy's treatise.

There is plenty of evidence that the method of stereographic projection is not Ptolemy's invention but antedates him by at least two centuries. It has often been stated, as if it were self-evident, that Hipparchus created the method and applied it to instrumental use, while others with equal confidence credited Eudoxus, two centuries before Hipparchus, with the invention. The scanty evidence upon which such statements rest (or do not rest) will be presented at the end of this section.[1]

The available material is by far too fragmentary to see clearly the historical consequences of the discovery of stereographic projection. I can only conjecture that the standard arrangement with the south pole of the equator as center of projection caused Hipparchus to use the simultaneously culminating points of the ecliptic[2] for the characterization of stellar positions. This is a coordinate which then plays an important role in Indian astronomy ("polar longitudes" in modern notation[3]) and re-appears in astrolabic treatises of mediaeval Europe as "*mediatio coeli.*"[4]

One of the main reasons for the early interest in stereographic projection probably lies in the possibility of dealing with problems of spherical astronomy by means of plane trigonometry, an aspect which is still very visible in Ptolemy's treatise but disappears from the later astrolabic literature. The interest shifts more and more from the theory of projection to the applications of the instrument. On the other hand one can imagine that the transformation of spherical figures to the plane contributed to obscuring the basic difference between great circles and small circles in spherical geometry and thus effectively delayed the development of spherical trigonometry.

Finally it should be mentioned that A. Rome observed[5] that in the commentaries of Pappus and Theon to the Almagest, representations of spherical figures seem to be influenced by the method of stereographic projection. In fact, the same type of figures is already well attested in the early treatises on spherical astronomy and thus more likely appears to be the forerunner of stereographic projection than its application[6].

The Preservation of Circles. Apollonius, Conic Sect. I, 5, proved that two families of circular sections exist on every oblique circular cone[1]: one consists of the

[1] Below V B 3, 7 B.
[2] Above p. 279.
[3] As far as I know introduced by E. Burgess (S. S., p. 320).
[4] Cf., e.g., Kepler, Epitome Astronomiae Copernicanae III, 5 (Werke 7, p. 217f.).
[5] Rome, CA I, p. 141, note.
[6] Cf. above IV D 3, 2.
[1] In Conic Sections I, 9 Apollonius shows that there are no other circular sections. For a straight circular cone the two families of circles coincide.

circles parallel to the given circular base while the second set is produced by parallel planes inclined in the following way to the planes of the first family: Let A be the center of the circular base, B the vertex of a cone (cf. Fig. 44); let BCD be the plane which contains the axis AB of the cone and which is perpendicular to the plane of the base; draw a line KH in the plane BCD such that the angle at K equals the angle at C (and therefore also angle at H = angle at D). We shall prove that a plane through KH and perpendicular to the plane BCD intersects the cone in a circle.

Indeed, let P be a point of the section which has KH as diameter (Fig. 45), $PQ \perp KH$ and $EF \parallel CD$ the diameter of the circle which is parallel to the base and which also contains PQ. Then $PQ^2 = EQ \cdot FQ$. But the triangle EKQ is similar to the triangle HFQ and thus $\frac{EQ}{KQ} = \frac{HQ}{FQ}$. Hence also $PQ^2 = KQ \cdot HQ$ which indicates that $PH \perp PK$, i.e. that P lies on a circle of diameter KH. This is a circle of the second family.

The "*stereographic projection*" is a mapping of the sphere (cf. Fig. 46) in which a point P of the sphere of center C is mapped on a point P′ of the plane by means of a straight line SPP′, where S, the center of the projection, is a point of the sphere such that the direction SC is orthogonal to the image plane to which P′ belongs. It is, of course, irrelevant whether we assume that this plane is tangential at T to the sphere (as in Fig. 46) or that it passes through C as assumed in Ptolemy's constructions.

We now show that circles on the sphere are mapped into circles in the plane. Let PQ be the diameter of a circle on the sphere (Fig. 47), thus SPQ the central section of an oblique circular cone, perpendicular to its base. The angles at Q and T are both denoted by α because they have the chord PS in common. Therefore the angle at S is $\bar{\alpha} = 90 - \alpha$; since $ST \perp TP'$ the angle α' at P′ also equals α, hence $\beta' = \beta$. Thus the plane P′Q′ intersects the projecting cone in the second family of circles, while PQ belongs to the first one.

A special case is represented by the circles on the sphere which contain the center of projection S. Such circles are mapped on straight lines. And vice versa: straight lines in the plane are mapped on circles on the sphere which go through S. For this reason it is convenient to adopt a terminology according to which the straight lines in the plane are also "circles" (of "infinite radius").

Conformality. The last remarks lead to a simple proof of the conformality of the stereographic projection.[2] Let t_1 and t_2 be two tangents to the sphere at a point P (cf. Fig. 48), defining the angle α between two curves on the sphere intersecting at P. The corresponding directions at P′ intersect at an angle α'. Conformality means that $\alpha' = \alpha$. Indeed, the plane defined by t_1 and S intersects the sphere in a circle which contains S and therefore maps t_1 onto a straight line t_1' through P′. The tangent τ_1 to this circle at S is parallel to t_1'. Similarly for the plane defined by t_2 and S: the tangent τ_2 in S to the circle which is mapped on t_2' is parallel to t_2'. Hence the angle between τ_1 and τ_2 is the same as the angle α' between t_1' and t_2'. But the angle between the tangents τ_1 and τ_2 at S between the two intersecting circles is the same as the angle α between the tangents t_1 and t_2 at P. Thus $\alpha = \alpha'$, q.e.d.

[2] The first published proof seems to be due to Halley [1696], p. 204 f.; cf. Lohne [1965], p. 26 f., also about predecessors (Harriot, Hook, de Moivre).

As far as I know the preservation of angles is nowhere mentioned in ancient or medieval astrolabic literature. This is surprising since the orthogonal coordinate systems of spherical astronomy appear again as orthogonal networks in the plane, as every instrument maker must have realized.

2. Auxiliary Theorems

Ptolemy's "Planisphaerium" is concerned with two main problems: to construct the images under stereographic projection of the principal circles of the celestial sphere and to determine the rising times at sphaera recta and at sphaera obliqua without making use of spherical trigonometry.

From now on we shall use as center of projection the south pole S of the celestial sphere, as image plane the plane of the celestial equator. Thus the latter will be mapped onto itself, the northern hemisphere on its interior (with the image N of the north pole as center), the southern hemisphere on its exterior. And it is taken for granted that all circles are mapped onto circles, except circles which contain S, which are mapped on straight lines; in particular circles of declination (i.e. hour-circles in modern terminology) appear as radii intersecting at N.

The first problem consists in finding the images of the ecliptic and of the horizon. Both are great circles whose position can be defined by their extremal declinations, $\pm\varepsilon$ for the ecliptic, $\pm(90-\varphi)=\pm\bar{\varphi}$ for the horizon. Consequently both circles can be constructed if we can find the image of a circle which touches parallel circles to the equator of given declination $\pm\delta$ in diametrically opposite points.

The solution of this problem makes use of methods which we would consider today as belonging to "descriptive geometry." That methods of this type were well developed in antiquity is not only shown by the two Ptolemaic treatises, the "Planisphaerium" and the "Analemma,"[1] but also by the writings of Heron,[2] of Vitruvius,[3] and by several fragments concerning the design of sundials and related problems.[4]

In our present case the plane of construction (Fig. 49) serves alternatively as the plane upon which the celestial sphere is to be mapped and as an orthogonal plane through the center of the sphere. Thus the circle ABCD represents either the celestial equator or a circle of declination with south pole D. In the second case the diameter AC is the intersection of the image plane with the circle of declination ABCD. If E is a point of declination δ, then E' is its image, and similarly F' is the image of F. Considering ABCD again as equator the image of the circle of declination δ is concentric with it and has the radius NE', whereas the circle with radius NF' represents the circle of declination $-\delta$. The image of a great circle which is inclined to the equator at an angle δ is therefore a circle which touches the circle NE' externally, NF' internally. Hence its diameter is E'F'. This circle represents the ecliptic if $\delta=\varepsilon$, the horizon if $\delta=90-\varphi$.

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[1] Above p. 839f.
[2] In the "Dioptra"; cf. p. 847f.
[3] In his "On Architecture"; cf. p. 844.
[4] Cf., e.g., above p. 849f., or below Fig. 67, p. 1395.

The construction in Fig. 49 also solves the problem of finding the stereographic projection of a point of given equatorial coordinates α and δ. The angles α of right ascension are measured on the equator and are not distorted by stereographic projection. The circles of declination are mapped on diameters through N while the amount of declination can be found as in Fig. 49 by means of the parallel circles constructed to the given δ.[5]

In Fig. 49 the circle E'F' intersects the equator in the diametrically opposite points B and D. Since two great circles on the sphere always intersect one another in diametrically opposite points the image of every great circle must meet the equator in the endpoints of a diameter.

In general: every straight line through N is the image of a declination circle (i.e. hour circle). Thus, if P' and Q' are the images of two diametrically opposite points of a great circle the straight line P'Q' must go through N (Fig. 51). And since we can always draw a circle of declination through two diametrically opposite points on the celestial sphere we see that the intersections of any pair of great circles must be mapped on two points which lie on a straight line through N. For example in Fig. 52 the equator is represented by the circle EAWV, the horizon by HEΔW, the ecliptic by VHAΔ. Then ENW is the declination circle which connects the points East and West, V is the vernal, A the autumnal equinox, H the rising, Δ the setting point of the ecliptic. The invisible semicircles of equator and ecliptic are represented by the dotted arcs in Fig. 52.

Ptolemy, in proving these relations, does not make use of the hour circles on the sphere which are mapped on diameters of the equator but he restricts himself – obviously as a matter of methodological principle[6] – to plane geometry by using a theorem for intersecting chords (Euclid III, 35) and its inverse (not in Euclid). That he was fully aware of the mapping of hour circles is not only self-evident but is also apparent in all applications that follow.

3. Right Ascensions

The next goal is the numerical determination of the rising times of the single zodiacal signs, first for sphaera recta, then for sphaera obliqua.

The initial step consists in computing the parameters for the image of the ecliptic on the basis of the construction shown in Fig. 53. Let F and G be points of declination $\pm\varepsilon$, D the south pole; we wish to compute NF' and NG' = NE'. Since $\alpha+\beta=90$ and $\beta+\gamma=90$ we have $\gamma=\alpha$. Hence

$$\frac{\mathrm{ND}}{\mathrm{NG'}}=\frac{\mathrm{BF}}{\mathrm{DF}} \tag{1a}$$

and

$$\frac{\mathrm{BF}}{\mathrm{DF}}=\frac{\mathrm{NF'}}{\mathrm{ND}}. \tag{1b}$$

[5] It follows from Fig. 50 that the radius NE' of the circle of constant declination δ is given by $\tan(45° - \delta/2)$, assuming that the radius of the equator is NB = 1.

[6] Cf., for a similar case, below p. 867.

For the numerical calculation we assume

$$\mathrm{ND}=R=60, \quad \varepsilon=23;51,20°. \tag{2}$$

Therefore, using the table of chords in Alm. I, 11

$$\mathrm{BF}=\mathrm{Crd}(90+\varepsilon)=100;33,28$$

$$\mathrm{DF}=\mathrm{Crd}(90-\varepsilon)=\ 65;29,\ 0.$$

Hence from (1 a) for the radius of the summer solstitial circle

$$\mathrm{NG}'=\mathrm{NE}'=39;4,19 \tag{3a}$$

and from (1 b) for the radius of the winter solstitial circle

$$\mathrm{NF}'=92;8,15. \tag{3b}$$

The image of the ecliptic has therefore the radius

$$R_\varepsilon=1/2(\mathrm{NF}'+\mathrm{NE}')=65;36,17 \tag{4}$$

and the center of this circle[1] has from N the distance

$$a=R_\varepsilon-\mathrm{NE}'=26;31,58. \tag{5}$$

This enables us to draw the circle BF′DE′ which represents the ecliptic.

The arrow BG in Fig. 53 indicates the direction of the daily rotation of the equator; hence B is the vernal equinox, E′ the summer solstice. In order to find the points which represent the longitudes $\pm 30°$, $\pm 60°$, etc., we construct the circles of constant declination which go through these points of the ecliptic. In other words we have first to take the declinations δ which belong to the endpoints of the zodiacal signs and then compute the radii of the corresponding parallels to the equator.

One could say that the table of declinations in Alm. I, 15 allows us to find δ for any given λ. This table, however, was computed by the use of spherical trigonometry which did not exist at the time that the methods of stereographic projection were developed. It is therefore important to remember that the required information is also obtainable by using only plane trigonometry as has been shown in the discussion of the "Analemma".[2] Thus we are, in any case, entitled to assume to know the values of δ which correspond to the longitudes $\lambda=\pm 30°$ and $\lambda=\pm 60°$ on the ecliptic, i.e.

$$\delta_1=\pm 11;39,59°, \quad \delta_2=\pm 20;30,9° \tag{6a}$$

respectively. Now we can repeat the preceding procedure, substituting the above values of δ for the arcs GA=AF in Fig. 53. In this way Ptolemy obtains the following radii R_δ[3]

[1] This is, of course, not the image of the center of the ecliptic.

[2] Cf. above p. 303 and Fig. 285 there.

[3] Recomputing these values I find only small deviations in the seconds, excepting the first value where I obtain 41;37,16.

$$\begin{array}{ll} \delta & R_\delta \\ +20;30,\ 9 & 41;39,15 \\ +11;39,59 & 48;52,42 \\ -11;39,59 & 73;39,\ 7 \\ -20;30,\ 9 & 86;29,42. \end{array} \tag{6b}$$

Circles with these radii and common center N are drawn in Fig. 54; their intersections with the ecliptic are the endpoints of the zodiacal signs.

The next step consists in finding the rising times $\alpha_1, \alpha_2, \ldots, \alpha_{12}$ of the single zodiacal signs. At sphaera recta the horizon passes through the poles of the equator. Therefore every straight line through N represents a possible position of the horizon. In Fig. 55 V represents the vernal equinox, VNA and H_1NH_2 two positions of the horizon, H_1 being the point ♓0°, H_2 ♍0° when the dotted circle H_1H_4 is the circle of constant declination $-\delta_1 = -11;39,59°$ (cf. (6b)), H_2H_3 of declination $+\delta_1$. The arc VH_1 of the ecliptic crosses the horizon in the same time as the arc VE of the equator. The arc $\alpha = VE$ is therefore the rising time α_{12} of ♓. In order to determine its value one draws ML perpendicular to H_1H_2. Then α appears also at M. The distance MN is known to be 26;31,58 (cf. (5), p. 862) and LN can be found from

$$LN = H_1N - 1/2\, H_1H_2$$

because we know (from (6b)) that $H_1H_2 = H_1N + NH_2 = 73;39,7 + 48;52,42 = 122;31,49$. Hence

$$LN = 73;39,7 - 61;15,55 = 12;23,12 \tag{7}$$

and

$$\sin\alpha = \frac{LN}{MN} = \frac{12;23,12}{26;31,58} = 0;28,0,38$$

hence

$$\alpha = 27;50°.$$

Since $H_1 =$ ♓ 0°, $H_2 =$ ♍ 0°, $H_3 =$ ♈ 30°, $H_4 =$ ♎ 30° we see that

$$\alpha_{12} = \alpha_6 = \alpha_1 = \alpha_7 = 27;50°. \tag{8}$$

This is exactly the value given as right ascension of Aries 30° in Alm. II, 8.

We now repeat the same process under the assumption that $H_1 =$ ♒ 0° and $H_2 =$ ♌ 0°. The angle α in Fig. 55 now represents the rising time of the total arc ♒ and ♓. Using for H_1N and H_2N the values known from (6a), p. 862 for $\pm\delta_2$ we find

$$\alpha = 57;44 \tag{9}$$

again in agreement with the right ascension given in Alm. II, 8 for the total of the first two signs. In order to obtain the rising time α_{11} by itself we have only to form with (8)

$$\alpha_{11} = 57;44 - 27;50 = 29;54.$$

Finally $\alpha_{10} = 90 - 57;44 = 32;16$ because the right ascension of each quadrant of the ecliptic is 90°. Thus we have found

$$\begin{aligned} \alpha_{11} &= \alpha_5 = \alpha_2 = \alpha_8 = 29;54°, \\ \alpha_{10} &= \alpha_4 = \alpha_3 = \alpha_9 = 32;16°. \end{aligned} \tag{10}$$

These arcs are shown in correct size in Fig. 54 (p. 1390).

4. Oblique Ascensions

For geographical latitudes $\varphi \neq 0°$ the horizon is no longer mapped on a diameter. In order to find its image we have only to remark that the extremal declinations of the horizon are $\pm(90-\varphi)$. Hence we can repeat the construction of Fig. 53, assuming $GA=AF=90-\varphi$. A circle which touches the circle of declination $-(90-\varphi)$ interiorly (at F'), the circle of declination $+(90-\varphi)$ exteriorly (at E'), is then a representation of the horizon of geographical latitude φ.

Ptolemy uses as example $\varphi=36$, the latitude of Rhodes. If r represents the radius of the horizon-circle, NO the distance of its midpoint O from the north Pole N, he finds (with $NV=60$)

$$\begin{aligned} r &= 102;4,45 \\ NO &= \ 82;35,3 \end{aligned} \tag{1}$$

(cf. Fig. 56 which is drawn to scale).

It is now easy to determine to given φ the longest daylight M. If $\rho_1, \rho_2, \ldots, \rho_{12}$ represent the rising times of the individual zodiacal signs from ♈ to ♓ we have

$$M=\rho_4+\rho_5+\cdots+\rho_9=360°-(\rho_1+\rho_2+\rho_3+\rho_{10}+\rho_{11}+\rho_{12}).$$

Fig. 57 shows that the ecliptic quadrant VH = ♑ + ♒ + ♓ crosses the horizon simultaneously with the equator arc VE. Thus

$$VE=\rho_{10}+\rho_{11}+\rho_{12}=90-\alpha.$$

By inverting the sense of rotation of the equator in Fig. 57 $A\Delta$ would represent the quadrant ♈ + ♉ + ♊ of the ecliptic and $AW=90-\alpha$ the corresponding rising time $\rho_1+\rho_2+\rho_3$. Hence we see that $\rho_1+\rho_2+\rho_3=\rho_{10}+\rho_{11}+\rho_{12}$ and

$$M=360-2(\rho_1+\rho_2+\rho_3)=180+2\alpha.$$

Since $H\Delta \perp VA$ the center O of the horizon must be located on the normal MO through the midpoint M of the image of the ecliptic where $NM=26;31,58$ according to (5), p. 862. On the other hand we have from (1) $NO=82;35,3$. Thus

$$\sin\alpha=\frac{26;31,58}{82;35,3}=0;19,16,36 \qquad \text{hence } \alpha=18;45.$$

This gives for the longest daylight

$$M=180°+37;30°=12^h+2;30^h=14\ 1/2^h \tag{2}$$

which is indeed the value for $\varphi=36$ (cf. Alm. II, 8). We also note that

$$\rho_{10}+\rho_{11}+\rho_{12}=90-\alpha=71;15°. \tag{3}$$

By a similar process we can determine the rising times of the individual zodiacal signs. We bring the horizon into such a position (Fig. 58) that H_1 = ♓ 0°, H_2 = ♍ 0°. The equator meets this horizon in E and W. Simultaneously with the ecliptic arc VH_1 the equator arc VE crosses the horizon. Thus VE is the rising time ρ_{12} of ♓. The diameter H_1NH_2 shows the position of the horizon at sphaera recta when H_1 is rising. Thus $VF=\alpha_{12}$ is the right ascension of ♓ (as VE in Fig. 55, p. 1390) and

$$\rho_{12}=\alpha_{12}-n$$

where $n = \mathrm{EF}$ is an "*ascensional difference*".[1] Since the circle of the horizon must contain the points $\mathrm{H}_1, \mathrm{H}_2$ and E, W, its center O must lie on the intersection of the perpendicular LMO to the chord $\mathrm{H}_1\mathrm{H}_2$ of the horizon and of NO which is perpendicular to ENW. Thus n can be found from $\sin n = \dfrac{\mathrm{LN}}{\mathrm{NO}}$ where LN is known from sphaera recta (Fig. 55) to be 12;23,12 (cf. (7), p. 863) and NO = 82;35,3 as radius of the horizon (cf. (1), p. 864). This leads to $n = 8;38$. Because $\alpha_{12} = 27;50$ (cf. (8), p. 863) we find

$$\rho_{12} = \rho_1 = 27;50 - 8;38 = 19;12^\circ \tag{4}$$

as rising times for ♈ and ♓ at $\varphi = 36^\circ$ (in agreement with Alm. II, 8). Because of the symmetry relations for rising times with respect to the solstices[2] we have

$$\rho_1 + \rho_6 = \alpha_1 + \alpha_6$$

and because $\rho_1 = \alpha_1 - n$ and $\alpha_1 = \alpha_6$ we find

$$\rho_6 = \rho_7 = \alpha_1 + n = 27;50 + 8;38 = 36;28^\circ. \tag{5}$$

Repeating the same construction with $\mathrm{VH}_1 = ♒ + ♓$ gives for n the value 15;46. The corresponding right ascension is (cf. (9), p. 863) $\alpha_{11} + \alpha_{12} = 57;44$. Therefore

$$\begin{aligned} \rho_{11} + \rho_{12} &= 57;44 - 15;46 = 41;58, \\ \rho_5 + \rho_6 &= 57;44 + 15;46 = 73;30. \end{aligned} \tag{6}$$

Since ρ_{12} and ρ_6 are known from (4) and (5) we have finally

$$\rho_{11} = \rho_2 = 22;46, \qquad \rho_5 = \rho_8 = 37;2. \tag{7}$$

Using (3) and (6) we obtain

$$\rho_{10} = \rho_3 = (\rho_{10} + \rho_{11} + \rho_{12}) - (\rho_{11} + \rho_{12}) = 71;15 - 41;58 = 29;17 \tag{8}$$

and similarly from $\rho_4 + \rho_5 + \rho_6 = 180 - (\rho_{10} + \rho_{11} + \rho_{12}) = 108;45$

$$\rho_4 = \rho_9 = 108;45 - 73;30 = 35;15. \tag{9}$$

This completes the determination of the rising times of the zodiacal signs for $\varphi = 36^\circ$ (cf. Fig. 56, p. 1391).

5. The Greatest Always Invisible Circle

What we have described so far can be called the theoretical part of Ptolemy's "Planisphaerium". It covers the construction of the parallels to the equator (and therefore of all points given by right ascension and declination) and with their help the construction of the longitudinal divisions of the ecliptic, in particular of the zodiacal signs and their rising times.

The second part completes these investigations by the construction of the circles orthogonal to the ecliptic and of the circles of constant latitude, such that every point with given ecliptic coordinates can be found. This, however, is done under

[1] Cf. above p. 36.

[2] Cf. (B), p. 35 and Fig. 29 there.

specific reference to the practical use in a "horoscopic instrument". Here the ecliptic system makes the so-called "spider", i.e. the movable network that carries the pointers which indicate the position of stars that are defined by ecliptic coordinates.

It is in view of this practical application that Ptolemy introduces the question of the size of the instrument. At a locality of geographical latitude $+\varphi$ no star will ever be visible whose distance from the south pole is $\leqq \varphi$ (cf. Fig. 59). Thus the circle of declination $\delta = -(90-\varphi)$ is the "greatest always invisible circle". The radius r_0 of its image under stereographic projection is readily constructed (Fig. 60); obviously this radius determines the desirable size of the instrument. Consequently the value of r_0 is the primarily given element for the instrument maker; Ptolemy therefore answers the question of how to find to a given radius r_0 of the greatest always invisible circle the radius R of the image of the equator.

The method consists, of course, in a reversal of the construction for finding r_0 from R (Fig. 60). In a circle of given radius r_0 one draws a chord which is subtended by the given angle φ (cf. Fig. 61). Its continuation intersects at a point C the tangent DC which is parallel to NA. If we make NE=DC then we will show that EA is the radius R of the equator which belongs to the given r_0 and φ.

Indeed, we repeat in Fig. 62 the construction of Fig. 61 and draw with center N the circle of radius NS=EA. Let T be the intersection of DS with this circle. By construction the angle at the south pole S (and also at T) is the same as the angle at A (and also at B); therefore the chord ST is subtended by the angle φ. Thus r_0 is the radius of the greatest invisible circle which belongs to the circle of radius EA=R as equator for the given geographical latitude φ.

6. Ecliptic Coordinates

The orthogonals to the ecliptic are easy to construct. One point is always the northern[1] pole of the ecliptic, located on the solstitial colure at a given distance ε from the north pole. If we use in Fig. 63 the point A as the south pole, Q as the actual pole of the ecliptic, then P is the image of Q. If we now wish to find the latitude circle which connects a given point Λ of the ecliptic with P we need only remember that this great circle must meet the ecliptic also in the diametrically opposite point Λ' and that $\Lambda\Lambda'$ goes through N.[2] The circle circumscribing the triangle $\Lambda P \Lambda'$ solves our problem.[3]

Also the mapping of the parallels to the ecliptic, i.e. of the circles of constant latitude β, presents no difficulties, at least in principle. We consider first the circle ESE' in Fig. 64 as the meridian, EE' as a diameter of the ecliptic, [A] [B] as diameter of a parallel circle of latitude β. Seen from the south pole S the diameter [A] [B] will be mapped on a diameter AB in the equatorial plane. Consequently the image of the circle of latitude β is the circle with diameter AB and midpoint C. In this way all circles of constant latitude can be constructed,

[1] The opposite pole would be mapped into the exterior of the always invisible circle and hence does not belong to the accessible area of an instrument.

[2] Cf. below p. 1388; Fig. 51.

[3] Every such circle meets the ecliptic at Λ and at Λ' at right angles.

provided one has the whole plane at one's disposal. For the construction of an instrument, however, only such circles and arcs of circles are of interest that lie within the greatest always invisible circle. This fact imposes some limitations to the procedure in Fig. 64.

Fortunately it is not necessary to obtain both endpoints of the diameter AB because the circle of latitude β must contain as a chord the diameter $FG \perp AB$ of the circle of constant declination whose center is the point K in which the circle of latitude β meets the axis of the celestial sphere (Fig. 66). Thus, even if A becomes inaccessible,[4] one still has three points (F, B and G) at one's disposal through which the visible arc of our circle must go. Indeed, if Fig. 65 represents the meridian of the celestial sphere, [A] [B] the diameter of the parallel circle to the ecliptic EE', then K is the point in which the axis NS meets the plane of the circle [A] [B]. This point K is also the center of a parallel circle DD' to the equator. Its diameter FG which is perpendicular to DD' is the line of intersection between the planes of the circle DD' and the circle [A] [B]. FG is therefore a chord in [A] [B], q.e.d.

Ptolemy's proof of this fact avoids all reference to the three-dimensional sphere and operates strictly in the plane, making use of Euclid III, 35 and its inverse.[5] Since we have used the much simpler demonstration on the sphere it will suffice here to repeat the construction of Fig. 64 with the addition of the circle of constant declination DKD' which is mapped into the circle of diameter FNG, which is perpendicular to BN (cf. Fig. 66). The image of the circle of constant latitude β indeed contains all four points F, B, G, and A.

For great negative latitudes these constructions become impractical. Ptolemy therefore describes another device that allows us to find the image of a circle of constant latitude belonging only in part to the visible domain. Let [A] [B] be the diameter of a circle of constant latitude (cf. Fig. 67) which enters the area of invisibility, that is to say a circle which has an arc inside the circle of center S and radius φ around the south pole. Let this circle be turned around its diameter [C] [T] [D] into the plane of construction. The line [E] [F] which is perpendicular to [C] [D] is the line of intersection of the plane of the circle [C] [F] [D] with the plane of the circle of constant latitude. Therefore [F] is the point at which the circle [A] [B] meets the circumference of the greatest always invisible circle of diameter [C] [D]. The point [F] divides the circumference of the semicircle [C] [F] [D] in a certain ratio which appears undistorted in our construction because the plane of the circle [C] [F] [D] is parallel to the equatorial plane. The circle [C] [F] [D] with center [T] is mapped into the circle CFD with center N without distortion of angles with vertex [T] or N. Consequently the image F of [F] is found by making NF parallel to [T] [F]. Thus the image of the given circle of constant latitude meets the greatest always invisible circle at F (and at the symmetric point F'). One more point of this circle of latitude is A, the projection of [A] from S. This suffices for the construction of the arc FAF' which belongs to the visible region at a geographical latitude φ.[6]

In a badly preserved final section Ptolemy speaks about alternative possibilities for the definition of stellar positions: either ecliptic or equator coordinates. For

[4] This is the case when $\beta < -(90-\varphi-\varepsilon)$.
[5] Cf. above p. 861.
[6] The arc FAF' degenerates to a straight line when [B] coincides with S, that is for $\beta = -(90-\varepsilon)$.

both cases he suggests drawing the coordinate lines at intervals of either 6°, or 3°, or 2° (depending on the size of the planisphaerium) since these numbers are also divisors of the 30° which constitute one zodiacal sign and the 24° which are a convenient approximation for ε.

With these remarks concludes the extant version of the "Planisphaerium."

7. Historical Remarks; Synesius

A. Introduction

The method of stereographic projection is the basis for the instrument known as the (plane) "astrolabe"; consequently the history of this instrument and the history of the theoretical background are not only intimately related but are also particularly involved since influences from very different directions were simultaneously at work.

For easy reference for what follows I give here a list of approximate dates which otherwise belong to quite different chapters.

Vitruvius	died after A.D. 27
Theon of Alexandria	born about 335
Hypatia	died 415
Synesius of Cyrene	died between 412 and 415
Proclus	died 485
Ammonius, pupil of Proclus	
Philoponus, pupil of Ammonius	died about 555
Severus Sebokht	died 665
Ya'qūbī	about 875
Suidas	about 1000
al-Majrīṭī	died 1007/8
Nicephoros Gregoras	died 1359

B. Earliest History; Hipparchus

It is evident from the writings of Vitruvius, to be discussed in the next section, that not only the theory of stereographic projection existed before Ptolemy but practical applications as well. We can only conjecture the accurate time of invention. From a purely mathematical viewpoint all necessary methods are attested for Apollonius and nothing would exclude the same for Euclid (who also wrote on conic sections) or even for the geometers of the fifth century.

What speaks against such an early origin of the discovery of stereographic projection is not the mathematical background but its astronomical significance. As far as we know the interest of the method lies exclusively in its application to astronomy. In particular stereographic projection is a tool for the solution of problems in spherical astronomy, e.g. the determination of right and oblique ascensions of the signs of the zodiac, problems presupposing an accurate numerical approach to astronomical phenomena. All we know about early Greek astronomy, seen under this aspect, speaks against the period before Apollonius or Hipparchus.

It favours the latter since we know nothing about numerical procedures or trigonometric tables with Apollonius, nor of a consistent use of the sexagesimal system, so intimately connected with all Greek numerical procedures. In other words, merely from our knowledge of the use actually made of stereographic projection, combined with our, however fragmentary, knowledge of the work of Hipparchus in contrast to all his predecessors the most plausible conjecture seems to be to consider Hipparchus as the inventor of the method.

Beyond this general argument we have at least one specific statement from antiquity which supports our conjecture. Synesius, who certainly still had access to works of Hipparchus through his teacher Hypatia, the daughter of Theon of Alexandria, says:[1] "The unfolding of a spherical surface, preserving identity of ratios within the diversity of figures, Hipparchus of olden times vaguely sketched and he was the first who applied himself to this problem." Since this was written in connection with an instrument based on stereographic projection we know that it is exactly our problem that Synesius had in mind, even if he did not quite understand Hipparchus, as will not surprise a reader of Synesius' own explanations.[2]

A passage in Vitruvius that gives a list of alleged *("dicitur")* inventors of sun dials was used as an argument for a pre-Hipparchian origin of stereographic projection[3]. There he says that "the spider (was invented) by the astronomer Eudoxus or, as some say, by Apollonius." Now we know from Ptolemy's treatise[4] that the movable parts of the planisphaerium were called "spider." But it is by no means certain that every "spider" must be part of a planisphaerium. On the contrary, we know from the continuation of Vitruvius' rapid historical summary that there existed a "conical spider" which, of course, cannot be related to stereographic projection.[5] Thus it seems that any network, mounted in front of the surface of an instrument, could have been called a "spider." Vitruvius' statement therefore carries very little weight, even if we ignore his cautious "dicitur."

C. *Vitruvius and the Anaphoric Clock*

Vitruvius in his famous work "De Architectura,"[1] written in the decades around the beginning of our era,[2] describes (in IX, 8) a mechanism designed to show for any day of the year the seasonal hours of day or night: the so-called

[1] Terzaghi, Syn. II, p. 138, 18–20; cf. also below p. 873.

[2] Cf. below p. 873ff. Hipparchus has been credited repeatedly with the invention of stereographic projection but often on insufficient grounds. Delambre, e.g., refers to Proclus, Hypotyposis, Chap. V (HAA II, p. 454). Actually the passages in question do not belong to Proclus but to an 800 years younger scholion to a treatise by Nicephoros Gregoras which was added by Giorgio Valla to his edition of Proclus (Venice 1498); cf. Tannery, Mém. Sci. IV, p. 244 f.

[3] Vitruvius, De Arch. IX, 8 (Loeb II, p. 255).

[4] Cf. above p. 866.

[5] R. Böker in RE 9 A, 1 col. 1209/10 has invented a whole string of unfounded hypotheses, leading to a reconstruction of a "spider of Eudoxus" consisting of a branch of a hyperbola and of a parabola. It is regrettable to see such fantasies published in an authoritative handbook.

[1] Latin text, edited by F. Krohn, Teubner 1912. The English translation in the Loeb series (by F. Granger, 2 vols., 1931 and 1934) is very unsatisfactory for the astronomical sections. The German translation of 1796 by August Rode is much better.

[2] Some passages presuppose conditions in Rome around 30 B.C., others at A.D. 27; cf. RE IX A, 1 col. 432.

anaphoric clock, i.e. the clock based on rising times. This clock consists of a disk which is engraved with a stereographic projection of the constellations on the celestial sphere, the ecliptic being represented by a sequence of holes which can carry a knob (*"bulla"*) indicating the sun at its proper longitude. As with a parapegma[3] someone has to set the knob by hand, into the hole for the given day. The disk with the constellation is made to turn one rotation per day by means of a float and a counterweight. The float is lifted by a slowly filling water container, the counterweight has to overcome friction.[4] In front of the disk a fixed "spider," made of wires, represents in stereographic projection the local horizon and the curves which connect the twelfths of the day- and night-arcs of the sun's daily circles. The position of the bulla with respect to the hour-lines of the spider allows one to read off seasonal hours.

The text of Vitruvius assures us that stereographic projection was known at least in the first century B.C. The application to a clock mechanism must have been fairly widespread since a fragment from a large anaphoric clock (about 40 inches in diameter) was found in such a provincial town as Salzburg,[5] another fragment in north-eastern France.[6] Both fragments belong to the period from the first to the third century A.D. Clearly, the principle of construction must have been well known among the "gnomonists" of the time.

D. Ptolemy

Only a distorted title is preserved through Suidas[1] of the original Greek text of Ptolemy's "Planisphaerium": *ἅπλωσις ἐπιφανείας σφαίρας*. This would mean

[3] Cf. above IV A 3, 3.

[4] For a model cf. Diels AT Pl. 18 (p. 217) or the drawing by Rehm RE VIII, 2 col. 2431.

[5] Found probably in, or near, a Roman cemetery which is dated by some coins of Vespasian (around A.D. 75). Epigraphic arguments suggest the first or second century A.D. for the inscriptions on the clock. Good photographic reproduction in the Jahreshefte des österreichischen archaeologischen Institutes in Wien 5 (1902) pl. V to p. 196f. Correct interpretation and reconstruction of the projection by Rehm [1903]. The spacing of the holes for the bulla corresponds to 2° of solar motion.

[6] In Grand ($\varphi=48;22$, 5;28 East), west of Neufchâteau, southwest of Toul (Dep. Vosges). Badly published by Maxe-Werly [1887], p. 170–178 and even worse in C.I.L. XIII 2, 1, p. 138, No. 5955 (1905). In neither one of these publications was it realized that the object was an anaphoric clock, based on stereographic projection.

The remnants of the disk (about one quarter) are now in the Musée des Antiquités Nationales in Saint-Germain-en-Laye which put an excellent photograph at my disposal. The disk is about 18 cm (7 inches) in radius and has a lip around the edge, obviously in order to hold a disk on which the constellations were painted. The back shows the dates (kalends, nones, ides) for the holes of the bulla. Under the inscription Aequinoct VIII K Oct is drawn a straight line which should be a diameter of the equator. Hence the autumnal equinox would be September 24, which is correct around A.D. 200/250 in agreement with the epigraphic evidence.

The workmanship of this clock is very poor. On the preserved arc of 77° one counts 43 holes which gives about 196 intervals for 360° instead of 183. The position of the equinoctial diameter corresponds to an eccentricity of the ecliptic circle of about 4.5 cm instead of 7 cm. However, neither the center of the ecliptic nor of the equator is still preserved on the fragment.

It is of interest that in Grand also were found, in a pit, two ivory diptychs, each of which depicts in five concentric rings (a) sun and moon, (b) the signs of the zodiac, (c) the number of degrees (in Greek numerals) of the planetary "terms" according to the "Egyptian" system, (d) the 36 decans, and (e) their names in Greek; wind goddesses occupy the four corners. These diptychs are presumably from the second century A.D.; cf. Archeologia 71 (June 1974), p. 27 to 29.

[1] Ptolemaeus, Opera II, ed. Heiberg, p. 226; from Suidas (ed. A. Adler IV, p. 254, 7).

"simplification of a spherical surface" but in all likelihood the first word is a mistake for ἐξάπλωσις "unfolding" i.e. projection,[2] a term also used by Synesius and common in the later astrolabic literature. Of Ptolemy's original terminology the Latin version has preserved the terms "*horoscopium instrumentum*" and "*aranea*" (spider).[3]

The term "astrolabe" is used by Ptolemy in the Almagest for the armillary sphere, described in detail in Book V, 1. Probably the "horoscopic astrolabe," recommended in the Tetrabiblos for accurate observations,[4] is a similar, if not the same, instrument. In the Geography (I, 2) "astrolabes" are mentioned as instruments for the determination of geographical coordinates,[5] thus most likely again referring to the armillary sphere which would serve this purpose well. Thus we can be fairly sure that the term "astrolabe" in the time of Ptolemy only means instruments for the observation of positions of celestial bodies, but not the planisphaerium.

The instrument to which Ptolemy refers at the end of his treatise[6] is adapted to only one geographical latitude, Lower Egypt.[7] As outer rim of the instrument appears the greatest always invisible circle,[8] thus again an element which depends on one given geographical latitude. Obviously Ptolemy's instrument was not yet equipped with the interchangeable disks upon which horizons and hour circles for different geographical latitudes are engraved. On the other hand it was fitted to show all risings and settings visible during the year at Alexandria. I think one can describe Ptolemy's "horoscopic instrument" as an anaphoric clock, but not of the Vitruvian type (preserved in Salzburg and Grand) but with the equator and horizon coordinates fixed and the stars with the ecliptic made movable.

It is also clear that the term "astrolabe," meaning to take, to observe, the positions of a star, has originally nothing to do with the planisphaerium which only performs the functions of a celestial globe in a convenient plane projection. Only when the central pivot which is needed for the spider is utilized to carry on the other side of the disk a diagonal ruler equipped with sighting holes the instrument becomes an "astrolabe." Held in vertical position the circular rim can be graduated for the observation of altitudes of stars or of sun and moon. We have no reason to assume such a combination with a "diopter" for Ptolemy's "horoscopic instrument"; but we will see that this transformation to an "astrolabe" was known to Theon.[9]

The text of the "Planisphaerium" as we have it is a Latin translation (by Hermann the Dalmatian, or Secundus, completed in Toulouse in the year 1143) of an Arabic version whose author is Maslama ben Aḥmed al-Majrīṭī (who died in the year 1007/8). A strong indication that Ptolemy's treatise contained more than the extant Latin version comes from a remark by Philoponus, in which he

[2] First suggested by Kaufmann, RE 2 col. 1801, 11.
[3] Ed. Heiberg, p. 249, 22/23.
[4] Ed. Boll-Boer, p. 110, 14/15; Robbins, p. 228/229.
[5] Ed. Nobbe, p. 6, 1.
[6] Above p. 866.
[7] Ed. Heiberg, p. 259, 9. The examples are computed for clima IV (Rhodes, $\varphi = 36°$) "because it is the middle one of the inhabitable climates" (Heiberg, p. 249, 16).
[8] Cf. above p. 865f.
[9] Below p. 878.

tells us that Ptolemy drew the hour lines only below the horizon in order to avoid confusion with the coordinate lines above the horizon.[10] No such discussion of the hour lines is, however, found in our text. This seems to indicate that more about the practical construction of an instrument had been said in the original version than in the extant text which indeed, toward the end is corrupt and gives the impression of incompleteness. Ptolemy's concluding remarks concern the alternative possibilities to draw networks of ecliptic or of equatorial coordinates.[11] This seems to point to a discussion of both types of anaphoric clocks, rather than of the astrolabe in the later sense of the word.

E. Synesius

Synesius was not competent as a scientist, but he was rich and well educated; the history of his life contributes much to our knowledge of the conditions which prevailed in late antiquity, in particular in North Africa, at the end of the fourth century. Repeated raids by the desert tribes and famine caused the city of Cyrene to send Synesius as a special envoy to Constantinople in order to obtain tax relief. This mission kept Synesius in the capital for the years 399 to 402. Whatever the result, it could not have brought lasting improvement. Increased raids from without, as well as corruption among the government officials within, created the necessity of self-defense and local leadership. Therefore, in 410, the office of Bishop of Ptolemais and thereby Metropolitan of the Pentapolis was thrust upon Synesius (who was baptized after his election!). In this new position he had to fight heretics as well as tribesmen, to excommunicate a praefect sent from Constantinople, to become a theologian of sorts and to resign the pursuit of his private life. He died sick and despairing, probably shortly before 415, the year that Hypatia was murdered by the Christian mob of Alexandria. In 436 the fourth Council of Carthage attended by 214 Bishops, decreed that a bishop should not read the heathen writings.[1]

With the history of the astrolabe Synesius is connected through a dedicatory letter which he wrote to a certain Count Paeonius in Constantinople.[2] Obviously in recognition of the support Synesius received during his mission to the capital from Paeonius, he sent him an "instrument" (*ὄργανον*, but never called "astrolabe"), clearly based on stereographic projection. Some treatise, now lost, probably accompanied the instrument. Let us hope that this exposition was less obscure than the letter of dedication. Nicephoros Gregoras states in his letters,[3] more than nine centuries later, that he had made use of Synesius' writings for his own treatise

[10] Hase [1839], p. 139; Drecker [1928], p. 29; Tannery, Mém. Sci. 9, p. 351.

[11] Above p. 868.

[1] The life of Synesius has been described repeatedly; cf., e.g., W.S. Crawford, Synesius the Hellene (London 1901); A. Fitzgerald, The Letters of Synesius of Cyrene (Oxford 1926); Charles Lacombrade, Synésios de Cyrène, Hellène et chrétien (Paris 1951); the article Synesios in RE 4 A, 2 col. 1362–1365 by v. Camphausen (1932). For the Greek text one has now the edition of Terzaghi (1944) which supersedes Migne PG 66, a reprint of the edition by Petavius of 1631. For the Council of Carthage cf. Mansi, Concil., Vol. 3, col. 952, No. XVI.

[2] Terzaghi, Vol. II, p. 132–142; translation in Fitzgerald, Synesius, p. 266.

[3] French translation of excerpts concerning the astrolabe cf. Delatte, AA II, p. 190; more complete in Guilland, Corresp., p. 95f. and p. 249f. Greek text in Bezdeki [1924], Nos. 23 (p. 252, 11–32) and 16 (p. 308, 32–309, 9).

on the astrolabe; but from the quotations given by Nicephoros it seems doubtful that he had more at his disposal than the letter to Paeonius.

From a strictly chronological viewpoint Synesius should perhaps not procede Theon whose daughter Hypatia was his "most revered teacher." Theon himself had written a treatise on the astrolabe, preserved for us only indirectly through Ya'qūbī, Severus Sebokht, and Philoponus.[4] Nevertheless these sources are sufficient to show that Theon's treatise contained a systematic discussion of the theory of the astrolabe and its applications, far superior to the disorganized and utterly insufficient presentation we find in Synesius. Yet Synesius, writing his letter to Paeonius probably soon after his return to Cyrene in 402, insists that he was the first after Ptolemy to deal with the problem. This seems inexplicable unless Theon's treatise was written after Synesius', i.e. shortly after 400. One does not know the date of Theon's death but one usually accepts 400 as plausible.[5] I think one may perhaps add a few years to this date and consider the treatise on the astrolabe as one of Theon's latest writings. On the other hand one must admit that Synesius' contact with the Alexandrians cannot have been very lively when he says in a letter to Hypatia: "I am sending you also my (essay) concerning the gift; this was produced long ago in my ambassadorial period."[5a]

In the following I give a translation of the astronomical passages of Synesius' letter to Paeonius.[6] I follow essentially the translation by Fitzgerald except for the technical terminology which he did not recognize as such.

The projection of a spherical surface, preserving identity of ratios within the diversity of figures, Hipparchus of olden time vaguely sketched and he was the first who applied himself to this problem. But we, if it is not more than befits us to say, have finished the weaving of this tissue even to the fringes and have perfected it, while the problem had been neglected in the long intervening	projection: here and in the following for lit. "unfolding." It is this introductory sentence which is cited, more or less verbatim, in the letters of Nicephoros Gregoras.[7]
time. The great Ptolemy and the divine band of his successors were content to have only the (practical) use (of the method of projection),	successors: one would think of Pappus and Theon but at least Theon's treatise went much farther than Ptolemy's Planisphaerium.[8]
since the sixteen stars, which only were transposed and inserted in the	16 stars: this number agrees well with actual astrolabe constructions[8a].

[4] Cf. below p. 877.
[5] K. Ziegler in RE 5 A, 2 col. 2076, 18, the only argument being that Theon is not mentioned in the stories of the fame and of the tragic end (in 415) of Hypatia.
[5a] No. 154, Fitzgerald, p. 254; Migne PG 66, col. 1557 A.
[6] Terzaghi, Syn. II, p. 138, 18–139, 5; 139, 18–141, 6; 141, 20–142, 5; 142, 10–15.
[7] Cf. Guilland, Corresp., p. 249, p. 95; also Delatte AA II, p. 190.
[8] Nothing is known of a treatise by Pappus on the astrolabe.
[8a] The Catalogue of Stars in the Almagest counts 15 stars of the first magnitude.

instrument by Hipparchus, sufficed for the night clock.[9]

transposed (μετατιθείς): transposition based on projection; cf. below (p. 875) "transposition of viewpoint" (μετάθεσις τοῦ θεωρήματος).

There follows a verbose paragraph on the virtue of philosophy, etc. Then Synesius returns to the description of his gift.

Considering the problem of projection worthy of study for its own sake we worked it out and composed a treatise and studded it thickly with the necessary abundance and variety of theorems. Then we hastened to translate our conclusions into a material form and produced a beautiful image of the cosmic width. This very approach gives us the means of cutting a plane surface and a uniform cavity in the same ratios. And since we think that any sort of concavity is more closely related to the completely spherical we have hollowed the width by pressing it in (?) and we have taken care of the rest such that the appearance of the instrument may remind the intelligent observer of the reality.

treatise: if ever published, now lost.

material form: we built an instrument according to theory.

I assume that "plane surface" means the instrument, "uniform cavity" the celestial sphere (seen from inside). Hence: we mapped the sphere onto a plane, maintaining proper ratios.
width: πλάτος, as above τοῦ κοσμικοῦ πλάτους for "cosmic width."
pressing in: an otherwise unknown word ὑπεμβολαία (variant: ὑπερβολαία).

The last sentence is the real crux of the whole description. Without it everything would fit a plane instrument based on stereographic projection. However, we are told that the width was hollowed, without explaining the meaning of "width" (or "breadth") of a circular instrument. Any deviation from a plane makes the projection meaningless and this could not have escaped Synesius. The only tentative suggestion I can offer is to assume that he introduced some insignificant modification to the plane star map. If "width" refers, in a loose sense, to the outermost parts of the disk we may perhaps conjecture that a flat circular part was surrounded by an outer rim, bent upwards such that the constellations drawn on the plane background would appear to lie in "some sort of concavity".

What follows agrees again well with stereographic projection, the constellations and their ecliptic coordinates providing the stationary background as in the Vitruvian anaphoric clock.[10]

[9] Νυκτερινὸν ὡροσκοπεῖον. A ὡροσκοπεῖον that shows seasonal hours is also mentioned by Theon in his commentary to the Handy Tables (Halma I, p. 33). Cf. Ptolemy's term ὄργανον ὡροσκόπιον (above p. 871).

[10] Cf. above p. 870.

And we have entered the stars which are classified in six magnitudes, preserving their relative alignments. And of the circles we drew some (fully) around, some we drew (only) across.

Since the ancients recorded some 1000 stars down to 6th magnitude (1022 in the Almagest) Synesius can only have given enough stars to outline the conventional constellations, distorted as they appear by the projection, but preserving the relative alignments. Within the limits of the given disk some circles which represent ecliptic coordinates are fully visible (the circles of constant positive latitude), others (the orthogonals) go only across, from rim to rim.[11]

But all (circles) we divided by degrees, making the 5-degree (division-) lines longer than the (single-)degree lines. We also made larger the lettering indicating the numbers at these (5-degree-)lines. And the black (of the writing) produces on the silver below the appearance of a book.

For the graduation of instruments cf. Rome [1927], p. 82.

But they (the lines of division) were not all cut equidistant, neither individually nor with respect to one another. Some (intervals) are cut equal in size, others irregular and unequally in appearance, yet in principle regular and equal. This had to happen so that different figures should agree.

Stereographic projection of the ecliptic system does not produce equidistant divisions on any individual circle, nor are the distortions the same on different circles.

By this reason also the great circles, drawn through the poles and the signs of the tropics, though in principle remaining circles, have become straight lines

This proves that the south pole was used, as usual, as center of projection.

by the transposition of the viewpoint. The antarctic circle has been inscribed larger than the great circles and the relative distances have been lengthened according to this projection.

transposition of viewpoint: from the center of the celestial sphere to its south pole; cf. for this terminology above p. 874.

The last sentence shows that the star map was bounded by the greatest always invisible circle, as in Ptolemy's Planisphaerium. This circle is the envelope of all possible positions of the horizon, i.e. of the images of all great circles tangential to the declination $90-\varphi$.

Synesius then mentions two epigrams which he had engraved on his instrument. One is the famous epigram, ascribed to Ptolemy in the Anthologia Palatina, praising the astronomer's participation in heavenly nourishment.[12] The second, of Synesius' own manufacture, is put at the end of his letter for the benefit of future readers. However, he still had something to say about the equatorial

[11] A similar remark is made by Philoponus: Hase [1839], p. 132, 21-24.

[12] Greek Anthol. IX, 577; Loeb Class. Libr. III, p. 320/321. Cf. also Heiberg, Ptol. opera I, 1 apparatus to line 5 and Vol. II, Prolegomena, p. CXLVII f. and above p. 835.

coordinates, presumably represented on the movable spider which turns around the north celestial pole.

It (the instrument) undertakes (to show) the positions of the stars, however, so it professes, not with respect to the zodiac, but with respect to the equator. For it has been shown by geometric construction that the former is impossible,

geometric construction (διὰ τῶν γραμμάτων): I assume the meaning "geometric drawings," not "in my work" as Fitzgerald takes it.

since the declinations are given, that is for the sections of the zodiac with respect to the section of the equator; and for all of them the simultaneous rising-times, that means the number of zodiacal degrees that cross with a number of equatorial degrees the same meridian.

declinations (λοξώσεις): literally "inclinations," "slants," but well known as technical term for declination, e.g. in Ptolemy (Opera II, p. 171, 2) and in Theon (Halma I, p. 54) in the introduction to the Handy Tables.

The accepted text here has the meaningless formulation "... that cross with a number of equatorial degrees the same equator." The edition of Terzaghi, however, indicates the existence of a variant[13] that makes sense: crossing the same meridian, thus representing right ascensions. In spite of Synesius' clumsy wording it is now clear that he wishes to find the equatorial coordinates of the stars by means of the instrument. Taking his text literally he seems to say that it is impossible to refer the stars to ecliptic coordinates. This is obviously nonsense, particularly since in the first part of his letter he is concerned with the projection of the ecliptic coordinates. What he means now must concern only the movable parts which make an "instrument" out of a star map. Since the projection is made from the south pole of the equator and since the ecliptic coordinates are engraved on the stationary background it is indeed impossible to also represent them on the movable spider. The rotating part has now to carry the network of circles of constant declination and of radii representing meridians. This obviously completes the "instrument" since no mention is made of horizon coordinates or hour lines which would make the instrument usable as a clock for a given geographical latitude.

The dedicatory letter to Paeonius concludes with the epigram composed by Synesius.[14] There we read that

[13] Terzaghi, Synes. II, apparatus to p. 142, 5.

[14] Counted also as Anthol. gr., App. IV, 74. Except for Fitzgerald I know only of a Latin translation (Cougny, Anthol. Pal. III, p. 408) and a French translation (Druon, Étude, p. 196) which makes no sense.

intelligence has arranged the curved surface of the sphere and cut equal circles in not uniform sections. Look at all the monsters, (extending) down to the rim, where Titan rules balancing night and daylight. Regard the (variable) declinations of the zodiac and do not miss the famous centers of the connection with the meridian.

monsters: the constellations, e.g. Hydra. Titan: the sun.

declinations (λοξωσίας): here used in the same technical sense as above λοξώσεις; "slantings" makes no sense.

Here Synesius refers again to the elementary problem of finding the declinations of points of the ecliptic (Almagest I, 14) and to two of the four astrological "centers" on the ecliptic, the upper and lower culmination.[15] The fact that he does not mention the two other centers, ascendant and setting point, confirms our conclusion that no horizon coordinates were shown on the instrument.

As a result of our detailed analysis we can now give a simple description of the "instrument." A celestial map in stereographic projection, (executed in silver) served as background, showing the constellations and the ecliptic coordinates. Some modification, probably near the outer rim, was supposed to create the impression of a hollowed surface. The central area was undoubtedly plane as required by stereographic projection. The rotating part, centered at the north pole, showed only equatorial coordinates, i.e. concentric circles of declination and radii as meridians. All that the instrument could do was to change the ecliptic coordinates of the star map on the background to right ascensions and declinations. Clearly this was a very expensive but completely useless showpiece.

The astronomical content of the letter of dedication is badly organized and obscured by philosophical verbosity and boasting. The historical remarks would be rather misleading, if we did not have independent sources, most of all Ptolemy's Planisphaerium. Nevertheless one may hope that the references to Hipparchus as the founder of the theory reflect a correct historical tradition. It is also clear that Synesius does not refer to any instrument that combines stereographic projection on one side with a diopter on the other side. This agrees with our analysis of Ptolemy's horoscopic instrument very well.

F. Theon, Severus Sebokht, Philoponus

The first evidence of a real "astrolabe," i.e. of the planisphaerium combined with a diopter, is found with Theon of Alexandria. We know from Suidas[1] that Theon had written a "treatise on the Little Astrolabe"[2] and that this information is confirmed by Islamic authors.[3] Evidently we have here the origin of the term

[15] The "centers" are, of course, also of purely astronomical importance; cf., e.g., Almagest II, 9. As to be expected they appear regularly in the later astrolabe literature, e.g. with Philoponus or Severus Sebokht.

[1] About A.D. 1000; ed. A. Adler II, p. 702, 15f.

[2] Discussion of this title by A. Rome CA I, p. 4, note.

[3] Earliest notice in the Fihrist (A.D. 987) on which later sources depend; cf. Neugebauer [1949], p. 242, notes 10 to 12.

"astrolabe," at first called "little" in contrast to Ptolemy's spherical astrolabe, a terminology still used by Proclus. Paulus of Alexandria, more or less Theon's contemporary, had already used the word "astrolabe" in referring to the plane instrument[3a]. The same terminology is accepted by Heliodorus in 503[3b] and finally by Philoponus in his treatise which established the medieval terminology.

Theon's treatise is not extant[4] but a detailed table of contents is preserved through Yaʿqūbī's "History of the World" (written about A.D. 880), available in a German translation by Klamroth.[5] This table of contents also fits a Syriac treatise, except for some very minor changes, which is fully preserved, written by the Bishop Severus Sebokht, obviously following Theon's treatise very closely but without saying so explicitly.[6] Ioannes Philoponus did practically the same thing about a century earlier (first half of the 6th century) though he handled Theon's treatise a little more freely, e.g. omitting a concluding geographical section.[7] Philoponus[8] mentions in his preface an earlier study by his teacher "the great philosopher Ammonius" but no such work has reached us.[9]

On the basis of Yaʿqūbī and the extant treatises of Philoponus and Severus Sebokht we can give a summary of Theon's work. An introductory section gives a description of the main parts of an astrolabe (disks for different climates, spider, diopter), their dismantling and assembly. This is followed by a discussion of the different curves, e.g. horizon, hour lines, ecliptic, etc. The main part of the treatise concerns the practical applications of the instrument: methods of checking its accuracy, the determination of the seasonal hours of daytime or night, longitudes of sun, moon, and planets; oblique and right ascension of the zodiacal signs, length of daylight and seasonal hours, rising and setting of stars and their arc above the horizon, precession (1° per century), and obliquity of the ecliptic. A concluding section concerns the determination of geographical data and discusses the zones of the terrestrial sphere.

It is quite clear that Theon's treatise constitutes the prototype for many later treatises which all describe the theoretical basis of the design of the instrument and discuss the problems one can readily solve with it, assuming the existence of proper tables for the motion of the sun, the moon, and the planets as well as the basic tables of spherical astronomy. From then on the "astrolabe" plays the role

[3a] Paulus, Apot., ed. Boer, p. 80, 13 and 20; cf. below p. 956.

[3b] Cf. below p. 1040.

[4] Houzeau-Lancaster, Bibl. gén. I, 1, p. 631, No. 3073 assume the existence of an Arabic and a Persian translation in unpublished manuscripts in Firenze. I find no trace of such texts in Asseman's Catal. Another, even more obscure, reference is given in M.P. Khareghat Memorial Volume II (Bombay 1950), Astrolabes, p. 117: "In Konar libraries are found treatises on the astrolabe attributed to Ptolemy, Theon of Alexandria, Ammonius, etc."

[5] Klamroth [1888].

[6] Edited, with French translation, by Nau, Sév. Sab.

[7] Greek text badly edited by Hase [1839]. Cf. Tannery, Mém. Sci. 4, p. 241–260 and p. 229 for corrections and Mém. Sci. 9, p. 341–367 for a French translation. German translation by Drecker [1928], inferior to Tannery's. For the comparison of Yaʿqūbī with Severus Sebokht and Philoponus cf. Neugebauer [1949].

[8] In Alexandrian parlance "Philoponus" indicates association with an activist group of "zealots". Cf., e.g., Patrol. Or. 2, p. 26, 9ff.; also Pétridès [1904].

[9] Many manuscripts which go under the name of Ammonius actually contain a treatise by Nicephoros Gregoras. Cf. the title reproduced above p. 857.

of a convenient mechanical computer, attached to a simple sighting instrument, to be used in conjunction with "Handy Tables." It is perhaps Theon's most original contribution to mediaeval astronomy, if he is the inventor of the astrolabe as universal instrument.

§ 4. Map Projection

In the following the term "projection" is used in a loose fashion for any "mapping" (in the mathematical sense of the term) of the geographical coordinates, longitude L and latitude φ onto a plane (with coordinates, e.g., x and y). In most cases this mapping is not a projection in a strict geometrical sense, that is to say it is not possible to obtain the mapping by connecting the points (L, φ) and (x, y) by straight lines through a fixed center ("eye"), be it at finite or infinite distance from the x-y-plane. Nevertheless the result will be called a "map" in the ordinary geographical sense of the term.

No attempt was ever made in ancient mapping to represent the whole terrestrial surface on one map; the restriction to the quadrant (more or less) of the "inhabited world" (the "oikoumene") conveniently avoided the intricate problems raised by a mapping of the whole surface of a sphere onto a plane.

The only "projection" in the strict sense of the word that occurs in geographical context is Ptolemy's perspective representation of one half of the terrestrial globe (cf. below V B 4, 4). Stereographic projection, which is indeed a mapping of the whole sphere onto a plane by projection from a fixed center (cf. above V B 3, 1) has rarely been used for geographical maps,[1] obviously because distances and areas are too badly distorted.

1. The Marinus' Projection

It is well-known that the work of Marinus of Tyre is one of the principal sources of Ptolemy's "Geography". In fact all that we know about this author, who wrote around 110 A.D.,[1] is based on Ptolemy's references to him, and these are of mostly polemical character, as was the style in ancient scientific literature — much like that of the "humanists" in the Renaissance.

Regarding the method of mapping adopted by Marinus, Ptolemy's description leaves no doubt that he operated with orthogonal coordinates such that distances on all meridians and on the parallel of Rhodes ($\varphi = 36°$) appear in the same scale and undistorted. Since distances reckoned in degrees of longitude on a parallel φ are of length $\cos\varphi$ times the corresponding lengths on the equator and since $\cos 36° \approx 0;48,32 \approx 0;48 = 4/5$ Marinus represents meridians by parallel lines at

[1] For some exceptions in the 16th century cf. Avezac [1863], p. 303ff. and p. 466; also Günther, Stud. Geogr., p. 304f. although he did not realize that Werner's map (1514) is based on the classical norm in which the south pole is the center of projection.

[1] His work takes into account conditions based on events as late as 107 A.D. but ignores changes which took place in 114/116 A.D. Cf. Honigmann in RE 14 (1930) col. 1767–1769 and Mžik, Ptol. Erdkunde, p. 24f., note 2.

distances 4/5 of the distances between parallels of latitude (cf. Fig. 68). Consequently longitudinal distances in this map are foreshortened between the equator and the parallel of Rhodes and increasingly enlarged north of it.

Ptolemy slightly improves on the parameters of Marinus in this type of mapping by assuming for one degree on the parallel of Rhodes 93/115 of the length of one degree on the meridians.[2] Indeed

$$93/115 = 0;48,31, \ldots$$

hence almost exactly $\cos 36°$.

Ptolemy took as the northern boundary of his maps the parallel through the Island of Thule, supposedly located at $\varphi = 63°$. If again 360 equatorial degrees correspond to 115 units then 360° of longitude at the parallel of Thule should be of the length

$$115 \cos 63° = 1,55 \cdot 0;27,14,22 \approx 52;12,32$$

rounded by Ptolemy to 52. The map of Fig. 68 enlarges these 52 parts of the northern boundary to 93, but it contracts to the same amount the 115 parts of the equator. All meridians are undistorted, thus containing between $\varphi = 0$ and $\varphi = 63$

$$\frac{1,55 \cdot 1,3}{6,0} = 20;7,30$$

parts as the parallel of Rhodes contains 93.

2. Ptolemy's First Conic Projection

In Geogr. I 24, 1 to 9 (Mžik: I 24, 1 to 8) Ptolemy describes a mapping of the oikoumene that corresponds (at least in its major part, i.e. north of the equator) to a "conic projection", according to modern terminology. Its advantage over the Marinus projection lies in the fact that it not only preserves distances on all meridians and on the parallel of Rhodes, but that the ratios of distances at the parallel of Thule ($\varphi = 63°$) and at the equator are correctly represented as well.

Ptolemy extends his map to the southern latitude $\varphi = -16;25°$, i.e. to a parallel symmetric to the parallel of Meroe ($\varphi = +16;25°$). He uses on this southern parallel (which we shall call the parallel of "Anti-Meroe") the same spacing for the meridians which one finds for the parallel of Meroe because the continuation of the conic projection would result in greatly increased distances (MN in Fig. 69 instead of ΦX) but he had to pay for this advantage by a sharp discontinuity in the direction of the meridians at the equator (PT in Fig. 69). This mapping on the southern section is, of course, no longer a conic projection. Our subsequent analysis concerns only the northern part.

Ptolemy adopts the following layout for his map (cf. Fig. 69): the whole map should approximately fit into a rectangular frame of width EZ = 97;25 and length AB = 195 ≈ 2 EZ. The actual map is bounded by parallels of latitude, represented

[2] Ptol., Geogr. I, 20 (Nobbe, p. 42, 27). For the reason of assuming 115 units as length of 360° on the equator cf. below p. 881 f.

by arcs of circles with center H, and meridians represented by radii. The following numerical data are assumed:

$$\begin{array}{llll} \text{Thule:} & r_1 = \mathrm{HO} = 52 & \varphi_1 = 63^\circ & \\ \text{Rhodes:} & r_2 = \mathrm{HK} = 79 & \varphi_2 = 36 & \\ \text{Equator:} & r_3 = \mathrm{H}\Sigma = 115 & \varphi_3 = 0 & \\ \text{Anti-Meroe:} & r_4 = \mathrm{HZ} = 131;25 & \varphi_4 = -16;25. & \end{array} \tag{1}$$

The units in which the r_i are measured are chosen such that the arc length

$$\Theta\mathrm{K} = \mathrm{K}\Lambda = 72 = 4/5 \cdot 90. \tag{2}$$

In this scale ΘK represents 90° of geographical longitude divided into 18 segments each of which therefore represent $5^\circ = 1/3^\mathrm{h}$. The meridian HZ is the meridian of Rhodes.

That distances are preserved on all radii is evident from (1) since, e.g., $r_3 - r_1 = 63 = \varphi_1$ and $r_3 - r_2 = 36 = \varphi_2$. It is also clear that the same distance scale is valid on the parallel of Rhodes since a quadrant on this parallel should have the length $90 \cdot \cos 36^\circ \approx 90 \cdot 0;48,32 \approx 4/5 \cdot 90$ in agreement with (2). Finally the ratio of distances on the parallel of Thule and on the equator is shown correctly. Indeed this ratio is expressed by

$$\cos\varphi_1/\cos\varphi_3 = 0;27,14,22/1$$

and

$$r_1/r_3 = 52/115 \approx 0;27,7,49.\ldots$$

is practically the same.

We have still to answer the question how Ptolemy determined the parameters r_i in (1). This can be done[1] by taking up, step by step, the desired qualities of the mapping.

Condition 1: preservation of length on all meridians requires

$$\Delta r = -\Delta\varphi \quad \text{or} \quad r = 90 - \varphi + c = \bar{\varphi} + c. \tag{3}$$

Condition 2: preservation of the ratio of lengths on two parallels r_1 and r_3. This leads to the determination of the constant c in (3) because it is now required that

$$\frac{\cos\varphi_1}{\cos\varphi_3} = \frac{r_1}{r_3} = \frac{\bar{\varphi}_1 + c}{\bar{\varphi}_3 + c}$$

hence

$$c = \frac{\bar{\varphi}_3 \cos\varphi_1 - \bar{\varphi}_1 \cos\varphi_3}{\cos\varphi_3 - \cos\varphi_1}. \tag{4}$$

If one chooses $\varphi_1 = 63^\circ$ (Thule) and $\varphi_3 = 0^\circ$ (equator) one obtains from (4)[2]

$$c = 25;22,\ldots \approx 25 \tag{5}$$

[1] A systematic analysis of all Ptolemaic mappings has been given by F. Hopfner in the Appendices to Mžik, Ptol. Erdkunde, p. 87–109. For a derivation of the basic parameters (1) from Hipparchus' table of chords cf. Toomer [1973], p. 23–25.

[2] $\cos\varphi_1 = 0;27,14,22$.

hence with (3)

$$\begin{aligned} r_1 &= 90-63+25=52, \quad & r_3 &= 90+25=115, \\ r_2 &= 90-36+25=79, \quad & r_4 &= 90+16;25+25=131;25 \end{aligned} \tag{6}$$

and furthermore for $\varphi=90$

$$r_0=c=25. \tag{7}$$

Hence the image of the north pole would be a point at a distance $r_0=25$ from H or 27 units (i.e. degrees) to the north from O; this is the same distance as from K to O.

Condition 3, preservation of lengths on the parallel of Rhodes, leads, as we have seen, to the norm (2), p. 881.

If the northern frame AEB of the map (Fig. 69) should go through the northern edges of the map one should have $HE=r_1 \cos\alpha$ where α is the angle ΘHK, which is about 52;13° since $K\Theta=72$ at $r_2=79$. Therefore one has $r_1 \cos\alpha = 52 \cdot 0;36,45=31;51$ instead of Ptolemy's HE=34. Consequently the northern frame cuts off two small corners of the map (as shown in Fig. 69).

Remark. It is tempting to improve on the preceding mapping by requiring preservation of lengths not only on the parallel of Rhodes (φ_2) but also for φ_1, φ_3, and for Meroe and Anti-Meroe.

Let r and α be polar coordinates with center H, $\alpha=0$ being the radius HZ which is also the zero meridian $L=0$. A point of longitude L at latitude φ is then located at a distance $L\cos\varphi$ from the zero meridian, the distance being measured on the parallel $\varphi=$const. The corresponding circular arc on the map must then satisfy the condition

$$r\cdot\alpha\cdot\frac{\pi}{180}=L\cos\varphi \tag{8}$$

or, with (3)

$$\alpha=\frac{180\cos\varphi}{\pi(\bar{\varphi}+c)}\cdot L=\kappa L. \tag{9}$$

Using for c the value (5) and $\pi\approx 3;8,30$ one obtains for the coefficient κ the following values:

Thule:	$\varphi=63°$	$\kappa=0;29,47$	$90\cdot\kappa=44;40°=\alpha$	for $L=90°$
Rhodes:	36	0;35, 2	52;23	
Meroe:	16;25	0;33,19	49;59	
Equator:	0	0;29,47	44;40	
Anti-Meroe:	−16;25	0;25, 1	37;32.	

The angles $\alpha=90\cdot\kappa$ determine the endpoints of the oikoumene ($L=\pm 90°$) on each of the five parallels as seen from H. This makes it very easy to construct the corresponding map (cf. Fig. 70) which avoids the sharp discontinuity in Fig. 70. Distances are now correct on all five parallels and on the meridian HZ but no longer on other meridians.[3] Distances remain correct on all radii ($\alpha=$const.) but these lines do not represent meridians (excepting $\alpha=0$).

[3] In Fig. 70 drawn at 15° intervals.

3. Ptolemy's Second Conic Projection

In Fig. 70 we have given a modification of Ptolemy's first conic projection, resulting in a much smoother representation of the meridians than in Ptolemy's Fig. 69. We have no evidence for such a step by Ptolemy but his second conic projection (Geogr. I 24, 10–29, Mžik: 9–22) follows the same direction by constructing a map in which the meridians are represented by circular arcs (cf. Fig. 73, p. 1397) while three parallels, Thule ($\varphi_1 = 63°$), Syene ($\varphi_2 = 23;50 = \varepsilon$), and Anti-Meroe ($\varphi_3 = -16;25$) are singled out for undistorted distances.

It is clear that Ptolemy strove here for a more "natural" appearance of his map than was obtained in the first conic projection. Let O be the center of the earth (cf. Fig. 71) and AEZ the central meridian of the oikoumene, i.e. the meridian through Rhodes and Syene. The point E, Syene, is chosen because it is approximately the midpoint of the meridian arc between Thule ($\varphi = 63$) and Anti-Meroe ($\varphi = -16;25$). It is furthermore assumed that the earth is viewed from a point somewhere on the straight line OE.[1] Then the central meridian is seen as a straight line and also the great circle BED is mapped onto a straight line, perpendicular to the image of the meridian (cf. Fig. 72).

What now follows is no longer the result of a real projection but a convenient "mapping" in the mathematical sense. It is assumed that distances are correctly represented not only on the central meridian but also on the perpendicular semi-circle BED. Thus in Fig. 72:

$$ZE = 23;50, \qquad EB = ED = 90 \tag{1}$$

in units which represent degrees; B, Z, and D are points of the equator. It is then decreed that the semicircle BZD of the equator (Fig. 71) is mapped onto the circular arc BZD defined by the three points in Fig. 72, H being the center. Ptolemy thus finds for the radius of the equator image

$$HZ = 181;50 \tag{2}$$

later rounded to 180.[2]

It is furthermore assumed that the northern and southern boundaries of the oikoumene, i.e. the parallels of Thule ($\varphi_1 = 63$) and of Anti-Meroe ($\varphi_3 = -16;25$), as well as the middle parallel through Syene ($\varphi_2 = 23;50$) appear as circles with center H while distances on these three parallels remain undistorted. Ptolemy wishes to draw 18 meridians on each side of the central meridian, i.e. meridians at 5° intervals.[3] The correct intervals at a parallel of latitude φ are therefore given by $5 \cos \varphi$, i.e. for the three aforementioned latitudes

$$\begin{aligned} &\varphi_1 = 63 && 5 \cos \varphi = 5 \cdot 0;27,14,22 = 2;16,11,50 \approx 2;15 \\ &\varphi_2 = 23;50 && 5 \cdot 0;54,53,\ 1 = 4;34,25,\ 5 \approx 4;35 \\ &\varphi_3 = -16;25 && 5 \cdot 0;57,33,14 = 4;47,46,10 \approx 4;50. \end{aligned} \tag{3}$$

In this way one obtains for each meridian three points of equal longitude $k \cdot 5°$ with respect to the central meridian.

[1] Actually the eye must be at infinite distance in order to see the endpoints B and D of the semi-circle BED.

[2] The value (2) is very accurate since $HZ = 181;50,1, \ldots$.

[3] The meridians in Fig. 73 are spaced 15° apart.

To complete the grid it is arbitrarily decided that the meridians between φ_1 and φ_3 be represented by circular arcs, passing through the three points defined by (3). The result is shown in Fig. 73.

Remark. Analysis of the mapping by Ptolemy's "second conic projection."[4] We introduce again polar coordinates (r, α) with the center H.

Condition 1: preservation of lengths on the central meridian ($\alpha=0$) and representation of all parallels $\varphi=$const. by circles $r=$const., hence

$$r=c-\varphi \qquad (c>0). \tag{4}$$

This implies, as before, preservation of lengths on all radii $\alpha=$const.

Condition 2: preservation of lengths on the three parallels listed in (3). This requires, as before p. 882 (9), that for each φ_i

$$\alpha=\frac{180 \cos \varphi_i}{\pi(c-\varphi_i)} L=\kappa(\varphi_i)\cdot L. \tag{5}$$

For $L=$const., i.e. for meridians, with (4):

$$\alpha=\frac{180 \cos (c-r)}{\pi r} L=\kappa L. \tag{6}$$

For meridians with $L\neq 0$ one can obtain from (6) a solution for $\alpha=0$ only for $\cos(c-r)=0$. Since r and c are positive quantities this means that

$$c>90 \tag{7}$$

is the necessary and sufficient condition that the meridians $L=$const. are mapped on curves which begin and end on the central meridian $\alpha=0$, i.e. in points which represent the terrestrial north and south poles.

Condition 3: Except for (7) the constant c can be chosen arbitrarily. It follows from (4) that larger values of c result in flatter arcs for the parallels $\varphi=$const. As we have seen before (cf. p. 883 and Figs. 71 and 72) Ptolemy represents the equator ($\varphi=0$) by the arc of a circle of radius $r_0=181;50$, hence he assumes in (4)

$$c=181;50\approx 180. \tag{8}$$

Condition 4: The relations (4), (8), and (5) completely define the images of the curves $\varphi=$const. (circular arcs) and $L=$const. (6). Ptolemy, however, assumes for his meridians not the curves (6) but circular arcs defined by three points determined by (3).

Nevertheless, Ptolemy's circular arcs are good approximations to the actual meridian curves (cf. Fig. 74) defined by (6). Table 11 gives for three meridians the angles α at H for the intersections of these meridians with ten parallels of latitude. The angles corresponding to Ptolemy's triples deviate only little from the theoretical values. The main deviation (between Meroe and Anti-Meroe) is caused by his postulate that EB=ED=90 (a requirement which has no relation to the remaining mapping), combined with the rounding HZ=180 instead of 181;50. This rounding furthermore changes EB=ED from 90 to 89;30. This then moves the intersection

[4] Based on Hopfner in Mžik, Ptol. Erdkunde, p. 100–105.

Table 11

	φ	r	κ	α for $L=$ 90°	60°	30°	Ptolemy: α for $L=$ 90°	60°	30°
N-Pole	90	90	0	0	0	0			
	75	105	0; 8,28, 1	12;42	8;28	4;14			
Thule	63	117	0;13,20,20	20	13;20	6;40	19;50	13;13	6;37
	60	120	0;14,19,25	21;29	14;19	7;10			
	45	135	0;18	27; 0	18; 0	9; 0			
	30	150	0;19,50,50	29;46	19;51	9;55			
Syene	23;50	156;10	0;20, 8, 8	30;12	20; 8	10; 4	30;16	20;11	10; 5
	15	165	0;20, 7,28	30;11	20; 7	10; 3			
Equ.	0	180	0;19, 5,53	28;39	19; 6	9;33			
	−15	195	0;17, 1,45	25;33	17; 2	8;31			
Anti-M.	−16;25	196;25	0;16,47,18	25;11	16;47	8;24	28;17	18;52	9;26

of the straight line BED with the correct meridians $L = \pm 90°$ to a distance of 86 from E.

All these deviations are negligible for practical purposes because they fall far below the order of magnitude of errors in the empirical data, extracted from travel reports which constitute the basis of all ancient maps.

Summary. In spite of some mathematical inconsistencies Ptolemy's conic projections represent a tremendous progress over the trivial rectangular grids of the Marinus type. Ptolemy for the first time approached the mapping of a substantial part of the terrestrial globe as a mathematical problem; his methods, in fact, initiated modern cartography.

It was not before the 16th century, however, that the significance of Ptolemy's methods was realized. Ptolemy himself had foreseen that his first conic projection would find a greater public than the second,[5] simply because its grid is so much easier to construct than the circular arcs for the meridians in the second type, in spite of the by far more natural appearance obtained for the area of the oikoumene by the latter method.

Ptolemy's prediction seems to be borne out by the still extant material. Looking, e.g., at the maps reproduced by Fischer (Ptol. Geogr.) one finds 4 Greek and 3 Latin maps which are based on the first projection, only one Greek and 3 Latin ones in the second projection. An amusing hybrid type is preserved in an Arabic map[6] which maintains the sharp discontinuity in the directions of the meridians at the equator while the meridians on both sides are drawn in a curved fashion, evidently under the influence of the second type of projection.

In the Renaissance for the first time new ideas were developed out of Ptolemy's theoretical discussions in Book I of his Geography. Obviously there was no reason to adhere to Ptolemy's restriction to a correct representation of distances on three parallels, introduced in order to construct circular meridians.[7] By avoiding this

[5] Geogr. I 24, 29 (Nobbe, p. 58, 4–9; Mžik: 24, 22, p. 75).

[6] Fischer, Ptol. Geogr. [I, 2] A et B*.

[7] Cf. above p. 882.

arbitrary choice for the form of the meridians and by applying Ptolemy's procedure to any number of equidistant parallels one could obtain a map correct on all these parallels and with meridians easily constructible as curves (or polygons) connecting points of equal longitudes on consecutive parallels.

This slight but extremely useful generalization of Ptolemy's second conic projection seems to have been made first by Bernardus Sylvanus[8] in a map at the end of a Latin version of the "Geography," published in Venice 1511.[9] Only three years later (1514) there appeared in Nuremberg a new translation of Book I of the Geography by Johannes Werner to which he added, among other topics, a theoretical discussion of two generalizations of Ptolemy's second conic projection.[10] The second one was designed to represent the whole surface of the terrestrial globe in one map.[10]

In Werner's "Propositio IV"[11] it is required that lengths are preserved on all parallels (represented by concentric circular arcs, $r=\text{const.}$) and on all radii ($\alpha=\text{const.}$). The mapping is furthermore normed in such a fashion that the image of the north pole is also the center of the polar coordinates r, α. Thus the mapping is defined by

$$r=90-\varphi=\bar{\varphi}, \quad r\alpha=cL\cos\varphi. \tag{9}$$

The constant c is determined by the requirement that the boundary of the oikoumene, $L=\pm 90$, should intersect the equator ($\varphi=0$ or $r=90$) at $\alpha=90$. Hence $c=90$ and

$$r=\bar{\varphi}, \quad \alpha=90L\cdot\frac{\sin\bar{\varphi}}{\bar{\varphi}}. \tag{10}$$

Table 12 gives the values for the points of the boundary $L=90°$ for $\bar{\varphi}=k\cdot 10°$, $k=0, 1, \ldots, 18$.[12] Werner's table[13] shows only occasional deviations in the last digits. Fig. 75 gives the resulting grid for the eastern half of the oikoumene.[14]

[8] *Eboliensis*, i.e. from Eboli, not far to the east of Salerno.

[9] Sylvanus' work bears the title "*Claudii Ptholemaei Alexandrini liber geographiae* cum tabulis et universali figura et cum additione locorum quae a recentioribus reperta sunt diligenti cura emendatus et impressus." This "additio locorum" apparently also implies an arbitrary rewriting of Ptolemaic coordinates, characterized by Lelewel (in his Géographie du moyen age, p. 151, p. 156) by "sous le titre de Ptolémée, il inventa un ouvrage remarquable, inutile et sans suites" or "un jeu d'esprit qui n'aboutit a rien." But the concluding world map is of interest not only as a document of contemporary discoveries (including the American continent) but also by its generalization of Ptolemy's mapping such that the whole globe could be represented with undistorted parallels. (For obvious reason latitudes are depicted only between $-40°$ and $+80°$, longitudes between 290° East ($=70°$ W) and 250° East of the Fortunate Islands.) On the other hand the map shows the traditional trimmings, familiar from Byzantine maps, e.g. zodiacal signs at the eastern margin beside latitudes for which the sun reaches the zenith (cf. below p. 936).

[10] The relevant section (fol. g V verso to h II verso) is entitled "*Libellus de quatuor terrarum orbis in plano figurationibus* ab eodem Ioanne Vernero nouissime compertis et enarratis." The whole of Werner's treatise has been reprinted (in part using the original sheets) by Petrus Apianus in his "*Introductio geographica ...*" (Ingolstadt 1533). I wish to gratefully acknowledge the help of the staff of the John Carter Brown Library of Brown University in making these rare sources available to me.

[11] Werner, fol. h I verso.

[12] The value for $k=0$ of $\sin\bar{\varphi}/\bar{\varphi}$ is $\pi/180$ since $\bar{\varphi}$ is reckoned in degrees.

[13] Werner fol. h I recto. The deviations in the table of $\sin\bar{\varphi}$ (fol. g VI recto, right half) could have been avoided by using the table of chords given on fol. i III recto to IV recto.

[14] Fig. 75 is only a slight modification of Werner's drawing fol. h I verso.

Table 12

φ	$r=\bar{\varphi}$	$\sin\bar{\varphi}$	last digit Werner	$\frac{\sin\bar{\varphi}}{\bar{\varphi}}$	$\alpha=2,15,0\cdot\frac{\sin\bar{\varphi}}{\bar{\varphi}}$	last digit Werner
90°	0	0		0;1, 2, 50	141;22°	[;20]
80	10	0;10,25, 8	9	0;1, 2,31	140;39	40
70	20	0;20,31,16		0;1, 1,34	138;31	23!
60	30	0;30		0;1	135;0	
50	40	0;38,34, 2	3	0;0,57,51	130;10	
40	50	0;45,57,46	45	0;0,55, 9	124; 6	4
30	60	0;51,57,41	49!	0;0,51,58	116;55	58
20	70	0;56,22,54	53	0;0,48,20	108;44	55!
10	80	0;59, 5,18	19	0;0,44,19	99;43	42
0	90	1		0;0,40	90; 0	
−10	100	0;59, 5,18	19	0;0,35,26	79;46	
−20	110	0;56,22,54	53	0;0,30,45	69;12	
−30	120	0;51,57,41	49!	0;0,25,59	58;27	41!
−40	130	0;45,57,46	45	0;0,21,13	47;44	43
−50	140	0;38,34, 2	3	0;0,16,32	37;11	8
−60	150	0;30		0;0,12	27; 0	
−70	160	0;20,31,16		0;0, 7,42	17;19	20
−80	170	0;10,25, 8	9	0;0, 3,40	8;16	17
−90	180	0		0	0	

The parallels of northern latitudes are mapped through (10) on arcs of more than 90° representing 90° of longitude. Consequently it is not possible to map the whole terrestrial globe by means of (10) without a partial overlapping for the northern hemisphere. This caused Werner to introduce another condition in his "Propositio V"[15] for the determination of c in (9). He now required that a quadrant on the equator has the same length as the radius between a pole and the equator. If, then, in Fig. 76, DA is the image of the quadrant of the central meridian between the north pole and the equator, AB the image of 90° of longitude on the equator, the arc AB should have the same length as DA. Now for $r=90$

$$\text{arc AB} = 90\frac{\pi}{180}\alpha_0.$$

Thus the condition arc AB$=90$ requires that

$$\alpha_0=\frac{180}{\pi}\approx 57;17,44°.^{16} \tag{11}$$

This determines c in (9) since for the point B must hold

$$90\alpha_0=c\cdot 90\cdot 1$$

thus

$$c=\alpha_0. \tag{12}$$

[15] Werner, fol. h II recto.

[16] Werner (fol. h II recto) has $\alpha_0=57;16°$, i.e. $\pi\approx 3;8,35$ instead of 3;8,30.

Table 13

φ	$r=\bar{\varphi}$	$\frac{\sin\bar{\varphi}}{\bar{\varphi}}$	$\alpha=\frac{\sin\bar{\varphi}}{\bar{\varphi}}\cdot 1,25,56;36$
90°	0	0;1, 2,50	90; 0°
80	10	1, 2,31	89;33
70	20	1, 1,34	88;11
60	30	1	85;57
50	40	0;0,57,51	82;52
40	50	55, 9	79; 0
30	60	51,58	74;26
20	70	0;0,48,20	69;14
10	80	44,19	63;29
0	90	40	57;18
−10	100	0;0,35,27	50;47
−20	110	30,45	44; 3
−30	120	25,59	37;13
−40	130	0;0,21,13	30;23
−50	140	16,32	23;41
−60	150	12	17;11
−70	160	0;0, 7,42	11; 0
−80	170	3,41	5;16
−90	180	0, 0	0

The new mapping is therefore given by

$$r=\bar{\varphi}, \quad \alpha=57;17,44\,L\cdot\frac{\sin\bar{\varphi}}{\bar{\varphi}}. \tag{13}$$

In particular the meridian $L=90°$ has the equation

$$\alpha=1,25,56;36\cdot\frac{\sin\bar{\varphi}}{\bar{\varphi}}. \tag{14}$$

The corresponding values for $\bar{\varphi}=k\cdot 10°$ are shown in Table 13 and are used for the construction of Fig. 77.[17]

The maps of Sylvanus (1511)[18] and of Werner (1514) are the first solutions of the problem to represent the whole surface of a sphere within a finite area of the plane. Since distances on radii ($\alpha=$ const.) and on parallels ($r=$ const.) are preserved this is also true for areas.[19] Sylvanus' map is the equivalent of the so-called "Bonne projection."[20] In it the north pole does not coincide with the common center of the circles which represent the parallels, thus reverting in part from Werner's to Ptolemy's arrangement.[21]

[17] Similar figure in Werner, fol. h II verso. The value of α for $k=0$ is 90°; cf. above p. 886, note 12.

[18] Cf. above p. 886.

[19] First demonstrated by Mollweide [1806].

[20] According to Avezac [1863], p. 285 this name is not used in France. Rigobert Bonne (1727–1795) is best known through his "Atlas encyclopédique" (2 vols., Paris 1787–1788).

[21] Both the Sylvanus and the Bonne projection contain an arbitrary parameter $f>0$ such that $r=\bar{\varphi}+f$. In other words the image of the north pole lies on the central meridian at a distance f below the center of the parallels. In the Bonne projection f is chosen in such a way that the radii $\alpha=$ const. touch the meridian curves always on a given parallel, e.g. the parallel of Paris (which is placed on the meridian $\alpha=0$). Sylvanus accidentally uses a similar value (about $f=10$ to judge from his printed map). For Werner $f=0$.

4. Visual Appearance of a Terrestrial Globe

Ptolemy in Book I of his "Geography" repeatedly refers to the possibility of representing the oikoumene on a terrestrial globe.[1] This procedure avoids all distortion but requires a sphere of considerable size if a sufficient amount of detail is to be shown. In Book VII, 6 and 7, he returns to a spherical representation of the "inhabited" part of the earth's surface but it is now projected onto a plane, i.e. to a representation corresponding, at least to a certain extent, to the visual impression of a terrestrial globe. He assumes an observer whose eye is located somewhere on the intersection of the plane of the parallel of Syene[2] ($\varphi=23;50°$) and the plane of the central meridian.

We must now explain in what sense the resulting picture is only "to a certain extent" a truly perspective representation of the globe as seen from an eye at a finite distance. Ptolemy considers the globe not only as a simple sphere but he thinks of it as equipped with outside rings which represent the equator, the ecliptic ($\varepsilon=24°$), and the solstitial parallels, such that the sphere is freely turnable below these rings about the polar axis. As always it is assumed that the oikoumene extends 180° in longitude and from Anti-Meroe ($\varphi=-16;25$) to Thule ($\varphi=63$) in latitude. It is furthermore assumed that the sphere is brought into such a position with respect to the surrounding rings so that no part of the map of the oikoumene is obscured by the rings. For an actual globe this will never be the case since the rings must be very close to the sphere to be useful – a requirement expressly made, e.g., in Book I.[3] Consequently, Ptolemy introduces for his pictorial representation of the globe the totally unrealistic assumption that the radii R of the rings are 4/3 of the radius r of the solid sphere. Moreover the principle of perspective representation is completely abandoned for the depiction of the map itself. The reason is obvious: a truly perspective picture of a map drawn on a spherical surface and extending over 180° in longitude would result in very serious distortions, in particular since the eye is placed quite near to the surface in order to remove the images of the rings from the map.

The details of the construction are as follows. We divide the radius $r=O\Pi$ of the solid sphere into 90 parts (cf. Fig. 78 which represents a cross-section in the plane of the central meridian) and make $OT=16;25$ (for the southern latitude of Anti-Meroe), $O\Sigma=23;50$ (φ of Syene), $OY=63$ (φ of Thule). The eye Ω is located on the straight line ΣE at such a point that the nearest point H of the ring which represents the Tropic of Cancer is projected into Y (or above it), the nearest point Δ of the equator into T (or below it). These conditions determine the distance $\Sigma\Omega$ and the radii R of the rings; one finds[4]

$$R\approx 120=4/3\,r, \qquad \Sigma\Omega\approx 290.$$

The position of Ω and the size of the rings being known one can now construct (accurately or approximately) the ellipses which are the perspective images of the rings with diameter HZ and ΔB.

[1] Cf., e.g., Geogr. I 20, 1 and I 22 and 23.

[2] Consequently the eye is not located on the radius OE as had been assumed in the construction of the "second conic projection" (above V B 4, 3 and Fig. 71, p. 1397).

[3] Geogr. I 22, 2 (Nobbe, p. 44, 8–11; Mžik: 22, 1); cf. also Alm. VIII, 3 for a celestial globe.

[4] For details cf. Neugebauer [1959, 1], p. 27; also Mollweide [1805] and Wilberg [1846].

There still remains to be drawn the map of the oikoumene. As in the case of the second conic projection one starts from the image ΠP of the central meridian on which the points Y, Σ, and T are located in correct distances. On these three parallels the endpoints $L = \pm 90°$ of the oikoumene are at distances $r \cos \varphi$ from the prime meridian, i.e.

$$90 \cdot \cos 63 \approx 41, \quad 90 \cdot \cos 23;50 \approx 82, \quad 90 \cdot \cos 16;25 \approx 86,$$

respectively. On these three parallels distances are undistorted. The middle parallel (Syene) is seen from Ω as a straight line; the two outer ones (Thule and Anti-Meroe) are probably slightly concave toward the middle.[5] For each given longitude between $-90°$ and $+90°$ can then be marked a point on each of the three parallels and hence a circular arc drawn which represents a meridian.

Roughly speaking the whole design can be described as a map reminiscent of the map obtained by the second conic projection,[6] framed, as it were, by the circumference of the sphere which in turn is surrounded by the perspective picture of rings, placed at such a respectful distance that they do not interfere with the map. From the viewpoint of mathematical consistency this is the least appealing of Ptolemy's constructions but it testifies again to his versatility in the design of new forms of geometrical representations. It may well be that this part of the "Geography" had some influence on the development of the art of true "perspective" in the Renaissance.[7]

5. Appendix. Precession-Globe (Alm. VIII, 3)

In his Catalogue of Stars (Alm. VII, 5/VIII, 1) Ptolemy defines the positions of 1022 stars[1] (arranged in 48 constellations) by ecliptic coordinates, the longitudes being reckoned from the equinox of the year Antoninus 1[2] (A.D. 137). In VIII, 2 follows a detailed description of the Milky Way, its variable density and branches, always in relation to fixed stars. Here the fixed stars serve as the invariant reference system, in contrast to the norm of the catalogue which is subject to the influence of precession.

In order to eliminate the time dependence of stellar coordinates later star catalogues[3] often give longitudes with respect to a fixed star, e.g. Regulus which lies almost in the ecliptic ($\beta = +0;10°$[4]), a point of reference used also in the Handy Tables[5] and in the Canobic Inscription.[6] In Alm. VIII, 3, however, Sirius

[5] Cf. Geogr. VII 6, 4.
[6] Excepting the different curvatures of the parallels.
[7] Cf. Samuel Y. Edgerton, Florentine Interest in Ptolemaic Cartography as Background for Renaissance Painting, Architecture, and the Discovery of America. J. of the Society of Architectural Historians 33 (1974), p. 275–292.
[1] Alm. VIII, 1 (Man. II, p. 64).
[2] Alm. VII, 4 (Man. II, p. 31).
[3] Regulus is used, e.g., in Vat. gr. 208 fol. 126v–129r (unpublished). Copernicus uses γ Ar as the origin of sidereal longitudes (De Revol. II 14, Gesamtausgabe II, p. 102, 16f.).
[4] Alm. VII, 5 (Man. II, p. 47).
[5] Cf. below p. 986.
[6] Heiberg II, p. 152, 2/3. Cf. below p. 915.

is considered as reference star in spite of its great distance from the ecliptic.[7] The change from Sirius to Regulus may well be taken as one of the many instances where a first step made in the Almagest is perfected in some later work of Ptolemy.

I do not know of any star map from antiquity which would show more than pictorial representations of the conventional constellations. The only real mapping of the celestial sphere and its coordinate systems is the stereographic projection, practically applied in the astrolabe.[8] It seems, however, that this method has never been used for drawing a star map with a network of coordinate lines, comparable to geographical maps of the terrestrial globe. Instead the globe had been used widely, not only because all distortions are avoided but also because a globe can readily be used to answer all questions of rising and setting of stars, simply by rotating it about a properly inclined axis. But here again precession intervenes since the poles of the axis of the daily rotation move slowly around the poles of the ecliptic (on a circle of radius ε which itself is subject to secular changes). A globe which is constructed to allow for changes in the position of the axis due to precession is called a "precession-globe." Ptolemy in Alm. VIII, 3 gives the first known description of such an instrument.

The principle of construction is simple enough. Two diametrically opposite points on the globe, P and P′ (cf. Fig. 79) represent the poles of the ecliptic while PAP' is a great circle drawn on the globe. On it are marked points A and Σ such that $PA = 90^\circ$ and $A\Sigma = 39;10^\circ$ in the direction toward P′. Then Σ represents Sirius[9] and the orthogonal great circle through A is the ecliptic, divided in 360 degrees, beginning at A. This fixes the axes of our sidereal coordinate system with Sirius as star of reference.

Next two wooden rings should be made, both representing great circles, one just touching the surface of the globe, the second only so much larger that the first ring can be fitted under the second. Furthermore in each ring should be made a central slit along half of its circumference, provided with marks for single degrees (cf. Fig. 80 A). The inner ring is the "latitude-ring" and is therefore marked from (+)90 at P, to 0, to (−)90 at P′. The outer ring is the "meridian-ring" and shows declinations (again from $+90^\circ$ to -90°). It is difficult to see how the technical requirements for such a construction could be met with the tools available in antiquity. A technically simpler arrangement, shown in the lower part of Fig. 80 A, seems not to be supported by the wording of the text.

The latitude ring is now fitted to the globe by means of pins at P and P′ which fit into the slit at its two ends. By turning this ring with respect to the globe stellar coordinates can be marked on its surface. Ptolemy obviously assumes that all stars of the catalogue should be shown on the globe, marked by yellow dots[10] (of different size according to the stellar magnitude) on the dark background of the surface, the boundaries of the constellations being marked only lightly and

[7] Alm. VIII, 1 (Man. II, p. 47) gives $\lambda = ♊\ 17;40$, $\beta = -39;10^\circ$.

[8] Cf. above V B 3 and, in particular, V B 3, 5 for the limits of the visible area (at given φ).

[9] Cf. above note 7.

[10] Excepting some stars of "characteristic color" (τὸ ἐπ' ἐνίων διασημαινόμενον χρῶμα). However, the only color mentioned in the catalogue is red or reddish (cf. below p. 598, n. 14) unless "bright" (λαμπρός) should be taken as "white". No star is called "blue".

by simple lines. The Milky Way is also to be depicted according to the description given in VIII, 2.

At a given time the longitudinal distance of the summer solstice from Sirius is known, eg. 12;20° for Antoninus 1.[11] If S marks this point on the ecliptic the latitude-ring in the position PSP′ contains the north pole N at the distance $PN = \varepsilon = 23;51°$. For any given time the latitude-ring has to be kept in the proper position PSP′, perhaps simply by the friction of the ring on the surface of the globe. The meridian-ring, however, is now to be attached to the latitude-ring with two pins at the ends N and N′ of its slit at bearings provided at a distance ε from P and P′ (cf. Fig. 80B).[12] If N and N′ are held in a fixed position the globe with its latitude-ring can freely rotate below the meridian-ring.

Finally a stand is made with a horizontal ring which holds the meridian-ring in a vertical position but allows it to be moved in its plane such that N can obtain any given altitude φ above the horizon (cf. Fig. 80C). With the latitude-ring going through the proper solstitial point S the rotation of the globe represents the motion of the celestial sphere for the given time and the given locality.

By turning the globe such that a given star obtains a position exactly under the slit of the meridian-ring one can determine directly the "polar latitude" and also the "polar longitude" of the star[13] since for the given time the position of the vernal point must be known. The fact of direct access through readings on the globe undoubtedly contributed to the survival of these "polar" coordinates deep into the Renaissance in spite of being consistently ignored by Ptolemy.

The terrestrial coordinates are fortunately purely equatorial and independent of precession. Ptolemy (who introduced the reckoning by degrees for both geographical coordinates[14]) could therefore dispense with the latitude-ring of his celestial globe and make the meridian-ring turn in close contact with the surface of the globe. The meridian-ring could then be used directly to draw on the globe the grid of meridians and parallels.[15]

§ 5. Optics

Ever since Galileo's "Sidereal Messenger" (1610) and Newton's invention of the reflecting telescope (1671)[1] and his "Opticks" (1704) the progress of astronomy is inseparably connected with the progress of optics. In antiquity and in the Middle Ages the relation between these two fields is much less intimate, although by no means wholly absent. It is even more remarkable that it is Ptolemy again whose work on optics constitutes a decisive progress both in theory and experimentation, and this not only in the area of strictly geometrical optics but far into

[11] Cf. above p. 891, n. 7.
[12] This construction, incidentally, shows that Ptolemy did not consider the possibility of secular changes in ε.
[13] Cf. for these coordinates below Fig. 12 (p. 1436); also above p. 279.
[14] Cf. below p. 934.
[15] Geogr. I, 22. For the history of globes in general cf. Schlachter, Globus.
[1] Newton, Corresp. I, p. 74–76.

the field of physiological optics through his experiments on binocular vision.[2] The width of Ptolemy's interest, his power of analysis and practical inventiveness makes him truly one of the greatest figures in the history of science.

Greek optics was based on three assumptions concerning the process of vision: (a) from the eye emanate "visual rays" whose incident on objects is basic for the process of seeing; (b) color is a quality inherent in the objects but (c) both color and light are necessary to make an object visible. Not before Ibn al-Haitham, in the 11th century, was the distinction between visual rays and light rays abolished.

Treatises on Greek optics are still extant—although in more or less fragmentary form or only through Arabic or Latin translations—from the time of Euclid (i.e. around 300 B.C.) to Theon (4th cent. A.D.). Euclid's "Optics"[3] were re-edited by Theon,[4] although long antiquated, much narrower in scope and more primitive in methodology than Ptolemy's "Optics." Perhaps it was just the restriction to purely geometrical procedures that recommended Euclid's work to Theon's didactic purposes, as against the experimental-empirical approach of Ptolemy, applied to new types of problems, e.g. binocular vision or refraction. Also ascribed to Euclid is a treatise on "Catoptrics",[5] i.e. on geometrical optics, in particular on reflection. In its present form it is certainly the work of a late compilator, though it may contain an authentic Euclidean nucleus.[6] The "Catoptrics" of Archimedes are lost[7]; fragments of a work perhaps due to Geminus[8] (first cent. A.D.) and an Arabic version of a treatise on "Burning Mirrors" by Diocles[9] give some glimpses of optical theory before Heron's "Catoptrics" and "Dioptra"[10] (first cent. A.D.). The last mentioned work is only loosely related to optics in the modern sense of the word since it deals mainly with sighting instruments and their use in problems of triangulation. We discussed above the Chap. 35 of the "Dioptra" on the determination of the distance Alexandria-Rome by "analemma" methods.[11] In Chap. 2 the usefulness of dioptra for "measuring the distances between stars and of the size and distance of sun and moon" is mentioned.[12]

Ptolemy's "Optics" has come down to us only in a Latin translation of an Arabic translation, beginning and end (i.e. Book I and the end of Book V) being lost.[13] The author of the Latin translation is the "Admiral" in the service of the Norman kings Eugene of Sicily (second half of the 12th cent.). Being of Greek extraction and fully familiar with Arabic he devoted his literary efforts to several

[2] Cf. Lejeune, Euclide et Ptol., p. 130–177.
[3] Euclid. Opera VII, p. 2–141 (ed. Heiberg).
[4] Euclid. Opera VII, p. 144–284.
[5] Euclid. Opera VII, p. 286–382, and Heron, Opera II, 1, p. 394–399 (ed. Nix-Schmidt).
[6] Cf. Lejeune, Catoptr. gr., p. 112–136, p. 146.
[7] For a fragment cf. Archimedes, Opera II, p. 549–551 (ed. Heiberg); also Lejeune, Catoptr. gr., p. 142–145.
[8] Cf. Schöne, Damianos, p. 22–31.
[9] Cf. Toomer [1975].
[10] Heron, Opera II, 1, p. 303–365, p. 368–373, p. 406–415 (ed. Nix-Schmidt) and Opera III (ed. Schöne; cf. also R.E. Suppl. 6, cols 1287–1290), respectively; also Lejeune, Catoptr. gr., p. 137–142.
[11] Above p. 847f.
[12] Heron, Opera III, p. 190, 6–8.
[13] This was already the situation in the time of Ibn al-Haitham; cf. Lejeune, Catoptr. gr., p. 17 and note 8 there.

translations,[14] helping, incidentally, with a Latin translation of the Almagest from the Greek.[15] Through the Arabic as well as through the Latin version Ptolemy's "Optics" had great influence on optical theory in the later Middle Ages, partly and indirectly through the writings of Ibn al-Haitham.[16]

One might be tempted to conclude from the reversibility of the direction of light that the theory of "visual rays" emanating from the eye would lead to the same results as the theory which operates with light rays falling into the eye.[17] In fact this is the case only in a very restricted sense. The whole methodology of Ptolemy's experiments is directed toward the question of what an eye would see when placed into a specific position, e.g. with respect to a plane, concave, or convex mirror. Within this approach one cannot reach the concept of an objectively measurable relation between object and image ("real" or "virtual") but one is faced with the far more difficult problem of investigating the rays of a cone of wide opening[18] which has its vertex in the eye.[19]

On the other hand Ptolemy's interest reaches far beyond the mere geometry of optics. As far as we know he was the first one to develop a theory of binocular vision and to design ingenious experiments to test the consequences in relation to our visual impressions.[20] He was concerned with the origin of color sensation[21] and with optical illusions, such as the apparent magnification of celestial objects near the horizon.[22] Of greatest historic interest is, however, his attempt to establish a law of refraction.

In Book V of his Optics[23] Ptolemy describes an experimental arrangement for the measurement of the angles of refraction. The results are displayed in our Table 14 in which α represents the angle of incidence in relation to the normal to the surface of separation between the two media, β the angle of refraction. The numbers listed under $\Delta\beta$ show that Ptolemy smoothened the actual results of his measurements to a difference sequence of second order which, except for the range $\alpha > 70°$, represents the facts quite well, in particular if one relies on naked eye observations of small markers on a graded disk instead of operating with narrow bundles of light as one would do it today.

[14] Cf. Lejeune, Catoptr. gr., p. 9ff. or Ptol. Opt., p. 10*ff. where additional literature is quoted. For biographical data cf. also E. Jamison Eug. of Sic. (rev. by Dölger, Byz. Z. 50 (1957), p. 444–446).

[15] Cf. Manitius, Alm., Vol. I, p. XV.

[16] Cf. for details Lejeune, Opt. Ptol., p. 31*ff. Also the summary of the development of ancient optics in the introduction to VerEecke, Eucl. Opt.

[17] For the different ancient theories concerning the process of vision cf. Haas [1907].

[18] Ptolemy assumed, probably under the influence of arguments a priori, that the visual cone had an opening of 90°. He supports this assertion by an experiment according to which an eye placed at one point of the circumference of a circle can see just half of the rim on the opposite side. This could have been suggested by experiences with the rings of the armillary sphere; cf. also Lejeune, Eucl. Ptol., p. 45ff.

[19] This difference in principle between Ptolemaic and modern optics has first been fully realized and discussed in detail by Lejeune, Catoptr. gr., p. 71ff.

[20] Cf. Lejeune, Catoptr. gr., Chap. V and VI.

[21] E.g. the mixture of colors on rotating disks (in II, 82 and II, 96, ed. Lejeune, p. 55, 2 and 60, 21). This experience Ptolemy related to the extended visibility of tracks of meteors (II, 96, ed. Lejeune, p. 61, 8). Cf. also Lejeune, Eucl. Ptol., p. 22ff., p. 75f.

[22] Cf. below p. 896.

[23] Lejeune, Opt. Ptol. V, 6–35 (p. 225, 21–246, 6); translated in Cohen-Drabkin, Source Book, p. 273–281.

Ptolemy investigated three media: air, water, and glass. His data from Table 14 can be used to check the accuracy of his measurements in detail. We know that actually

$$\sin\alpha/\sin\beta = n_1/n_2 \tag{1}$$

for the transition from the first to the second medium and similarly for the other cases, n_1, n_2, n_3 being constants. From correct observations one should find that

$$\frac{n_1}{n_2}\cdot\frac{n_2}{n_3}=\frac{n_1}{n_3}$$

or, in other words, that these constants satisfy the relation

$$\log\frac{n_1}{n_2}+\log\frac{n_2}{n_3}=\log\frac{n_1}{n_3}. \tag{2}$$

Since we have Ptolemy's values for the angles α and β we can compute the values in (2). Obviously $\log\frac{n_1}{n_2}$ should be a constant, independent of α and the same should be true for the two other ratios. Fig. 81 shows that this is not at all the case; nor is the curve "3" the sum of "1" and "2" as (2) would require. But the latter relation is nearly satisfied for α between about 50° and 70° since the deviation is at most 0.005. For $\alpha = 60°$ one finds with Table 14

$$\begin{aligned} &\log\sin 60-\log\sin 40;30=0.12499 \quad &&\text{hence } n_1/n_2=1.33\\ &\log\sin 60-\log\sin 49;30=0.056648 &&n_2/n_3=1.14\\ &\log\sin 60-\log\sin 34;30=0.18444 &&n_1/n_3=1.53\end{aligned}$$

whereas the first two values would give as total 0.18147 i.e. $n_1/n_3 \approx 1.52$ instead of 1.53. The observations in this range are therefore nearly exact; this is confirmed by the correctness of the refractive index 1.33 for air/water and the perfectly possible value 1.52 for air/glass. The third value is then automatically correct since (2) is satisfied.

Having thus found the refractive indices of the media used by Ptolemy in his experiments we can compute the correct value of β for all cases and find that Ptolemy's smoothened values (Table 14) are about 1/2° to 3/4° greater than they

Table 14

	air/water		water/glass		air/glass	
α	β	$\Delta\beta$	β	$\Delta\beta$	β	$\Delta\beta$
10°	8°		9;30°		7°	
20	15;30	7;30°	18;30	9°	13;30	6;30°
30	22;30	7	27	8;30	19;30	6
40	29	6;30	35	8	25	5;30
50	35	6	42;30	7;30	30	5
60	40;30	5;30	49;30	7	34;30	4;30
70	45;30	5	56	6;30	38;30	4
80	50	4;30	62	6	42	3;30

should be, except for $\alpha = 80°$ (very difficult to observe) where the error reaches about 2 1/2°.

Obviously Ptolemy had no inkling of the actual law of refraction. He even gives a reason for the fact that no attempt was made to determine the effect of the phenomenon on astronomical observations: it is unknown where the dividing surface between "aether" and air is located,[24] an argument obviously invalid since even under the assumption of a homogeneous air and a sharp boundary the aether's side would remain inaccessible. Thus only discrepancies between theoretical positions and observations could have helped to establish a correction which was very near, if not below, the limit of ancient instrumental accuracy.[25] It should be remembered how difficult the problem still appeared to Brahe and Kepler when it was again taken up around 1600.[26]

In the Almagest Ptolemy mentions twice the optical illusion that the celestial bodies near the horizon appear larger than at greater altitude.[27] This is a classical topic in Greek optics, found, e.g., in a fragment from an early treatise, the papyrus P. Paris. 7733 of the Ptolemaic period.[28] The traditional explanation which also Ptolemy offers in Alm. I, 3 is obviously not relevant: the moist atmosphere near the horizon makes objects appear larger in the same way as immersion into water increases the apparent size of an object, increasingly so as it sinks deeper. Obviously such a superficial and incorrect analogy antedates Ptolemy's own serious study of optics. Indeed, in the "Planetary Hypotheses" this explanation is no longer upheld and the said phenomenon is recognized as an optical illusion, caused by wrongly estimating size in relation to nearby terrestrial objects,[29] a topic further studied in his "Optics."[30] But Ptolemy's discoveries in all branches of optics came too late to have any influence on astronomical theory and observational practice.

§ 6. The Tetrabiblos

In the late Middle Ages Ptolemy's name would have been connected first and foremost with his great astrological work, the "Tetrabiblos"—latinized "*Quadri-*

[24] Lejeune, Opt. Ptol. V, 30 (p. 242, 2–7).

[25] The inaccuracies of ancient clocks makes it practically impossible to establish the excess of the observed length of daylight over the theoretical value. Lunar eclipses with both sun and moon visible above the horizon seem to have been known since Hipparchus but remained unused for an estimate of the effect of refraction. For Hipparchus cf. Pliny, NH II, 57 (Budé, Vol. II, p. 25 f.); for Cleomedes II, 6, ed. Ziegler, p. 218, 8–226, 6 (trsl. Heath, Gr. Astr., p. 162–166, also Cohen-Drabkin, Source Book, p. 284 f.).

[26] Brahe assumed different refraction for sun, moon, and stars (Opera II. p. 64, p. 136, p. 287 and III, p. 377, respectively; cf. Fig. 82); cf. also Kepler, Epit. Astr. Cop. I, 3 (Werke 7, p. 58). For Kepler's own theory cf. Werke 2, p. 217 and p. 411–415. In the Astron. Nova, Chap. 15 (Werke 3, p. 144; trsl., p. 136) the excess of refraction for the sun over the stars is explained by the turbulence of the air caused by the opposition of Mars and sun. The correct relation (1) seems to have been first found by Thomas Harriot in 1602 (cf. Lohne [1959]), then again by Snellius (around 1620). Neither Harriot nor Snellius published their discovery. Descartes knew the correct law in 1632 and published it in his Dioptrice, Chap. II (1637), suspected by Huygens and by Leibniz of plagiarism (cf. Adam, Descartes, p. 194–197).

[27] Alm. I, 3 (Man. I, p. 9) and IX, 2 (Man. II, p. 96).

[28] Wessely [1891], p. 314, II 1, 12–15; also Egger [1870], p. 465 (col. III).

[29] Plan. Hyp., Goldstein [1967], p. 9 (No. 7).

[30] Lejeune, Eucl. Ptol., p. 95 ff. and Catoptr., p. 22 f.

partitum."[1] A modern edition of the Greek text is now available in Ptol. Opera III, 1[2] (1940) and in the Loeb Classical Library (1940), with English translation. Ptolemy's name is also attached to a compilation of 100 astrological sentences (thus called "*Centiloquium*"), no doubt without any real basis. Because of its traditional association, however, it has been included in Ptol. Opera III, 2.[3] The original title was καρπός, fruit, thus Latin "*Fructus*."

The Almagest was certainly written before the Tetrabiblos.[4] Astrological references in the Almagest are almost non-existent, except for a few remarks concerning the significance of "inclinations" for weather prognostications,[5] a subject also discussed in Ptolemy's "Phaseis."[6] In Tetrab. I, 13 (14 B.-B.) the parallelism between "aspects" and harmonies is mentioned, again a topic to which he returned in a special treatise, the "Harmonics."[7] On the whole, the Tetrabiblos stands alone and has very little contact with other works of Ptolemy; his very outspoken gift for systematic arrangement has reduced duplications in all his writings to a minimum.

The vast amount of astrological material available today, e.g. through the CCAG,[8] has by no means been exhausted for strictly astronomical material. More systematic attention has been paid to geographical data because they provide information about the origin and transmission of doctrines by reflecting situations as existed, e.g., only during the Persian period in Mesopotamia before Alexander.[9] On the basis of geographical data Boll uncovered the dependence on earlier sources of these sections of the Tetrabiblos[10] and, incidentally, showed[11] that the Tetrabiblos was written before the "Geography."

It was again Boll who realized[12] that interesting elements of observational astronomy are embedded in Tetrab. I, 9 where fixed stars and planets are astrologically associated on the basis of empirical data concerning the color of stars and planets, assembled by "older" astronomers.[13] We touch here upon problems

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[1] The original title was perhaps "Apotelesmatica," the technical term for astrological theory. Cf. the introduction to Robbins' edition in the Loeb series and E. Boer in R.E. 23, 2 col. 1831–1838.

[2] The "index verborum" to the Tetrab. was printed in Opera III, 2 (1952), p. 70–120, but omitted from edition of 1961 and not included in the edition of 1954 of Opera III, 1 (ed. Boll-Boer = B.-B.).

[3] Edited by E. Boer, p. XVII–XXXIV, p. 37–69; cf. also R.E. 23, 2 col. 1838 f.

[4] The Almagest is quoted in Tetr. I, 1 (p. 3, 4 B.-B.); Tetr. II, (7) 8 (p. 82, 5 B.-B.) refers to Alm. VIII, 4.

[5] E.g. Alm. VI, 11; cf. p. 141.

[6] Cf. below V B 8, 1 B.

[7] Cf. below p. 933. The aspects are also mentioned in passing in Alm. VIII, 4.

[8] Mainly thanks to the foresight and energy of F. Cumont.

[9] Cf., e.g. Cumont, La plus ancienne géographie astrologique, Klio 9 (1909), p. 263–273. For evidence of transmission of Babylonian astrology to Egypt during the Persian period (around 500 B.C.) cf. Parker, Vienna Pap., p. 30 ff.

[10] Boll [1894], p. 181 ff. Cf. also K. Trüdinger, Studien zur Geschichte der griechisch-römischen Ethnographie (Basel 1918), p. 81–89 where he disproves Boll's attempt to make Posidonius an early representative of astrological geography.

[11] Boll [1894], p. 206.

[12] Boll [1916].

[13] Tetrab., p. 30, 7 (B.-B.). Ptolemy's list (conveniently tabulated in Boll [1916], p. 32–47) enumerates 48 constellations, beginning with the 12 zodiacal constellations (starting with Aries), followed by 21 northern and 15 southern constellations. Cf. concerning this arrangement the remarks made above p. 285 f.

of interest to modern astrophysics, in particular the question of reddishness of Sirius.[14]

For mathematical astronomy the Tetrabiblos is not very informative. In I, 20 Ptolemy criticizes as trigonometrically inaccurate the arithmetical scheme of oblique ascensions which we call "System A" (adapted for Alexandria).[15] This is only added evidence for the importance of Babylonian arithmetical methods in ancient astrology,[16] a fact which is emphasized by their transmission to India.[17]

A more involved application of spherical trigonometry occurs in Tetrab. III, 10 (11) in connection with a doctrine for the determination of the length of life. Its basic method consists in assigning to a person a "starting point" P in the ecliptic (τόπος ἀφετικός) and similarly a "destructive point" Q (τόπος ἀναιρετικός). The astrological considerations of how P and Q are chosen are fortunately of no concern to us here.[18] We need only to know that the ecliptic arc PQ is transformed into a certain arc of right ascension and that the number of degrees thus obtained is supposed to signify the number of years allotted to the person's life; it is only the transformation $\Delta\lambda \to \Delta\alpha$ which interests us in the present context.

Ptolemy computes four examples in which always $P = ♈ 0°$, $Q = ♊ 0°$ and the longest daylight $M = 14^h$ (Alexandria). The four examples are chosen such that (1) P is rising, (2) P is setting, (3) P is culminating, (4) P is 3^h beyond the meridian.

We follow now Ptolemy's procedure in his first example: $P = ♈ 0°$ rising, $Q = ♊ 0°$ (cf. Fig. 83). Let α represent right ascensions, ρ oblique ascensions for $M = 14^h$, taken from the tables in Alm. II, 8.[19] Ptolemy first determines the length of one seasonal hour, assuming the sun at Q:

$$AQ = \alpha(Q) = \alpha(♊ 0°) = 57;44°$$
$$BQ = \rho(Q) = \rho(♊ 0°) = 45;\ 5$$
$$\text{thus} \quad \alpha - \rho = 12;39,$$

hence

$$1 \text{ seas. h.} = 15° + 1/6\ 12;39 = 17;6,30° \quad (\text{Ptolemy: } \approx 17°).$$

The time for Q to reach the meridian is given by

$$\alpha(Q) + 90° = 57;44 + 90 = 147;44° \quad (\text{Ptolemy: } \approx 148°)$$

whereas

$$6 \text{ seas. h.} = 17;6,30 \cdot 6 = 102;39° \quad (\text{Ptolemy: } \approx 6 \cdot 17 = 102°)$$

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[14] For the literature on this topic cf. Nallino, Batt. II, p. 283–289; Boll [1916]; See [1927]. The following facts can be considered as established beyond doubt: (1) Sirius is called "red" (ὑπόκιρρος) in the Almagest (VIII, 1, p. 142, 12f. Heib.) together with 5 other stars: α Boo, α Tau, α Sco, α Ori, β Gem; the first three of these five are again called "red" in Tetr. I, 9. (2) The same term is used in association with Mars (Tetrab. II, 10 B.-B. p. 19, 21; cf. also Boll [1916], p. 20f. (3) In Tetrab. I, 9 (B.-B., p. 29, 12f.) Sirius is associated "with Jupiter and, in a less degree (ἠρέμα), with Mars" (cf. also Boll [1916], p. 78/79, No. 12). (4) There are many passages in ancient literature, independent of Ptolemy, which support a description of Sirius as reddish (cf. the evidence collected by See [1927]).

[15] Cf. p. 368.

[16] Cf., e.g., Neugebauer-Van Hoesen, Gr. Hor., p. 127, p. 129, et passim.

[17] Cf. p. 371.

[18] Cf., e.g., Bouché-Leclercq AG, p. 411 ff.; also Neugebauer-Van Hoesen Gr. Hor., p. 12 s.v. Starter.

[19] Cf. above I A 4, 1 and I A 4, 2.

therefore

$$BQ = 147;44 - 102;39 = 45;5° \qquad (\text{Ptolemy: } 148 - 102 = 46°).$$

This is the desired quantity, i.e. Ptolemy would predict 46 years as length of life.

Obviously this result could have been reached without any computation whatever. Since the quantity in question is the arc of right ascension which is required to bring Q into the same position at which P started, i.e. into the horizon, we simply have $BQ = \rho(Q) = 45;5°$ as tabulated in Alm. II, 8; it is only due to inconsistent roundings that Ptolemy obtained 46°. Indeed his whole procedure described above can be condensed into the identity

$$BQ = (\alpha(Q) + 90) - 6(15 + 1/6(\alpha(Q) - \rho(Q))) = \rho(Q).$$

It is difficult to see why Ptolemy chose such a roundabout way to reach this result.[20]

In the next example, $P = ♈ 0°$ culminating, he is as short as he should be: in order to bring Q into the meridian, from which P started, one needs the time

$$\alpha(Q) = \alpha(♊ 0°) = 57;44°$$

which Ptolemy rounds to 58° (thus 58 years of life expectancy).

In the third example his method is again more complicated since he refers to data obtained in the first example. In fact he could have said that the setting time of $Q = ♊ 0°$ is the rising time of ♎ + ♏, i.e.

$$\rho(♐ 0°) - 180° = 250;23 - 180 = 70;23°,$$

rounded by Ptolemy to 70°.

In the last example P starts 45° past the meridian, i.e. 1/4 of its day arc. In order for Q to reach a point 1/4 of its day arc past the meridian it has first to reach the meridian and then travel the arc of 3 seasonal hours i.e. $3 \cdot 17;6,30 = 51;19,30°$ (Ptolemy: $\approx 3 \cdot 17 = 51°$). The arc from the initial position of Q to the meridian is, of course

$$\alpha(Q) - 45° = 57;44 - 45 = 12;44° \qquad (\text{Ptolemy: } \approx 13°),$$

hence the arc in question $51;19,30 + 12;44 = 64;3,30°$, in agreement with Ptolemy's 64°. But to reach this result he also determined the culminating point of the ecliptic in the initial position (♉ 18°) without making any use of it.

Another doctrine which was fascinating to human thought for many centuries was the idea that the different ages of life are ruled in succession by different planets.[21] This theme is treated in the last chapter of the Tetrabiblos (IV, 10) and there one finds the following number of years associated with the celestial bodies

Moon	4 years
Mercury	20 years
Venus	8 years
Sun	19 years
Mars	15 years
Jupiter	12 years.

Saturn, the cold and destructive planet, rules over the remaining years of life.

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[20] One cannot assume a simple oversight because in his general text (p. 135, 15-18 B.-B.) he gives the correct rule: $BQ = \rho(Q)$.

[21] Cf. Boll, Lebensalter.

In the above numbers one recognizes the usual synodic periods of 8 and 12 years for Venus and Jupiter, respectively. The 19 years of the sun are taken from the luni-solar intercalation cycle—in another context the sun is often associated with the number 1461 of the Sothic cycle,[22] the moon with the 25-year cycle.[23] The 15 years for Mars and the 20 years for Mercury are very common in astrological literature,[24] being based on approximate synodic periods.[25] The 4 years for the moon, however, seem to have no astronomical significance, being chosen simply because of the 4-year cycle of the Alexandrian intercalations as stated by Ptolemy.[26] The indiscriminate mixture of astronomically significant data with utterly trivial numbers from which to construct a basically absurd pattern is characteristic for the whole astrological literature.

§ 7. "Planetary Hypotheses" and "Canobic Inscription"

1. Introduction

Ptolemy's "Planetary Hypotheses" are arranged in two "Books," only the first of which is partially preserved in Greek—referred to in the following as "Part 1" of Book I (or as I, 1). There exists, however, an Arabic translation of the complete work, made by Thābit ibn Qurra (late 9th cent.), as well as a Hebrew translation, based on the Arabic version, made by Kalonymos (early 14th cent.). The Greek text of I, 1 was edited by Heiberg in the Opera astronomica minora (1907)[1] of Ptolemy, faced by a German translation of the corresponding Arabic version by L. Nix. The Greek text and a French translation by Halma had been published in 1820[2]; the Arabic text of both Books was first published by Goldstein [1967], with an English translation of the remaining part (I, 2) of Book I. A German translation of Book II, by L. Nix, was included in Ptolemaeus, Opera II, p. 110–145.[3]

A Latin translation by Bainbridge of I, 1, made with real understanding of the astronomical contents, was published in London 1620. Most of Bainbridge's results were ignored for the "critical" edition by Heiberg and Nix[4]; much of the following is a return to Bainbridge. The implausible sequence of events which led Goldstein to the discovery of I, 2 will be described presently in Sect. 6.[5] Tables which completed Book II seem to be definitively lost.

[22] Cf., e.g. CCAG 1, p. 163, 18.
[23] Cf., e.g. CCAG 1, p. 163, 19; CCAG 5, 2, p. 119, 36/37. For the 25-year cycle cf. above III, 2.
[24] Cf., e.g. Bouché-Leclerq, AG, p. 410; Neugebauer-Van Hoesen, Gr. Hor., p. 10f.; CCAG 5, 2, p. 116, 17; p. 119, 36.
[25] For the 20-year period of Mercury cf. also Almagest IX, 10 (Heiberg, p. 293).
[26] Tetrab. IV, 10, p. 206, 6 B.-B.; cf. also Boll, Lebensalter, p. 33ff.
[1] Opera II, p. 70–106; cf. also p. VI–X, p. XV, and p. CLXVI–CLXXIV.
[2] Halma, Hypoth., p. 41–56; cf. for this edition Heiberg in Ptol., Opera II, p. CLXXIV.
[3] Cf. also the preface (p. XVI f.) by Heegaard.
[4] Cf. Heiberg in Ptol., Opera II, p. CLXXIII f.
[5] Cf. below p. 918.

The "Canobic Inscription" is preserved in three Greek manuscripts,[6] edited by Heiberg (without translation) in Ptolemaeus, Opera II, p. 148–155, again preceded by Halma (text and French translation, 1820). The text claims to reproduce an inscription on a stele in Canopus,[7] erected by Ptolemy in the year Antoninus 10 (A.D. 146/7),[8] recording the basic parameters of astronomy. As these parameters agree well with the data in the Almagest I do not doubt the date or the authenticity of the Inscription.

Since the latest date mentioned in the Almagest for an observation falls in the year Antoninus 4 (A.D. 141)[9] the "Canobic Inscription" must be near to the completion of the Almagest; hence it belongs to the earliest period of Ptolemy's work. The "Planetary Hypotheses," however, show a close relationship to the Handy Tables and are therefore a late, if not the latest, work of the author.

In the subsequent discussion of parameters "years" without further specification always means Egyptian years (365^d). For numerical parameters I consistently use what seems to be the best value available; the incorrect versions are mentioned in the notes. The printed text illustrates the futility of restoring the original version of a text by purely philological procedures; the obviously correct numbers are sometimes found in the Greek text adopted in the edition, sometimes relegated to the variants, or only given in the Arabic version.

2. Sun and Moon

The only parameter contained in the first four sections of the "Planetary Hypotheses" is the value $\varepsilon = 23;51,20°$ for the obliquity of the ecliptic, well-known from the Almagest.[1] Sect. 5 states that

$$300^y + 74^d = 300 \text{ tropical years} \tag{1}$$

which is the equivalent of

$$1 \text{ tropical year} = 365;14,48^d, \tag{2}$$

again as in the Almagest.[2] Similarly it is assumed that the fixed stars and the planetary apogees are subject to a motion of 3° in 300 tropical years, hence completing one rotation in 36000 tropical years $= 36024^y + 120^d$. Accordingly the longitudes of all fixed stars and of the apogees of the planets increase by 0;0,36° each tropical year.

Sect. 6 tells us that[3]

$$8523 \text{ trop. y.} = 8528^y + 277;20,24^d = 105416 \text{ syn. m.} \tag{3}$$

[6] Ptolemy, Opera II, p. X, p. CLXXV.

[7] Not far from Alexandria, to the north-east.

[8] Opera II, p. 155, 3.

[9] Above p. 834.

[1] Cf., e.g., above p. 734; the same value in the Canobic Inscription (below p. 913).

[2] Cf. above I B 1, 1.

[3] Variants: 8523]: 8528 Greek; correct in Arab. 277]: 257 in Arab. B. 105416]: correct in Greek, following Bainbridge, but 109416 in AB, 106416 Arab.

The first equation is a consequence of (1) or (2). The second relation implies that

$$1 \text{ syn. m.} = 29;31,50,8,48,\ldots^{d}. \tag{4}$$

Both relations (3) and (4) seem not to be attested elsewhere in exactly this form, though they closely approximate well-known ratios.

Equally unattested is the relation

$$3277 \text{ syn. m.} = 3512 \text{ anom. m.} \tag{5}$$

or

$$1 \text{ anom. m.} = 0;55,59,6,41,\ldots \text{ syn. m.} \tag{6}$$

Sect. 7 deals with the planets. For each of them a relation of the form

$$a \text{ synodic periods} = s \text{ sidereal years} = n^{y} + m^{d}$$

is given. The second half leads in all cases to the same length of the sidereal year:

$$1 \text{ sid. y.} = 365;15,24,31,22,27,7^{d} \tag{7}$$

without involving any rounding. This is an interesting result since it gives a basic parameter with full accuracy.

The remaining Sect. 8 to 14 deal with the elements for the orbits of sun, moon, and planets. These parameters concern the dimensions and relative positions of the eccenters and epicycles in terms of the radius $R=60$ of the deferent. Each section ends with the values of these parameters at epoch, for which is chosen "the first year after the death of Alexander, Thoth 1, Alexandria noon," i.e. the epoch of the Era Philip. The same era is used in the Handy Tables, whereas the Canobic Inscription is based on the Era Augustus. The Era Philip begins 424 years after the Era Nabonassar, the Era Augustus 294 years after the Era Philip. The corresponding julian dates are: -746 Feb. 26, -323 Nov. 12, -29 Aug. 31, respectively.

For the sun the following data are given:

$$\begin{array}{llll} \text{radius of the eccenter:} & R=60, & \text{eccentricity: } e=2;30 & \\ \text{apogee:} & \lambda_A = 65;30^\circ, & 150 \text{ anom. y.} = 150^{y}+37^{d} & (8) \\ \text{at epoch: anomaly:} & \bar{\kappa}=162;10^\circ, & \text{long. of Regulus: } 117;54^\circ. & \end{array}$$

Eccentricity and apogee are the same as in the Almagest and in the Canobic Inscription.[4] Since the relation for the anomalistic year, given in (8), is the exact equivalent of (1), p. 901 the value (2) is also valid for the anomalistic year. This is, of course, a consequence of Ptolemy's assumption that the solar apogee has a fixed tropical longitude.

For the mean motion of the sun during 424^{y} the tables in the Almagest give $256;55^\circ$. Since the anomaly for Nabonassar 1 was $265;15^\circ$ one finds for the new epoch an anomaly of $162;10^\circ$ as stated in (8).

[4] Opera II, p. 150, 2. The value $\lambda_A = 65;31$ (Opera II, p. 152, 7) is a scribal error, perhaps caused by the subsequent numbers which end in 11.

The position of Regulus can be obtained from the Canobic Inscription which gives for Augustus 1 the longitude 120;50°.[5] Since precession during 294^y amounts to 2;56,24° we obtain for Philip 1 for Regulus the longitude 120;50 − 2;56 = 117;54°.[6]

Turning to the moon, Sect. 9 first states the basic parameters

$$\begin{array}{ll} R=60, & \text{eccentricity: } e=12;30 \\ \text{rad. of epicycle: } r=6;20, & \text{inclination: } i=5^\circ. \end{array} \tag{9}$$

The extremal latitude of 5° is the generally accepted value.[7] The values for e and r are only formally different from the parameters 10;19 and 5;15, respectively, in the Almagest.[8] The latter uses the norm (cf. Fig. 84) OM + MC = 60, whereas the Canobic Inscription and the Planetary Hypotheses make MC = 60. Accurate conversion would lead to $e=12;27,32,\dots$ and $r=6;20,24,\dots$ rounded in the Canobic Inscription to $e=12;28$ and $r=6;20$, respectively.[9]

Next come the parameters which define the motion of the center C of the epicycle, based on the refined Ptolemaic model: (a) the retrograde motion of the nodes or the equivalent motion of the "northern limiting point" N of the inclined orbit[10]; (b) the retrograde motion of the center M of the eccenter; (c) the direct motion of C on the eccenter (cf. Fig. 84).

The motion of N is the excess of the moon's mean argument of latitude $\bar{\omega}'$ over the mean longitude $\bar{\lambda}$:

$$\lambda_N = \bar{\lambda} - \bar{\omega}'. \tag{10}$$

The amount of this retrograde motion is defined by

$$2 \text{ rotations } (+0;1^\circ) \text{ of N in } 37^y + 88^d \tag{11}$$

which is equivalent to a daily motion of $-0;3,10,41,10,53,\dots^{\circ/d}$ as compared with $-0;3,10,41,15,\dots^{\circ/d}$ in Alm. IV, 4.

Here, as in similar relations later on, the text mentions a small correction in the form "*n* rotations, or more accurately, 1 minute more" – corrections which amount at most to 4 minutes of arc. These small increments seem to be purely fictitious since they have either no effect at all on the relevant digits or lead to a less accurate result. It seems strange that Ptolemy himself should have made such senseless additions, but since they are found not only in the Arabic but also in the Greek versions one is led to the conclusion that the Greek text had been tampered with (scholia?) before the Islamic period.[11]

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[5] Opera II, p. 152, 3. Heiberg divided the sentences in lines 2 and 3 incorrectly and took the wrong version (120;8) into the text, removing 120;50 to the apparatus.

[6] The same result can be derived from Alm. VII, 2 (p. 15, 3, Heiberg) where the longitude of Regulus for Antoninus 2 (A.D. 139) is given as 122;30°.

[7] E.g. Alm. V, 7 et passim.

[8] Cf. above p. 88.

[9] Opera II, p. 150. 5 and 12. For $e=12;28$ cf. also Pappus' Comm. to Alm. VI, ed. Rome CA 1, p. 171, 13.

[10] Cf. Fig. 72, p. 1228.

[11] As an example of modifications of Ptolemaic parameters by 0;1° which have no effect whatever one can point to Bar Hebraeus in his report on planetary latitudes, written in A.D. 1279 (L'asc. VI, 6, p. 59 trsl. Nau). Cf. also below p. 907, n. 4.

The retrograde motion of the center M of the eccenter is described as the excess of the double elongation $2\bar{\eta}$ over the argument of latitude (cf. Fig. 84)

$$\omega'_M = 2\bar{\eta} - \bar{\omega}' \tag{12}$$

in the amount of

$$203 \text{ rotations } (-0;2^\circ) \text{ of M in } 17^y + 348^d. \tag{13}$$

The corresponding daily motion is $11;9,7,43,\ldots^{\circ/d}$ with respect to the direction ON.[12]

Finally the center C of the epicycle is supposed to complete

$$490 \text{ rotations } (+0;4^\circ) \text{ in } 19^y + 300^d, \tag{14}$$

i.e. $24;29,16,29,\ldots^{\circ/d}$ with respect to the direction OM. This motion is said to be the sum of the two preceding ones. In fact, we have

$$24;29,16 = 2(12;11,27 + 0;3,11)$$

which is twice the sum of the daily elongation and the nodal motion. Since C is moving away from M at the rate of the double elongation $2\bar{\eta}$ and since (13) represents the rate of $2\bar{\eta} - \bar{\omega}'$ we should have added $\bar{\omega}'$ to the latter in order to obtain $2\bar{\eta}$. Apparently the motion of the node and the motion with respect to the node were confused. Again it is hard to believe that Ptolemy committed such an elementary error.

The motion of the moon on the epicycle, the anomaly, is determined by[13]

$$348 \text{ rotations } (-0;1^\circ) \text{ in } 26^y + 99^d. \tag{15}$$

The corresponding daily motion is $13;3,53,54,\ldots^{\circ/d}$ and is not attested elsewhere in Greek sources[14]; Almagest and Canobic Inscription have $13;3,53,56,\ldots^{\circ/d}$.

For the epoch, Philip 1, Thoth 1 Alexandria noon, the following positions are given[15]

$$\left.\begin{array}{ll} \text{longitude of N:} & \lambda_N = -230;19^\circ \\ \text{apogee of eccenter from N:} & \omega'_M = -82;40^\circ \\ \text{center of epic. from apogee of ecc.:} & 2\bar{\eta} = 260;40^\circ \\ \text{anomaly:} & \bar{\alpha} = 85;17^\circ. \end{array}\right\} \tag{16}$$

The Greek version is corrupt here as the omission of a whole sentence, concerning ω'_M, shows. In the Arabic version all four parameters are correct since they agree with the Handy Tables where exactly the same epoch values are given.[16] But one should replace ω'_M in the Arabic version by

$$\mu = -82;40^\circ. \tag{17}$$

[12] Cf. Alm. V, 2 (Manitius I, p. 261); from Alm. IV, 4 one obtains $11;9,7,42,51,\ldots^{\circ/d}$. For the Canobic Inscription cf. below p. 914 (2).

[13] Variant: 348]: 347 Arab.

[14] Bailly, Astr. Ind., p. 422, Table V for 15 days leads for the anomaly to $13;10,34,52 - 0;6,40,58 = 13;3,53,54^{\circ/d}$. These tables are based on the Sūrya-Siddhanta tradition.

[15] Variants: 230;19]: Heiberg accepted the incorrect version 230;13; $\omega'_M = -82;40$]: missing in the Greek version; 260;40]: 261;32 Greek versions; 85;17]: 85;36 Greek versions.

[16] Cf. below p. 987; Halma HT II, p. 66/67.

On the other hand the sequence of parameters enumerated in (16) is probably the original one since the tables mentioned at the end of Book II[17] had four consecutive columns for $-\lambda_N$, $-\omega'_M$, $2\bar{\eta}$, and $\bar{\alpha}$, respectively, for which (16) should provide the epoch values. That this is not the case is additional evidence of serious tampering with the text.

It is easy to reconstruct these concluding tables since only mean motions are involved. The epoch value for the solar anomaly, 162;10°, is also found in the Handy Tables and since it was determined by the tables in the Almagest we can also find the differences in the same fashion; the result is simply the solar mean motion tables of the Handy Tables.[18] The same holds for the lunar tables, except for the arrangement of the columns and changing one table:

Plan. Hyp.:	$-\lambda_N$	$-\omega'_M$	$2\bar{\eta}$	$\bar{\alpha}$
Handy Tables:	μ	$2\bar{\eta}$	$\bar{\alpha}$	$-\lambda_N$

The table for the argument of latitude ω'_M of the center M of the eccenter can nevertheless be restored easily. The epoch value follows from

$$-\omega'_M=\lambda_N-\mu=-230;19+82;40=-147;39\equiv 212;21°$$

whereas the Almagest provides us with the differences for 25 year steps through the relation (12), p. 904:

$$\omega'_M=2\bar{\eta}-\bar{\varpi}'=21;7,54-117;49,40=263;18,14$$

and similarly for the shorter intervals. Hence the second lunar table in the Planetary Hypotheses must have begun as follows

Philip	1	212;21
	26	115;39
	51	18;57
	76	282;16

No mention is made of tables which would lead from the mean positions to the true positions. One cannot assume that Ptolemy has left his work at this point although the text gives a formal ending.[19]

3. Planets; Periods and Longitudes

In the preceding section we made use of the second part of a relation[1] given for each planet in the form

$$a \text{ synodic periods} = s \text{ sidereal years} = n^y + m^d. \tag{1}$$

[17] Opera II, p. 144, 16–29; cf. below p. 913 for the planets.
[18] Halma HT II, p. 66–67.
[19] Opera II, p. 145; cf. also below p. 913.
[1] Above p. 902, from Sect. 7 of the Planetary Hypotheses.

Table 15

	Text		
	a anom. rot.	s sid. years $= n^y + m^d$	Length of synodic period
☿	3150	993 s.y. $= 993^y + 255;\ 0,54,\ 0,\ 4,46,51^d$	$115;\ 8,34,18,10,17,14,\ 2,\ldots^d$
♀	603	964 s.y. $= 964^y + 247;34,\ 2,45,23,40,28^d$	$583;55,34,43,51,22,57,\ 9,\ldots$
♂	473	1010 s.y. $= 1010^y + 259;22,50,56,16,27,50^d$	$779;56,\ 6,57,27,18,27,\ 7,\ldots$
♃	706	771 s.y. $= 771^y + 198;\ 0,\ 9,18,\ 0,26,57^d$	$398;53,\ 6,58,55,14,29,\ 8,\ldots$
♄	313	324 s.y. $= 324^y + 83;12,26,19,14,25,48^d$	$378;\ 5,35,55,50,39,11,16,\ldots$

Table 15[2] shows the results one obtains from the first half of (1) for the lengths of the synodic periods. None of these parameters is known in exactly this form from any other source.

In the Sect. 10 to 14 of the Planetary Hypotheses we find a set of relations of the form

$$a \text{ sidereal rotations} = n^y + m^d. \tag{2}$$

If we convert $n^y + m^d$ to a total of t days the quotient $a \cdot 360/t$ represents the sidereal mean motion of the planet. In order to change to the customary mean motion in longitude (reckoned from the vernal point) we add to the sidereal motion of $a \cdot 360°$ the amount of precession which corresponds to the time interval t, at a rate of 1° per century. The resulting data are shown in Table 16[3]; the values for the sun are, of course, the same given for the center of the epicycle of Mercury and Venus.

Table 16

	Time	Precession	Number of rotations	Total motion
☉	$144^y + 37^d = 14,36,37^d$	1;26,28°	144	14,24,1;26,28°
☿	$208^y + 174^d = 21,\ 8,14^d$	2; 5, 5°	865	1,26,30,2; 5, 5°
♀	$35^y + 33^d = 3,33,28$	0;21, 3	57	5,42,0;21, 3
♂	$95^y + 361^d = 9,43,56$	0;57,36	51	5, 6,0;57,36
♃	$213^y + 238^d = 21,39,43$	2; 8,12	18	1,48,2; 8,12
♄	$117^y + 330^d = 11,57,15$	1;10,45	4	24,1;10,45

Computing the corresponding mean velocities one finds very close agreement with the values in Alm. IX, 4 (cf. Table 17). For the inner planets we must take the sum of mean motion in longitude and in anomaly in order to obtain the motion

[2] Table 15 gives the correct numbers. Variants in the Text: Mercury, fractions: Greek: ;0,54,46,51, omitting 0,4 in the middle; Arab. A: ;54,0,4,46,51, omitting first 0; Arab. B: ;0,54,0,4,40,51, i.e. 40 instead of 46. Venus: Greek: ;33,2,45,23,40,28, i.e. 33 instead of 34; Arab. A: ;34,2,45,23,57,28; Arab. B: ;34,2,45,23,48,28. Jupiter: Greek B and Arab.: 770 instead of 771.

[3] Variants: Mercury: Arab.: 250^y; Arab. A: 194^d; Greek correct. Jupiter: both Arab. and Greek have 240^d instead of the correct 238^d given by Bainbridge, perhaps without textual basis. In this case the deviation in Table 16 would be 0;0,0,0,27, ... Saturn: The German translation p. 105, 3 gives without explanation a number 119 in parenthesis beside the correct number 117: the first number comes from Arab. A, the second from B [Goldstein].

Table 17

	1 Planet. Hyp.	2 Almagest		1−2
☉	0;59, 8,17, 8, ...$^{\circ/d}$	0;59, 8,17,13,12,31	longitude	−0;0.0.0.5
☿	4; 5,32,24,12,16, ...	0;59, 8,17,13,12,31 3; 6,24, 6,59,35,50 4; 5,32,24,12,48,21	longitude anomaly	−0;0.0.0.0.32
♀	1;36, 7,44,37, ...	0;59, 8,17,13, ... 0;36,59,25,53, ... 1;36, 7,43, 6, ...	longitude anomaly	+0;0.0.1.31
♂	0;31,26,36,55,19, ...	0;31,26,36,53,51,33	longitude	+0;0.0.0.1.28
♃	0; 4,59,14,26,43,35, ...	0; 4,59,14,26,46,31	longitude	−0;0,0.0.0.3
♄	0; 2, 0,33,31,25,57, ...	0; 2, 0,33,31,28,51	longitude	−0;0,0,0.0.3

with respect to the direction toward the vernal point. The comparatively poor agreement in the case of Venus is not caused by an error in the numbers given since exact agreement with the values in the Almagest would only require an increase of r by $0;3^d$. In other words, the periods chosen are too small to give sufficiently accurate mean values. There can be no doubt that the parameters of the Almagest are at the basis of all the numbers in Table 16.[4]

We are confronted with quite a different situation when we look at the arrangement of the planetary models themselves. Table 18 shows that eccentricities and epicycle radii agree with the values in the Almagest, except for Mercury.[5] In the Almagest as well as in the Planetary Hypotheses the equant is at a distance $e_1 = 3$ from the observer and the center of the eccenter rotates around a mean position such that it coincides with the equant once in each rotation. The radius e_2 of this circular motion is in the Almagest also 3 units but 2;30 in the Planetary Hypotheses[6] (cf. Fig. 85) and in the Canobic Inscription.[7] The radius of the epicycle is slightly altered from $r = 22;30$ to 22;15 (cf. Table 18). Proclus in the Hypotyposis gives even $r = 21;30$[8] – probably a simple mistake since the other parameters agree with the Almagest.

Ptolemy himself says in his introduction to the Planetary Hypotheses[9] that he had made changes with respect to the Almagest, based on new observations. Obviously the new value for the arcus visionis of Mercury[10] in the Canobic Inscription is a case in point. The most important changes, however, do not consist in small corrections of the basic parameters but in a new approach to the

[4] The numbers listed in Table 16 ignore small corrections, supposedly due to "accurate computation" (cf. above p. 903) which range between $-0;3°$ and $+0;4°$, having no effect on the first three significant figures in column 1 of Table 17.

[5] Variants: Venus: Greek A and B: 0;15, Arab. 1;0.

[6] Opera II. p. 86. 25.

[7] Opera II. p. 150. 4. Since only one number is mentioned this could also mean $e_1 = e_2 = 2;30$.

[8] Manitius. p. 166, 11.

[9] Opera II. p. 72/73, Sect. 2.

[10] Cf. below p. 1017.

Table 18

	Eccentricity e		Rad. r of Epicycle	
	Plan. Hyp.	Alm.	Plan. H.	Alm.; Can. Inscr.
☿	$e_1=3$ $e_2=2;30$	$e_1=e_2=3$	22;15	22;30
♀	$e_1=e_2$	1;15	43;10	
♂		6	39;30	
♃		2;45	11;30	
♄		3;25	6;30	

theory of latitudes, visible not only in the text of the Planetary Hypotheses but also incorporated in the Handy Tables; we turn to it in the next section.[11]

4. Planetary Latitudes

A. Angles of Inclination

Using the same notation as in our discussion of the theory of planetary latitudes in the Almagest we call i_0 the angle between the ecliptic and the plane of the deferent; the angle between deferent and epicycle is i_1 for an outer planet and i_1 or i_2 for an inner planet following the scheme in Fig. 219 of (p. 1279).

The parameter given in the Planetary Hypotheses show equality of i_0 and i_1 for the outer planets. In the Canobic Inscription three parameters are marred by scribal errors, the remaining three are the same as in the Almagest.[1]

	Almagest		Canobic Inscr.		Plan. Hyp.	modern (A.D. 100)
	i_0	i_1	i_0	i_1	$i_0=i_1$	
♄	2;30	4;30	0; 0!	9, 5,0!	2;30	2;33
♃	1;30	2;30	1;30,0	1, 0!	1;30	1;25
♂	1; 0	2;15	1; 0	2;15	1;50	1;50

Identity of i_0 and i_1 implies a motion of the plane of the epicycle parallel to itself. As we have seen (p. 207ff.) this is not exactly correct since the eccentricities place the earth slightly outside a fixed deferent plane. Nevertheless the resulting deviations are very small and do not outweigh the practical advantages of computing under the assumption of strictly parallel planes.

[11] Also, in greater detail, below V C 4, 5 B.

[1] Variant: Mars 1;50]: Arab. 4;50, probably caused by a scribal error in the Greek original.

In the case of the inner planets the Almagest assumed for the plane of the deferent a periodically varying inclination, with +0;10° and 0° as limits for Venus and −0;45° and 0° for Mercury (cf. p. 214). A similar model can be restored for the Canobic Inscription if one accepts some slight emendations in the text, indicated by [ ] in the next tabulation. In the Planetary Hypotheses, however, a fixed inclination of the plane of the deferent is assumed,[2] as is also the case in the "Handy Tables."[3]

i_0	Alm.	Can. Inscr.	Pl. Hyp.
♀	+0;10°	[+0;]15°	0;10°
☿	−0;45	[−]0;40	0;10

Parallelism of the epicycle to itself during the motion along the deferent would require $i_1 = i_2$ and, strictly speaking, $i_0 = 0$. For the angles i_1 and i_2 we find

	Almagest		Canobic Inscr.		Plan. Hyp.	modern (A.D. 100)
	i_1	i_2	i_1	i_2	$i_1 = i_2$	
♀	2;30°	3;30°	2;30°		3;30°	3;33°
☿	6;15	7; 0	7;0	2;30 !	6;30	6;58

In the Canobic Inscription it is tempting to emend the obviously incorrect value $i_2 = 2;30°$ for Mercury to $i_2 = 7;0$ and thus to obtain $i_1 = i_2$ for both planets as in the Planetary Hypotheses.

B. Precession

A comparison of epoch values from different works of Ptolemy clearly shows that apsidal lines as well as nodal lines of the planetary orbits are subject to the same slow increase in longitude, corresponding exactly to the motion of 1° per century assumed for precession.[1] In other words: apsidal lines and nodes are sidereally fixed, in contrast to the solar apogee which has a fixed tropical longitude.

Although these statements undoubtedly represent the assumptions made by Ptolemy one finds some numerical discrepancies which are very difficult to explain. We have to consider the following epoch dates:

	Era Nabon.	used in	Δt	$\Delta\lambda$ caused by precession
Nabonassar 1	1	Almagest		
Philip 1	425	Plan. Hyp.; Handy Tables	424ʸ	4;14,24 ≈ 4;14°
Augustus 1	719	Canobic Inscription	294	2;56,24 ≈ 2;56
Antoninus 1	885	Cat. of Stars (Alm. VII, VIII)	166	1;39,36 ≈ 1;40
		Total:	884	8;50,24 ≈ 8;50

[2] Opera II, p. 84, 29 and p. 90, 13; Goldstein [1967], p. 6a.

[3] Below p. 1010.

[1] Cf. also Ptolemy's explicit statements, e.g., Alm. IX, 6 and end of IX, 7.

If we denote by A the position of the apogee of the deferent, by N its point of extremal northern latitude and by an arrow a change of longitude in agreement with the above given differences $\Delta\lambda$ caused by precession, 1° per century, then we find the data shown in Table 19.[2]

Jupiter,[3] Mars, Venus and Mercury present no problems; obviously the positions for earlier epochs were computed backwards, starting from integer values found at the time of Ptolemy[4]; the same holds for the apogee of Saturn.

For reasons unknown Ptolemy modified the relative position of A and N for Saturn:

$$\nu = \lambda_N - \lambda_A = \begin{cases} -50^\circ & \text{Almagest} \\ -40^\circ & \text{Plan. Hyp. and H.T.} \end{cases} \tag{1a}$$

With this change may be connected the fact that in the Canobic Inscription[5] the angle ν amounts to only 27°:

$$\lambda_N - \lambda_A = \text{♎}\, 24;20 - \text{♏}\, 21;20 = -27^\circ. \tag{1b}$$

Starting from $\lambda_N = \text{♎}\, 8;24$ at Philip 1 one would obtain $\lambda_N = \text{♎}\, 13^\circ$ for Antoninus 1, or $\lambda_N = \text{♎}\, 3^\circ$ assuming with the Almagest $\nu = -50^\circ$. This would explain the roundings $\lambda_N \approx \text{♎}\, 0^\circ$ and $\lambda_A \approx \text{♏}\, 20^\circ$. The angle (1b), however, seems to be attested nowhere else.

C. Epoch Values

We now can describe the conditions which prevailed according to the Planetary Hypotheses at the beginning of the Era Philip.

Some basic assumptions are common to all planets: (a) the nodal line of the plane of the deferent contains O, the place of the observer; perpendicular to it is the line ON to the northernmost point N of the deferent from where arguments of latitude ω are counted; (b) the line OE to the equant E also determines the apogee A of the eccenter[1]; (c) both N and A are sidereally fixed[2]; (d) the planes of the epicycles move essentially parallel to themselves, in the case of the inner planets such that their nodal lines are always perpendicular to the nodal line of the deferent, for the outer planets parallel (cf. the schematic Figs. 86a and b[3]); (e) the radius CB of the epicycle perpendicular to its nodal line defines on the epicycle a "northernmost" point B such that also on the epicycle an "argument of latitude" Ω can be introduced.

The last described arrangement leads to a new form of defining the position of the planet on the epicycle, not by its anomaly $\bar{\alpha}$ as in the Almagest, but by the argument of latitude Ω. In order to determine the position of the radius CB Ptolemy gives the angle θ between B and the apogee F of the epicycle (cf. Fig. 87). Obviously the relation

$$\bar{\alpha} = \Omega + \theta \tag{1}$$

[2] Variants: Mars ♋ 20;54]: Arab. correctly 110;54, Greek 110;44.
[3] The epoch value for A in Nab. 1 should have been rounded to 2;10°, similar to all other cases.
[4] Normed, e.g., for Antoninus 1 as in the Catalogue of Stars (Alm. VII, 4, Heiberg, p. 36, 14f.)
[5] Cf. below p. 916, Table 23 (where, however, longitudes are counted from Regulus).
[1] OE contains also the midpoint M of the deferent, excepting the case of Mercury where M is rotating.
[2] Cf. the preceding section.
[3] Fig. 86 is drawn for the case that the center C of the epicycle is at N; cf. Ptolemy's formulation Opera II, p. 131, 32–132, 3.

Table 19

	Saturn		Jupiter		Mars		Venus	Mercury	
	A	N	A	N	A	N	A=N	A	N
Nabonassar 1 (Alm. XI, 11)	♏ 14;10		♍ 2; 9		♋ 16;40		♉ 16;10	♎ 1;10	
	↓		(↓)		↓		↓	↓	
Philip 1 (Pl. Hyp., H. T.)	♏ 18;24	♎ 8;24	♍ 6;24	♍ 26;24	♋ 20;54	♋ 20;54	♉ 20;24	♎ 5; 24	♈ 5; 24
	↓		↓	↓	↓	↓	↓	↓	↓
Augustus 1 (Canob. Inscr.)	♏ 21;20	♎ 24;20	♍ 9;20	♍ 29;20	♋ 23;50	♋ 23;50	♉ 23;20	♎ [8;]20	♈ [8;]20
	↓		↓	↓	↓		↓	↓	↓
Antoninus 1 (Alm. IX, X)	♏ 23		♍ 11	(♎ 1)	♋ 25;30		♉ 25	♎ 10	♈ 10
rounded (Alm. XIII)	♏ 20	♎ 0	♍ 10	♎ 0	♌ 0	♌ 0	♉ 25	♎ 10	♈ 10

Table 20

		☿	♀	♂	♃	♄	
	λ_A	185;24	50;24	110;54	156;24	228;24	
	λ_N	5;24	50;24	110;54	176;24	188;24	
1 Plan. Hyp.	$\bar{\kappa}$	42;16	177;16	356;20	292;43	210;38	
	θ	−132;16	−87;16	−176;20	−92;43	−70;38	
	Ω	346;41	168;35	296;46	231;16	219;16	
	$\bar{\alpha}$	214;25	81;19	120;26	138;33	148;38	Philip 1
	$\bar{\lambda}$	227;40	227;40	107;14	89; 7	79; 2	
2 Alm.	$\bar{\alpha}$	214;40	82; 1	120;39	138;57	148;16	Nab. 425
	$\bar{\lambda}$	227;40	227;40	107; 1	88;43	79;24	
1−2	$\Delta\bar{\alpha}$	−0;15	−0;42	−0;13	−0;24	+0;22	
	$\Delta\bar{\lambda}$	0	0	+0;13	+0;24	−0;22	

is a consequence of this definition (θ being negative if counted clockwise). The position of the center C of the epicycle, i.e. the mean longitude $\bar{\lambda}$, is given, as always, through the mean anomaly $\bar{\kappa}$:

$$\bar{\lambda} = \lambda_A + \bar{\kappa}. \tag{2}$$

Finally we must express the aforementioned relationship (d) between the apsidal lines of the deferent and of the epicycle. This leads to requiring that θ satisfies the following relations[4]

$$\theta = -(\bar{\kappa} + \lambda_A - \lambda_N) + \begin{cases} 90^\circ \\ 180^\circ \end{cases} \text{for the} \begin{cases} \text{inner} \\ \text{outer} \end{cases} \text{planets}. \tag{3}$$

Using these three formulae one can compare the epoch parameters given in the Planetary Hypotheses with the values one derives from Alm. IX, 4 for the year Nab. 425 = Philip 1.[5] The results are shown in Table 20[6]; the fact that the deviations $\Delta\bar{\alpha}$ and $\Delta\bar{\lambda}$ for the outer planets show opposite signs but equal absolute values is a consequence of the relations (2) and (3). Similarly $\Delta\bar{\lambda} = 0$ for the inner planets results from the fact that also for the sun $\Delta\bar{\lambda} = 0$.[7]

At one point the relation between the Greek and the Arabic version is of some interest. For Mars both have $\theta = -176;20$ which should lead to $\bar{\kappa} = 180 - \theta = 356;20$ as is indeed found in the Greek version. Thābit, however, has 356;7, a

[4] For Venus and Mars $\lambda_A - \lambda_N = 0$, for Mercury 180°, for Jupiter and Saturn −20° and +40°, respectively.

[5] For Saturn's $\bar{\lambda}$ at Nab. 1 one must use 296;43 as given in the tables of Alm. IX, 4 (Manitius II, p. 104). In Opera I, 2, p. 425, 14 Heiberg changed the ;43 to ;44 in order to obtain better agreement with the computations in Alm. XI, 8. Actually a change to ;45 would be still better and produce agreement with the epoch value 296;45 used in the Handy Tables.

[6] Variants: Mercury: Greek: $\bar{\kappa} = 52;16$; Venus: Greek: $\bar{\kappa} = 177;12$, apogee F: $\theta = -87;12$; Mars: $\lambda_A = 110;44$. All these values are incorrect. Mars Ω: Greek A and B lost the initial 200 (cf. below p. 916, n. 20 and 21); Jupiter $\bar{\kappa}$: Greek and Arab. agree in θ, Greek gives the consistent value $\bar{\kappa} = 292;43$ but Arab. 292;23, thus reducing the deviation from the Almagest to ±0;4°. Jupiter Ω: Greek A and B: 31;31, Arab. correctly 231;16.

[7] Above p. 902.

value which would correspond to $\bar{\lambda} = 107;1$ which is exactly the value derived from the Almagest. Thus the Greek version is in itself consistent but differs by $\pm 0;13°$ from the Almagest; the Arabic version is inconsistent, agreeing for θ with the Greek version, for $\bar{\kappa}$ with the Almagest. The origin of such variants is difficult to explain.

It should be noted that all these parameters discussed so far are based on relative dimensions only, i.e. on $R = 60$, exactly as in the Almagest. There is no reference made to a possible arrangement of the single models within a set of nested spheres.[8]

D. Tables

According to the concluding remarks of Book II of the Planetary Hypotheses[1] the (lost) tables for each of the five planets contained four columns only, corresponding to the parameters listed in our Table 20, p. 912.

The first column gives λ_A, the longitude of the apogee of the eccenter; thus the differences must be the increments of precession, i.e. only 0;15° for the 25-year intervals. The second column concerns $\bar{\kappa}$, thus determining the position of the center C of the epicycle. The third column tabulates $-\theta$, i.e. the angle on the epicycle from its apogee F to its northernmost point B. Finally the fourth column determines the position of the planet P on the epicycle by giving the argument Ω.

Exactly as in the case of the tables for sun and moon[2] no mention is ever made of any tables which would allow one to determine the true longitudes from the mean positions. Since neither the Almagest nor the Handy Tables operate with the same set of variables as the tables described here one must expect also different tables for the equations. Ptolemy's silence on this question is difficult to explain.

5. The Canobic Inscription

The text[1] is headed by the words "From the stele in Canobus: Claudius Ptolemaeus to the Savior God, the Principles and Hypotheses of Astronomy"; it ends with the date: "Erected in Canobus in the year 10 of Antoninus."[2] I see no reason to doubt Ptolemy's authorship, except, perhaps, for the concluding section on cosmic harmonies.[3]

The list of parameters begins with $\varepsilon = 23;51,20°$ for the obliquity of the ecliptic[4] and with the length of the mean solar day ("nychthemeron"), expressed in time degrees: 360;59,8,17,13,12,31, i.e. one sidereal rotation plus the mean motion of the sun during one day, using Alm. III, 2. Then follow the ratios for the eccentricities with respect to the deferent $R = 60$, beginning with the sphere of the fixed stars (of course zero) down to the moon; next are given the radii of the epicycles, all

[8] Cf. for this model below Sect. 7 (p. 922ff.).
[1] Opera II, p. 144, 29–145, 9.
[2] Cf. above p. 905.
[1] Ptol., Opera II, p. 149–155.
[2] A.D. 146/7; cf. above p. 901.
[3] Cf. below p. 934, n. 14.
[4] As in the Planetary Hypotheses; cf. above p. 901.

agreeing with the Almagest and the Planetary Hypotheses,[5] except for the eccentricity of Mercury.[6]

A list of mean daily motions, to six sexagesimal places, follows, beginning again with the fixed stars which are given a motion of

$$p = 0;0,0,5,55,4,7^{\circ/d}. \qquad (1)$$

This, of course, is the constant of precession, 0;0,36,0,0,2,35 in one Egyptian year, thus 1;0,0,0,4,18,20° per century, computing rigidly.

The subsequent parameters for the motion of the planets (longitude and anomaly) and for the sun agree exactly with the Almagest (IX, 4[7]). At the end of this group we find the following mean motions for the moon:

$$\begin{array}{ll} \text{node:} & \lambda_N = \ 0;\ 3,10,41,48,20,51^{\circ/d}\,{}^{8} \\ \text{epicycle:} & \bar{\omega}' = 13;13,45,40,21,51,21 \\ \text{eccenter:} & \omega'_M = 11;\ 9,\ 7,42,18,44,37^{9} \\ \text{anomaly:} & \bar{\alpha} = 13;\ 3,53,56,17,51,59.^{10} \end{array} \qquad (2)$$

The last value is the same as in Alm. IV, 4. The other parameters must be transformed in the same way as in the Planetary Hypotheses[11] in order to be comparable with the Almagest. Thus we use the following relations:

$$\begin{aligned} \lambda_N &= \bar{\lambda} - \bar{\omega}' \\ \omega'_M &= 2\bar{\eta} - \bar{\omega}'. \end{aligned} \qquad (3)$$

Substituting in these formulae for $\bar{\lambda}$ and $2\bar{\eta}$ the values from the Almagest, for $\bar{\omega}'$ the value in (2), one finds for λ_N and ω'_M exactly the values given in (2). This proves that (2) is a consistent set of parameters.

The crucial value in these computations is the value of $\bar{\omega}'$ which in the Almagest IV, 4 is given as

$$\bar{\omega}' = 13;13,45,39,48,56,37^{\circ/d} \qquad (4)$$

for which (3) would give a nodal motion of

$$\lambda_N = 0;3,10,41,15,26,7^{\circ/d}. \qquad (5)$$

Hence we see that the (retrograde) motion of the lunar nodes has been increased by $0;0,0,0,32,54,44^{\circ/d}$ from the value (5) in the Almagest to the value (2) in the Canobic Inscription. During one "Saros"[12] this correction increases the motion of the nodes by 0;1,0,12°, corresponding to about 7 hours of lunar motion. Thus it would be quite possible to derive such a correction from eclipse observations.

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[5] For a change in the norm of the lunar parameters cf. above p. 903.
[6] Cf. above p. 907.
[7] The last digit for the longitude of Jupiter should be 31, not 33 (misprint?).
[8] Text incorrectly 0;3.0.41, ...
[9] The text inserts by mistake an additional digit 9 between 42 and 18.
[10] Text incorrectly 13;13, ...
[11] Cf. above p. 903 and Fig. 84 there.
[12] I.e. 1,49,45[d] (cf. above p. 503 (10)); 18 Egyptian years would give 0;1,0,4°.

Table 21

	Mean longitude $\bar{\lambda}$					Mean anomaly $\bar{\alpha}$				
	1 Can. Ins.	2 from Pl. H.	3 from Alm.	1−2	1−3	1 Can. Ins.	2 from Pl. H.	3 from Alm.	1−2	1−3
☉						90;41	90;41	90;41	0	0
☿	156;11	156;11	156;11	0	0	234;32	234;17	234;32	+0; 5	0
♀						359;34	358;52	359;34	+0;42	0
♂	183;52	184; 5	183;52	−0;13	0	332;19	332; 6	332;20	+0;13	−0;1
♃	8;35	8;59	8;35	−0;24	0	147;36	147;12	147;36	+0;24	0
♄	72;12	72;41	73; 4	−0;29	−0;52	83; 6	83;30	83; 8	−0;24	−0;2

☾	1 Can. Ins.	2 from Pl. H.	3 from Alm.	1−2	1−3
$\bar{\lambda}$	55;40	56;27	55;51	−0;47	−0;11
$\bar{\alpha}$	248;40	248;55	248;52	−0;15	−0;12
ν	115;31	115;37	115;35	−0; 6	−0; 4
ω'_M	256;42	256;49	256;31	−0; 7	+0;11

In adopting the era Augustus for the Canobic Inscription Ptolemy followed contemporary common practice which we know, e.g., from the writings of Vettius Valens.[13] Computing the corresponding parameters both from the Almagest (i.e. for Nabonassar 1) and from the Planetary Hypotheses (Philip 1) one obtains the data shown in Table 21.[14] It is obvious that the Almagest was the source of the planetary parameters, except for some modification in the $\bar{\lambda}$ of Saturn. The discrepancies in the case of the moon are not surprising in view of the change of the nodal motion.[15]

In Table 19[16] we made use of the Canobic Inscription for the comparison of epoch positions of the points A and N. These data, referring to the year Augustus 1, are expressed in the Canobic Inscription not as tropical longitudes but as elongations from Regulus, both for A and for the ascending node, 90° distant from the point N. This norm is chosen because of the assumption of sidereally fixed apogees and nodes such that these parameters do not change with a change of epoch. Indeed we find in the Introduction to the Handy Tables[17] and in the Canobic Inscription the same distances from Regulus.

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[13] Cf. above V A 1, 3; also p. 810ff. or p. 826.

[14] Variants: Sun $\bar{\alpha}$: 90;41]: copyist error 6;41; Saturn $\bar{\alpha}$: 83,6]: correct in C but Heiberg accepted the incorrect variant 83;36.

[15] The equation of time in reference to Nab. 1 amounts to only about −0;4,30^h and thus reduces $\bar{\lambda}$ and $\bar{\alpha}$ in column 3 by only about 0;2°.

[16] Above p. 911.

[17] Opera II, p. 169, 7–13.

Table 22

	Apogee of eccenter			
	Plan. H.	+2;56	−120;50	Can. Ins.
	Philip 1	August. 1	from Reg.	Handy T.
♄	228;24	231;20	110;30	110;30
♃	156;24	159;20	38;30	38;30
♂	110;54	113;50	353; 0	353; 0
♀	50;24	53;20	292;30	292;30
☿	185;24	188;20	67;30	67;30

To check these elements (cf. Table 22) we start from the longitudes of A for the year Philip 1, given in the Planetary Hypotheses. Adding 2;56° for precession[18] we find the longitudes at Augustus 1 and subtracting from the result 120;50° we obtain the longitudes with respect to Regulus[19]; the results agree exactly with the data in the Canobic Inscription.[20]

Table 23

	Canobic Inscr.				ν		
	asc. node	1 N	2 A	1−2 = ν	Alm.	Pl.H.	Handy Tables
♄	353;30	83;30	110;30	−27;0	−50	−40	
♃	328;30	58;30	38;30	20;0		20	
♂	[2]63; 0	353; 0	353; 0	0		0	
♀	202;30	292;30	[2]92;30	0		0	
☿	163;30	253;30	63;30	190;0		±180	
	1[57];30	2[47];30	67;30	180;0			

Table 23 in the first column displays the distances of the ascending nodes from Regulus as listed in the Inscription[21]; adding 90° gives the position of N which we compare with A from Table 22. The result agrees again with the Almagest, except for the obviously garbled data for Mercury and the position of the ascending node for Saturn, discussed in an earlier section.[22]

[18] Cf. above p. 909.

[19] Cf. above p. 902.

[20] Variants: Venus: initial 200 lost (cf. p. 912, n. 6), correct in the Introd. to the H.T. (Opera II, p. 169, 12); Mercury: 63;30 instead of 67;30 as in Introd. to the H.T. (p. 169, 13) and in Heiberg, Anon., p. 115, 8.

[21] Variants: Mars: initial 200 lost (cf. p. 912, n. 6); Mercury: 163;30 probably influenced by the (incorrect) number 63;30 for A (cf. n. 20). The words ἀπὸ τοῦ p. 152, 23 should be deleted.

[22] Cf. above p. 910.

The concluding sections of the Canobic Inscription touch upon a variety of topics. Still related to planetary theory proper are values for the normal arcus visionis,[23] in agreement with Alm. XIII, 7 except for Mercury (10;30° instead of previously 10°).

There follows a statement[24] about the apparent diameters of sun and moon at mean distance:

$$d_{\odot}=d_{\mathbb{C}}=\frac{90}{162} \tag{6a}$$

while the earth's shadow covers the fraction 1/65 of one quadrant:

$$2u=\frac{90^{\circ}}{65}. \tag{6b}$$

We have discussed these estimates in connection with Hipparchus' procedures.[25]

What follows flatly contradicts the results for the distances of the luminaries obtained in the Almagest:

$$R_{\mathbb{C}}=64=(2^3)^2=(2^2)^3, \qquad R_{\odot}=729=(3^2)^3=(3^3)^2 \tag{7}$$

in contrast to 64;10 and 1210 earth radii, respectively, found in Alm. V, 15. These values are also essential for the cosmic system of the Planetary Hypotheses which presupposes 1160 earth radii for the distance of the sun.[26]

The rest of the text completely departs from rational astronomy and gives data for cosmic harmonies in a form which cast doubts on a Ptolemaic authorship.[27]

6. The "Ptolemaic System"

In the Almagest and in the Handy Tables, in the Canobic Inscription and in the first part of the Planetary Hypotheses all planetary orbits are treated separately, assuming for each one of them a radius $R=60$ for the deferent. Only for the moon (through parallax) and for the sun (through eclipses) had Ptolemy established absolute distances, reckoned in earth radii. For the planets parallaxes as well as transits or occultations escaped ancient observational techniques. But an incredible numerical accident seemed to prove that the models for Mercury and Venus, as constructed in the Almagest, could be fitted into the space between moon and sun, such that the maximum geocentric distance of the moon coincided with the minimum distance of Mercury, whose maximum distance would determine the minimum distance of Venus, which at its maximum distance would reach the solar orbit. Nothing could seem more plausible than to extend this pattern beyond the sun to the outer planets and thereby obtain an estimate for the distance of the fixed stars just beyond Saturn.

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[23] Opera II. p. 153, 9–15; cf. also below p. 1017.
[24] Opera II. p. 153, 16–20.
[25] Cf. above p. 313.
[26] Cf. below Table 24 (p. 920).
[27] The numerical data are garbled but can easily be emended; cf. below p. 934, n. 14.

This picture of a universe, consisting of a sequence of nested planetary spheres, dominated Islamic and western medieval astronomy[1] and still remained the guiding principle for Kepler, both in his "Mysterium cosmographicum" and in the "Harmonice mundi." It is this model of contiguous planetary spheres that is usually meant when one speaks about the "Ptolemaic System." Nevertheless it was not until recently that one had incontrovertible evidence for associating Ptolemy's name with this cosmic hypothesis.[2] Indeed there were arguments which cast serious doubts on the assumption of Ptolemy's authorship, in particular Proclus' way of speaking about this theory without any reference to Ptolemy. We shall return to this point presently.[3]

This situation was changed drastically again by an unpredictable sequence of accidents. For the edition of the "Planetary Hypotheses" in the "Opera minora" of Ptolemy[4] L. Nix was entrusted with translating into German the Arabic version of Book I and II, extant in manuscripts in Leyden and in the British Museum. Nix died when he had completed only the translation of the part that corresponds to the still extant Greek text (now called I, 1) while he left a preliminary translation of Book II, to be later revised and completed by Buhl and Heegaard.[5] Inexplicably these two scholars missed, as we know now, ten pages of Arabic text at the end of Book I (now called I, 2).

In 1964 W. Hartner suggested that Arabic astronomers meant Ptolemy's "Planetary Hypotheses" when they were referring to a "kitāb al-manshūrāt" in which a system of nested planetary spheres was described[6]; he had naturally assumed that this section had been lost at the end of Book II. Hartner's paper induced B. Goldstein to investigate a Hebrew version of the Hypotheses, the existence of which had been established by Steinschneider in 1893.[7] Since this version contained the discussion of the nested spheres at the end of Book I (not II!) Goldstein went back to the Arabic manuscripts used by Nix and found to his surprise that these too contained the missing section at the end of Book I, overlooked by Buhl and Heegaard and never mentioned by any of the editors.

Now, since it can no longer be doubted that the system of nested planetary spheres was proposed by Ptolemy in his Hypotheses some passages of Proclus must be seen in a new light. In his commentary to the Timaeus he says explicitly[8] that Ptolemy "in the Planetary Hypotheses does not concern himself with the (planetary) distances, nor does he give proof for them" whereas he mentions in

[1] Cf., e.g., Kohl [1922] (from Ibn al-Haitham ≈ 1000) or Dreyer, Plan. Syst., p. 257ff. (from Jagmīnī ≈ 1200(?) and Bar Hebraeus 1279).

[2] In Book II of the Planetary Hypotheses it is said that Mercury and Venus should be fitted between moon and sun because this space would be useless otherwise (Opera II, p. 118, 10–20); also the description of mechanical models for contiguous planetary spheres (cf. next section) presupposes this system. Nevertheless, since Book II is only preserved in Arabic one could still think of Islamic additions to Ptolemy's work. This view could be supported by noting certain features which can hardly be attributed to Ptolemy (cf., e.g., above p. 903–905).

[3] Cf. below p. 919.

[4] Cf. above p. 900.

[5] Cf. Opera, II, preface p. IX.

[6] Hartner [1964], in particular p. 278ff. for the meaning of this title.

[7] Steinschneider, Hebr. Übers., p. 538 (§ 333).

[8] Proclus, Comm. Tim., ed. Diehl III, p. 62, 22–24; trsl. Festugière IV, p. 85; cf. below p. 920).

the Hypotyposis[9] the fitting of the spheres of Mercury and Venus between moon and sun as the opinion held by "some," an expression that certainly does not include Ptolemy whom he otherwise mentions by name on almost every page. It seems to me that one must conclude from these statements that Proclus in the fifth century only had an incomplete version of the "Hypotheses" at his disposal which covered only what we still have in our Greek text. How this can be reconciled with his reference to "the Books" of the Hypotheses[10] I do not know.

In placing the spheres of Mercury and Venus below the sun Ptolemy must have changed his mind about the problem of parallaxes. In the Almagest[11] he said that no planetary theory should place the inner planets near enough to the earth to result in an observable parallax. Now, in the Hypotheses,[12] he remarks that according to his new model Mercury and Venus should display some parallax and even Mars at perigee should show the sun's parallax at apogee. Of course, since the solar parallax (of less than 0;3°) was only computed[13] no parallax of Mars could be actually observed. But Mercury near maximum elongation should have a horizontal parallax of the order of magnitude of 1/2°, fully within the ancient limits of observation[14]. But it is not surprising that a cosmological theory of such impressive internal consistency was not conducive to serious scrutiny.

It still remains to discuss the numerical data upon which the "Ptolemaic System" rests as described in the second part of Book I of the Planetary Hypotheses.[15]

All distances are reckoned by Ptolemy at first in earth-radii, only to be transposed at the end to absolute values counted in stades.[16] The innermost shell is assumed (cf. Table 24) to have the radius 33, a slightly rounded value for the minimum distance $m = 33;33$ of the moon.[17] The maximum distance $M = 64;10$, rounded to 64, serves as inner radius of the space for Mercury. In order to obtain the radius of the outer sphere for Mercury we have to multiply $m = 64$ with the ratio M/m of the extremal distances, approximately 88/34 according to Ptolemy; this gives $M = 166$ for the maximum distance of Mercury, hence also for the minimum distance of Venus. For Venus M/m is taken to be 104/16 which leads to $M = 1079$, a distance only a little smaller than 1160, the minimum distance of the sun[18] (cf. the scale drawing in Fig. 88). Ptolemy remarks that this gap may be closed by a slight increase in the distance of the moon that results in a decrease of the sun's distance.[19] He could have saved himself this trouble, however, by

[9] Proclus, Hyp. VII, 20ff. (Manitius, p. 223/225). Manitius (p. 307) computed incorrectly the minimum distance of Mercury as 34;30 instead of 33;4 as given correctly by Proclus (cf. also above p. 164 (3)).
[10] Proclus, Comm. Rep., ed. Kroll II, p. 230, 14f. = Ptol. Opera II, p. 110; trsl. Festugière III, p. 185.
[11] Alm. IX, 1 (Manitius II, p. 94).
[12] Goldstein [1967], p. 9a.
[13] Cf. above p. 112.
[14] Cf. Fig. 99 (p. 1236), value of c_3.
[15] Goldstein [1967], p. 7.
[16] Cf. below p. 921.
[17] Above p. 113.
[18] Above p. 110 (9).
[19] The numerical details are not worked out but qualitatively easy to see in the well-known geometric procedure for determining the sun's distance (cf. Fig. 98, p. 1236). If one increases EM, without changing the apparent diameter of the moon, one must also increase EM'. But also σ increases with r_m because of (4), p. 109. Consequently the point F of the sun comes nearer to E.

Table 24

		1	2	3	4	5
		$R=60$ Alm. and Hypotyp.	Ptolemy		Proclus	
			Almagest	Plan. Hypoth. I, 2	Hypotyposis	Comm. Tim.
Moon	m		33;33	33		33
	M		64;10(=1,4;10)	64	64;10	64
Mercury	m	33;4	[1,4;10]	64	64;10	64
	M/m	[2;46,1,41]	[2;46,1,41]	88/34 [≈2;35,17,39]	91;30/33;4	83/32 [=2;35,37,3
	M	1,31;30	[2,57;33,28]	166 (=2,46) [≈2,45;39]	177;33	166
Venus	m	15;35	[2,57;33,28]	166 (=2,46)	177;33 (=2,57;33)	166
	M/m	[6;42,2]	[6;42,2]	104/16 [=6;30]	104;25/15;35	104/16 [=6;30]
	M	1,44;25	[19,49;44]	1079 (=17,59)	1190 (=19,50)	1079 (=17,59)
Sun	m	57;30	[19,19;35]	1160 (=19,20)	1160 (=19,20)	1076 (=17,56)
	μ	1,0	1210 (=20,10)			
	M/m	[1; 5,13]	[1; 5,13]			[1;10,15]
	M	1, 2;30	[21, 0;25]		1210 (=20,10)!	1260 (=21,0)

Numbers in [ ] are not mentioned in the texts. Columns 2 to 5: $r_e = 1$

rounding the ratios M/m for the parameters given in the Almagest (cf. Table 24, columns 1 to 3) less crudely; he then would have found that the maximum distance of Venus slightly exceeds the minimum distance of the sun (by only 30 earth-radii) instead of leaving a gap of 81.[20] This is indeed the result obtained by "some" astronomers, according to Proclus' story in the Hypotyposis[21] (Table 24, column 4). In the Commentary to the Timaeus,[22] however, Proclus quotes Ptolemy for the extrema found in the Planetary Hypotheses (I, 2; cf. Table 24, column 5), although the ratio M/m for Mercury is slightly altered from 88/34 (≈2;35,17,39) to 83/32 (=2;35,37,30). Where Proclus found 1076 as the minimum distance of the sun is unknown, if it is not simply a scribal error.[23]

The extension of this pattern to the outer planets is trivial. Computing with the ratios derived from the Almagest one reaches for the maximum distance of Saturn almost 20800 earth radii (cf. Table 25, columns 1 and 2 and Fig. 89, p. 1402). Ptolemy, using again more convenient roundings (column 3) remains a little below 20000. His numbers cannot be checked in Proclus who neither in the Hypotyposis nor in the Timaeus commentary gives numerical data for the outer planets. But Thābit ibn Qurra (column 4) fully confirms the numbers from the Planetary Hypotheses.[24]

[20] All units in Table 24, columns 2 to 5, are earth-radii.

[21] Proclus, Hyp. VII, 23 (Manitius, p. 224, 12f.). That the extremal (not the mean) distance of the sun is given as 1210 is a slip of the pen without consequences. The same error is found also in the scheme of the MS P[7] (Manitius, p. XXXIV).

[22] Proclus, Comm. Tim., ed. Diehl III. p. 62, 22–63, 20; trsl. Festugière IV, p. 85–87.

[23] All other distances appear also in Thābit b. Qurra's "De hiis" Nos. 43–47 (ed. Carmody, p. 137). Cf. also above p. 110, n. 11.

[24] Thābit, ed. Carmody, p. 137. The maximum 17865 for Saturn is a misprint; the correct value 19865 is given on p. 130.

Table 25

		1	2	3	4
		$R=60$ Alm. and Hypotyp.	Ptolemy		Thābit b. Qurra
			Almagest	Plan. Hypoth. I, 2	De Hiis, 45–47
Sun	M		21,0	1260 (=21,0)	1260
Mars	m	14;30	21,0	1260 (=21,0)	1260 (=21,0)
	M/m	[7;16,33]	[7;16,33]	7	
	M	1,45;30	[2,32,47;33]	8820 (=2,27,0)	8820
Jupiter	m	45;45	[2,32,47;33]	8820 (=2,27,0)	8820 (=2,27,0)
	M/m	[1;37,22]	[1;37,22]	37/23 [≈1;36,31]	
	M	1,14;15	[4,7,56;54]	14187 (≈3,56,28)	14187
Saturn	m	50;5	[4,7,56;54]	14187 (=3.56,27)	14187 (=3,56,27)
	M/m	[1;23,45]	[1;23,45]	7/5 [=1;24]	
	M	1,9;55	[5,46,5;40≈20766]	19865 (≈5,31,2)	19865 (=5,31,5)

Columns 2 to 4: $r_e=1$

In this way Ptolemy must have felt he had reached an estimate for the distance of the fixed stars. In any case, even if nature were to admit the existence of "useless" empty space, the 20000 earth-radii would be a lower limit for the size of the universe.

Finally all of the previously mentioned distances are converted to stades. Assuming for the length of a terrestrial great circle $360 \cdot 500$ stades[25] Ptolemy reckons the earth radius as 2;52 myriads of stades.[26]

Ptolemy does not stop at expressing the planetary distances in terrestrial units of length. By estimating the apparent diameters of the planets he is able to do the same with the actual sizes of the celestial bodies. He quotes two Hipparchian determinations according to which the smallest fixed stars would measure 1/30 of the solar diameter, whereas Venus, seemingly the largest planet, would come to 1/10 of the solar diameter. Similarly Ptolemy gives the following fractions, apparently representing his own estimates:

$$\begin{array}{llllll} \text{Mercury:} & 1/15 & \text{Jupiter:} & 1/12 & \text{fixed star of} & \\ \text{Mars:} & 1/20 & \text{Saturn:} & 1/18 & \text{1st magn.:} & 1/20. \end{array} \qquad (1)$$

Since one now has extremal distances m and M for each planet (cf. Table 26, col. 2) we can compute the mean distances $\mu = 1/2(m+M)$. If we use the fractions d from col. 1 we have in μd (col. 4a) the diameter in units for which $\mu_{\odot} = 1210$. But it is known from the Almagest[27] that the sun has a diameter 5 1/2 times the diameter of the earth. This, then, provides us with the actual diameters (col. 4b) reckoned in terrestrial diameters.

The true diameter of the sun is about 20 times the ancient value and the distances of the planets vary in a fashion very different from the system of nested spheres. Consequently not even the relative order of the sizes of the planets is correct.[28]

[25] Cf. below p. 935.
[26] Hence $\pi \approx 3;8,22$ (cf. p. 140, n. 3). For the numerical data cf. Goldstein [1967], p. 11.
[27] Above p. 110 (7).
[28] Cf. Table 26, columns 5 and 7.

Table 26

	1	2		3	4a	4b	5	6	7
	appar. diam. d	distance ($r_\varepsilon = 1$)		mean dist. ($r_\varepsilon = 1$) μ	diameter		order of size	actual	
		m	M		$\mu \cdot d$	$r_\varepsilon = 1/2$		radius ($r_\varepsilon = 1$)	order
Earth							5	1	4
Moon	4/3	33	64	48	64	1/4+1/24=0;17,30	7	0;16,22	8
Mercury	1/15		166	115	8	1/27=0;2,13,20	8	0;23,29	7
Venus	1/10		1079	622 1/2	62	1/4+1/20=0;18	6	0;58,20	5
Sun	1	1160	1260	1210	1210	5 1/2=5;30	1	109;5	1
Mars	1/20		8820	5040	252	1 1/7=1;8,34, ...	4	0;31,59	6
Jupiter	1/12		14187	11504	959	4 1/3+1/40=4;21,30	2	10;57	2
Saturn	1/18		19865	17026	946	4 1/4+1/20=4;18	3	8;59	3
Fixed Stars 1st m.	1/20			≈20000	1000	4 1/2+1/20=4;33			
Fixed Stars 6th m.	1/30								

7. Book II of the Planetary Hypotheses

The second Book of the "Planetary Hypotheses" is a rather sad affair. Looked at superficially it seems to suggest some mechanism which connects the motions of the planets within a larger cosmic system. In fact nothing of this kind is achieved. Ptolemy ascribes to each planet the voluntary decision to behave exactly according to the rules laid out in the Almagest. No planet is influenced by the motion of any other one and the only unifying principle is their confinement into contiguous but strictly separated compartments whose order and size was established in Book I.[1]

Also the mathematical part of the discussion is trivial. Each planetary model from the Almagest is enlarged into a space configuration by putting spheres over the circles of the original models. All that is new is the position of the planes of the diagrams perpendicular to the planes of the deferents and of the ecliptic that makes it possible to show the small (fixed) angles of inclination between the ecliptic and the orbital planes. The spheres themselves have no real function excepting to carry the axes for the rotations in the circular orbits.

In principle the arrangement in nested spheres was to show that the planetary system was not leaving "useless" empty space since the extremal geocentric distances of each planet can be made to meet the predecessor and the follower, respectively. The space of each of these planetary shells is now stuffed with colossal spheres (of unexplained material consistency), only one of which carries near its surface one miserable little planet.

Ptolemy must have felt the absurdity of this structure and therefore considered the possibility of cutting away all material from each side of the ecliptic that is

[1] Cf. above V B 7, 6.

never reached by the inclined orbital plane of the planet. What remains of the solid spheres of the epicycles are now "tambourins," moving inside of ring-formed zones or "wheels"; obviously this means in fact a return to the plane figures of the Almagest and leaves us again with a factually empty cosmos.

For our discussion of the details of Ptolemy's description of the consecutive planetary models the main problem consists in a proper reconstruction of the illustrations of which only incomplete remnants are preserved in one Arabic manuscript.[2] The reconstructions offered in the edition in Ptolemy's "Opera" contain too many basic errors to be useful.

The first ten sections of the second book of the "Hypotheses" contain a philosophical discussion of the causes of planetary motion, obviously in opposition to the Aristotelian system of concentric spheres which receive their motions from one outermost moving sphere. Ptolemy, instead, leaves control over its motion to each planet, independent of its neighbor – just as birds do not fly through contact with other birds that would merely hinder their movement. For the transmission of the daily rotation to all stars and planets he postulates shells (of unspecified thickness) of a mysterious "ether" between the contiguous planetary spheres.

In Sect. 11 Ptolemy begins with the discussion of the motion of the sphere of the fixed stars. He assumes an outermost spherical shell of "ether" which causes the daily rotation of the heavens around an axis AB (cf. Fig. 90) without telling us what supports this axis. The next inner shell (again of unknown thickness) carries the fixed stars; it is connected by an axis CD not only with the outer ether-shell but also with an inner one. Around this axis CD the shell of the fixed stars is capable of moving in such a fashion that its motion is exactly the same with respect to the outer and to the inner ether-shell. Consequently both ether-shells remain to each other in a fixed position such that Saturn inside the inner ether-shell is exactly in the same relation to it as the fixed stars to the outermost ether-shell. Saturn's compartment in turn is pivoted in the same fashion in relation to an interior ether-shell on the boundary toward Jupiter and thus transmits the daily rotation one step farther down, and so forth, until the shell of the moon is reached.

This accounts for 8 "moving spheres"[3] which cause the daily rotation of the fixed stars and of the seven planets. The direction CD, common to all planetary shells, is the axis of the ecliptic about which the mean motion of the planets take place (crudely speaking) and the slow motion of precession of the fixed stars. The angle between AB and CD is, of course, the obliquity of the ecliptic.

From here on we can ignore the common axis of the daily rotation. All that remains to be discussed concerns the individual planets whose motion inside their

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[2] Ptolemy usually speaks simply of "spheres" where I, for the sake of greater clarity, distinguish "solid spheres" from "spherical shells," the latter meaning the space between two concentric spherical surfaces. There remain "complements" of spheres when one removes eccentric shells from a volume between concentric surfaces.

In order to indicate relative movability of adjacent shells I leave some space between the "contiguous" surfaces, showing the pivots that correspond to their axis of rotation.

Only three of these figures have some parallel in the Arabic manuscript A (Goldstein [1967], p. 43 and p. 47f.). The spaces for the two remaining figures, for Mercury and moon, are left blank. All figures are missing in B.

[3] Using Ptolemy's terminology, Opera II, p. 141, 28/29.

shells is strictly arbitrary. Their relative order is no longer of any interest, assuming the proper dimensions of their shells is determined in advance.

The mechanism describing the solar motion is easy to unterstand (cf. Fig. 91). The outer circle belongs to the inner surface of the ether-shell that imparts to the sun its daily rotation; the innermost circle represents the outer surface of the ether-shell which provides the same service to Venus. The center of the universe is A, the axis of the ecliptic has the direction $B\Gamma$. The eccentricity of the solar orbit is AZ, the diameter of the solid solar sphere is $N\Xi$ which thus determines the thickness of the spherical shell $N\Pi$. Fig. 91 depicts the situation when the sun is in the apogee of its orbit. The shell into which the sun is embedded rotates about its diameter ΘK as axis with a velocity corresponding to the sun's mean motion in longitude. This requires pivots resting in the "complements" which fill the remaining space inside the solar cage.

The mechanism for the three outer planets is similar to the sun's with the difference that the rotating shell $N\Xi$ (cf. Fig. 92) has its axis inclined to the axis $B\Gamma$ of the ecliptic (in the amount by which the plane of the eccenter is inclined to the ecliptic) and that it carries on its equator a spherical cavity EK which contains the epicycle on which the planet itself is mounted. In Fig. 92a the distance AZ is again the eccentricity, the epicycle inside EK is shown at the apogee of the eccenter while the orbital plane in which the center of the epicycle moves intersects the plane of the diagram in the line EZA. The speed at which the epicycle moves in its plane is uniform with respect to a point F (located such that AZ=ZF) i.e. the familiar "equant" of the Ptolemaic theory.[4]

No mechanical support is provided for the points Z and F while A is at least the earth. It may be noted that the centers of the eccenters of the outer planets are located within the shell of Venus (i.e. within the small area at the left end of Fig. 89, p. 1402) since their distances, according to the model of nested spheres, would be about 504 earth radii for Mars, 527 for Jupiter, 970 for Saturn, while the radius of the outer shell of Venus is about 1080 (cf. Table 24, p. 920). The centers of the eccenters for Venus and Mercury lie within the moon's shell.

The epicycle itself is conceived as a solid sphere into which the planet Π is embedded close to the surface (cf. Fig. 92b[5]). In order to have the planet move on an epicycle, whose plane is parallel to the ecliptic, the solid sphere $\beta\gamma$ is enclosed in a concentric shell which fits into the hollow sphere EK. The axis $\nu\xi$ of this shell is parallel to the axis $N\Xi$ of the eccenter. By rotating this shell in a sense opposite to the rotation of the large shell about $N\Xi$ one eliminates the effect of this rotation on the solid sphere that carries the planet Π. This solid sphere, finally, rotates about its pivots $\beta\gamma$ which define a direction perpendicular to the ecliptic. Hence the planet eventually rotates on a circle parallel to the ecliptic, thus obeying the rule for the position of the epicycle, as stated in the first part of the "Planetary Hypotheses".[6] Perhaps this rule was made under the influence of the possibility for a simple cinematic realization rather than based on refined observations.

[4] In our figures all eccentricities are greatly exaggerated; for the text cf. Opera II, p. 131, 26–32 (not understood by the editors).

[5] Enlarged with respect to Fig. 92a.

[6] Cf. above p. 908.

Ptolemy, by referring to an earlier publication,[7] shows that he prefers not to discuss details of his theory of planetary latitudes in connection with the simplified models of Book II. He says that the arrangement for the outer planets is also valid for Venus, with the provision that for Venus and Mercury the slope of the epicycle is directed at right angles to the apsidal line.[8] He does not indicate, however, which modification in the epicycle mechanism must be made to satisfy the special rules for the inner planets. For the moon, on the other hand, the situation becomes simpler because the lunar epicycle always lies in the plane of the deferent. Hence one can spare the shell that surrounds the solid epicycle sphere in Fig. 92b and use the axis $\nu\xi$ directly (cf. below Fig. 94).

For Mercury, Ptolemy requires two more "spheres" surrounding the center A of the universe in order to account for the rotation of the center Z of the eccenter around a point H of the apsidal line (cf. Fig. 93[9]) $\mathrm{BA}\Gamma$ is the axis of the ecliptic, $\mathrm{NZ}\Xi$ the axis of the shell which carries the housing EK for the epicycle in the inclined orbital plane; but since the axis $\mathrm{N}\Xi$ rotates about the parallel axis $\Sigma\mathrm{HT}$ we have to provide two more complementary shells of maximum thickness $2e_2$, where $e_2 = \mathrm{HZ}$.

From the model for Mercury we can make an easy transition to the model for the moon. In both cases the center Z of the eccenter rotates in the orbital plane, i.e. about an axis perpendicular to the direction EA. In the case of Mercury the center of this rotation is a point H on the apsidal line; in the case of the moon Z rotates about A.[10] Hence we have again two parallel axes, one $\mathrm{NZ}\Xi$, the other $\Sigma\mathrm{AT}$ (cf. Fig. 94). The fact that the second axis goes through the center A of the universe produces as inner and outer surface for the complements of the eccentric shell (i.e. of axis $\mathrm{N}\Xi$) two concentric spheres. The inner one is the boundary against the sublunar elements and is no longer needed for transmitting the motion of daily rotation into a lower sphere; therefore no pivots are necessary to reach into this inner space.[11] Towards Mercury, however, the axis $\mathrm{B}\Gamma$ of the ecliptic must find a support; this necessitates an outer concentric shell. Thus the moon requires four "spheres": the solid sphere of the epicycle (of axis $\nu\xi$) and three "spheres" that surround A: the outer concentric shell, the rotating shell that carries the epicycle, and the outer complement between these two. Since the inner complement is no longer necessary it is not counted.

This brings us to Ptolemy's final account for the number of "spheres" required by his model for the universe:

"moving" ether-spheres:	8	
fixed stars:	1	
outer planets and Venus:	20 (3 each, surrounding A, plus 2 for epicycle)	(1)
sun:	3	
Mercury:	7 (5 surrounding A, plus 2 for epicycle)	
Moon:	4 (3 surrounding A, plus 1 for epicycle).	

[7] Opera II, p. 131, 20–24, presumably referring to the Almagest.

[8] Opera II, p. 132, 1–3; cf. for this scheme p. 1402, Fig. 86.

[9] In our previous terminology the radius HZ is the eccentricity e_2 (cf. e.g., Fig. 85); Ptolemy does not say whether he assumes also $\mathrm{AH} = 2e_2$ or not. For the basic model cf., e.g., Fig. 239 (p. 1288).

[10] Cf., e.g., Fig. 78 (p. 1229).

[11] Opera II, p. 140, 11 f.

This would give a total of 43; Ptolemy, mysteriously giving the sun only one sphere, finds 41.[12] Since he is not sure that the planets need the outside moving ether-spheres, instead of participating in the daily rotation on their own initiative, he is willing to reduce the number of required spheres to $41-7=34$. By some speculations replacing the complete "spheres" by "rings" and "tambourins"[13] he comes as low as 22 necessary pieces. Copernicus, in the "Commentariolus" again counts the circles of his universe:

outer planets and Venus:	20	
Earth:	3	(2)
Mercury:	7	
Moon:	4.	

"Altogether, therefore, 34 circles suffice to explain the entire structure of the universe and the entire ballet of the planets."[14]

§ 8. Additional Writings of Ptolemy

1. The "Phaseis"

Only the second book of a treatise by Ptolemy entitled "Phases of the fixed stars and collection of weather prognostications" is preserved[1]. It contains, following an introductory text, a list of dates, tabulated for the climates II to VI, for the phases of 30 stars of first and second magnitude and their significance for the weather. The introduction to this book fortunately includes a short summary of Book I.[2] It is enough to show that in it matters of spherical astronomy had been discussed, similar to topics also found in the Almagest, in particular in VIII, 6. We therefore begin our summary with this chapter of the Almagest.[3]

The motive for the tabulation of the heliacal risings and settings of the fixed stars is the belief that these phases are "indicative" of the weather. This theory seems to have its origin, or at least its systematisation, in the fifth century B.C., with Meton and Euctemon. Ptolemy was not firmly convinced of its value but he wished that at least its astronomical basis were solid.[4] Therefore he reduced the number of stars from a multitude of constellations considered by his predecessors

[12] Opera II, p. 141, 29.
[13] Cf. above p. 923.
[14] Rosen TCT[(2)], p. 90.
[1] Cf. for the Greek text Ptolemy, Opera II, p. 1–67 and p. III–V, p. CL–CLXV. Greek text with French translation: Halma, Chron., p. (13)–(54). Boll ([1894], p. 168) accepted as "plausible" a conclusion by Wachsmuth (in his Lydus, De ostentis, p. XLVIII, ed. of 1868, p. LVI/LVII, ed. of 1897) according to which the "Phaseis" were written around A.D. 137/8 since one finds there Thoth 28 as date of the autumnal equinox, a date which agrees with an observation made by Ptolemy (in A.D. 139, recorded in Alm. III, 1). Unfortunately Thoth 28 is reckoned in the Alexandrian calendar and therefore remains valid for decades.
[2] Heiberg, p. 3.
[3] For the general definition of the "phases" cf. below VI B 5, 2.
[4] Cf. Phaseis No. 8 (Heiberg, p. 11 f.) and the concluding remarks in Alm. VIII, 6.

to just 30 stars of first and second magnitude and gave exact numerical data for the phases. As a result his work lies entirely outside the main stream of the popular tradition and should be counted as one of his independent achievements.[5]

The association of the periodic climatic changes with the stars instead of with dates in the civil calendar makes good sense in a society which operates with a variety of local lunar calendars with almost no fixed rules for intercalations. It is therefore not surprising that devices became popular which indicated the progress of the sun and related it to the observable phases of the fixed stars and finally to the expected meteorological changes.[6]

This generally accepted hypothesis of stellar (and lunar) influences on the weather undoubtedly prepared the ground for astrological belief in general. It is not surprising to find it mentioned among the primary arguments in favor of astral influences.[7] The combination with the phases of the moon and of the planets easily leads to the consideration of the geometric "aspects" in the position of the celestial bodies (e.g. oppositions) and the "directions" (*προσνεύσεις*) towards points in the horizon, e.g. at eclipses[8] or for the lunar phases.[9]

A. Alm. VIII, 6

In order to find the date of a phase of a certain fixed star, e.g. of its heliacal rising, it suffices to determine the corresponding position of the sun because the solar theory allows us to convert longitudes into calendar dates. Furthermore, if one uses the Alexandrian calendar these dates can be considered as practically fixed because the difference between the tropical year and the Alexandrian year is too small to be significant for such ill defined phenomena as first and last visibility, at least within reasonable limits of time.[1]

Ptolemy at first discusses the parameters which influence the distance of a star Σ from the sun at the moment of, e.g., heliacal rising (cf. Fig. 95). Obviously, at a given locality, such parameters are the brightness of the star (i.e. its magnitude), its latitude β, and the inclination ν of the ecliptic to the horizon. These data will vary greatly from star to star and will also differ for different phases. Hence Ptolemy associates with a given star and a specific phase a characteristic parameter, the depression h of the sun below the horizon (the "*arcus visionis*" in modern terminology) at which the phenomenon takes place. In order to find h at a given geographical latitude φ_0 (e.g. Alexandria) one must establish, by actual observation, the date and hour of the phase, elements from which one can determine trigonometrically the simultaneously rising point H of the ecliptic[2] and the culminating point M.[3] From the observed date and moment one finds the longitude $\lambda_\odot$ of the sun and thus its distance σ from H.

[5] Cf. Vogt [1920], p. 17, p. 31, p. 51.
[6] For details cf. Rehm, RE Par., RE Ep., and [1940]; also Boll, Gr. Kal. I to V.
[7] E.g. in the Tetrabiblos I, 2.
[8] Cf. also above I B 6, 7.
[9] Cf., e.g., Tetrab. II, 14, p. 102, 3 Boll-Boer (= II, 13, p. 215, Loeb). Consult also the indices verborum in Ptolemy. Opera III, 2, p. 107 and Vettius Valens, p. 383, p. 411.
[1] Cf. Heiberg, p. 10, 8: *ἐπὶ πολὺν χρόνον*.
[2] Cf. above I A 4, 5 and p. 41.
[3] Cf. above p. 42.

One now considers the Menelaos configuration with vertex Z and the arcs ZS, Z⊙, SG, ⊙M for which one finds with Theorem II[4]:

$$\frac{(h)}{(90^\circ)}=\frac{(\sigma)}{(\mathrm{HM})}\cdot\frac{(a)}{(90^\circ)} \tag{1}$$

where the altitude a of M is known from

$$a=90-(\varphi_0+\delta(\mathrm{M})) \tag{2}$$

δ(M) being the declination of M. Hence h can be found from (1).

Ptolemy makes the simplifying assumption that h as determined for φ_0 is practically independent of the geographical latitude, or rather he hopes that this is so. For more northerly locations the decreasing values of v would require increasing h but this might be compensated for by the diminishing brightness of the sun in northern climates such that the previous h might again suffice. At any rate he has no other choice because of the scarcity of observational data for other climates.

Hence he considers h from now on to be known, independent of φ, for each star and each phase in question. Then one can determine σ from (1) since, as before, H can be found to given Σ and therefore M and thus HM. Similarly a follows from (2), replacing φ_0 by the new φ. Hence σ can be computed and thus $\lambda_\odot$. In this way the date of the phase is predictable for any climate.

B. The "Phaseis" Book II

The second, and only preserved, book of Ptolemy's "Phases of the fixed stars" begins with a long introductory section.[1] This is followed[2] by a "calendar," i.e. a list of dates at which the phases take place for 15 stars of the first magnitude (this is also the total in the Almagest) and for 15 stars of second magnitude. With the phases go prognostications for the weather in regions within the five climata between 13 1/2^h and 15 1/2^h of longest daylight.

As an example I give the following section from the beginning of the "calendar"[3]:

Thoth 1. 14 1/2 hours: The (star) in the Tail of Leo (β Leo) rises (Γ).
According to Hipparchus: the Etesian winds end; according to Eudoxus; rain, thunder, the Etesian winds end.

2. 14 hours: The (star) in the Tail of Leo (β Leo) rises (Γ) and Spica sets (Ω).
According to Hipparchus: indicative.

3. 13 1/2 hours: The (star) in the Tail of Leo (β Leo) rises (Γ).
14 hours: The (star) called Goat (α Aur) rises in the evening (Θ_1).
According to the Egyptians: The Etesian winds end; etc.

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[4] Cf. above p. 28.
[1] Heiberg, p. 3 to 13.
[2] Heiberg, p. 14 to 65.
[3] Heiberg, p. 14.

In other words we are told that

β Leo, Γ:	on Thoth 1 in clima $M = 14\,1/2^h$, on Thoth 2 in clima 14^h, on Thoth 3 in clima $13\,1/2^h$.
α Vir, Ω:	on Thoth 2 in clima 14^h.
α Aur, Θ_1:	on Thoth 3 in clima 14^h.

In this way the calendar runs through the whole (Alexandrian) year. In the prognostications the frequent term "indicative" (e.g. with Thoth 2) requires explanation. The Greek term ἐπισημαίνει would normally refer to an object which is "indicated" by the phase.[4] As a meteorological technical term, however, it means that the weather might change at the phase.[5] One could almost translate "the day is ominous."

In the calendar are also mentioned the Alexandrian dates of the equinoxes and solstices. Beginning with the autumnal equinox these dates are

I, 28   IV, 26   VII, 26   XI, 1.

For the length of the seasons, beginning with the vernal equinox at VII, 26 (= March 22), one has here the following intervals:

95   92   88   90

days. These are obviously round numbers, taken from the Hipparchian-Ptolemaic solar theory.[6]

A short epilogue concludes Ptolemy's treatise. There he repeats the list of the 30 stars and gives the names and regions for the observers. It is here that Bithynia, not Rhodes, is associated with Hipparchus.[7] Except for Caesar, Hipparchus is the most recent authority mentioned. Democritus, Meton, Euctemon belong to the 5th cent. B.C., Eudoxus, Philip, and Callippus to the 4th, Conon, Dositheos, and perhaps Metrodoros to the third. "The Egyptians" remain anonymous but there seem to be good reasons to place them before Hipparchus.[8]

These "Egyptians" are the only ones who observed in clima III (14^h, i.e. Lower Egypt). All the other authorities operated in the climata IV and V (14 1/2 to 15^h) in Greece, Asia Minor, or Macedonia. Only Eudoxus, Conon, and Metrodoros observed in Sicily and Italy, Eudoxus also in "Asia," Caesar in Italy.

The names of the five climata are in part at variance with the terms familiar from the Almagest and the "Geography":

$13\,1/2^h$	Syene-Berenike (usually only Syene)
14	*Αἰλίου*(?) *Αἰγύπτου*,[9] slightly south of Alexandria-Cyrene (usually Lower Egypt)
14 1/2	Rhodes
15	Mid-Hellespont
15 1/2	Aquileia(?)-Vienne (usually Mid-Pontus).

[4] The Latin term is *significat*; cf. also below p. 999, n. 27.
[5] Cf. Rehm RE Ep., col. 180–183.
[6] Alm. III, 4 (Manitius I, p. 167 and p. 170, respectively); cf. also above p. 58.
[7] Cf. above p. 275.
[8] Rehm, RE Par., col. 1351; cf. also above p. 562.
[9] Cf. the apparatus in Heiberg, p. 4, 9. The emendation to Heliopolis(!) seems most implausible.

At the last entry at least the text is corrupt[10] and hardly suited to provide the basis for conclusions about Ptolemy's geographical terminology.[11]

Ptolemy considers four phases, i.e. first and last visibility at sunrise and sunset, respectively,[12] but he distinguishes furthermore "true" and "apparent" phases. The true phases are not observable since star and sun are supposed to be simultaneously in the horizon. Only the apparent phases correspond to actual first and last visibility. Obviously the true phases can be determined exactly by spherical trigonometry alone (cf. Alm. VIII, 5). Ptolemy, by introducing for each phase and each star a definite empirical arcus visionis also brings the apparent phases into the reach of mathematical discussion although the actual facts may show considerable fluctuations around the ideal pattern.

For the stars near the ecliptic the sequence of the phases is Ω, Γ, Θ_1, Θ_2 but Ptolemy is well aware that permutations are possible for stars with greater northern or southern latitude.[13] A similar situation occurs for Venus near inferior conjunction (Ω, Γ) and finds special attention in the Handy Tables.[14]

In the preserved part of the "Phaseis" no values are given for the arcus visionis of the single stars and phases. Not only the method of determining h had been discussed in Book I but perhaps the specific observations and resulting values for h had found their place in Book I as well. The only way of recovering this information consists in analysing the final results as found in Book II and by extracting the underlying parameters. This has been done in an excellent study by H. Vogt, published as No. V of Boll's series "Griechische Kalender" (1920). Ptolemy's text covers a total of 480 phases.[15] For 30 stars this represents a comparatively large body of material on which Vogt could base his analysis. From the given data he determined values of h for all phases and the 5 climata. Combining them to mean values he found,[16] e.g.,

	Γ	Ω	Θ_1	Θ_2
β Leo	11.1°	12.4	6.6	6.0
α Vir	12.2	11.8	5.6	5.9
α Aur	11.3	11.0	7.6	6.6

for h. In this way it is now securely established that Ptolemy associated with each of his 30 stars four values of the arcus visionis which he then utilised for the determination of the dates in the four remaining climata, while Lower Egypt provided the initial empirical elements. The comparatively large fluctuations of the restored values of h suggest, however, that many determinations were based

[10] Cf. the apparatus in Heiberg, p. 4, 18. In Ptolemy, Geogr. III 1, 29 VIII 8, 6 and II 10, 11 VIII 5. 7, respectively both places are given $\varphi = 45°$ or $M = 15\ 1/2^h$.

[11] Honigmann, SK, p. 59, draws attention to similar combinations used by Hipparchus as reported by Strabo (Geogr. II 5, 36 and 38), i.e. mention of Berenike and of Cyrene, respectively.

[12] In the notation of VI B 5, 2 the phenomena Γ, Ω, Θ_1, and Θ_2. Cf. also Alm. VIII, 4.

[13] Heiberg, p. 8–10 (No. 6). Cf. also Fig. 56 (p. 1368).

[14] Cf. above p. 241.

[15] Opera II. p. 66. 23.

[16] Vogt [1920]. p. 54–57. For the coordinates of the stars Vogt used, of course, the catalogue of stars in the Almagest. That this catalogue is normed for the year Antoninus 1 (A.D. 137/8) tells us nothing about the date of writing of the "Phaseis" in relation to the Almagest.

on readings on a globe, but not on accurate trigonometric computations which would have been extremely laborious. Globe-readings were a well established practice in Greek astronomy, e.g. with Hipparchus, as has long been assumed.[17]

When Ptolemy in the "Phaseis" was striving to obtain reliable data for the visibility phenomena of the brightest stars one wonders how he could assign the same arcus visionis $h=15°$ in the "Planetary Hypotheses" to all stars of first magnitude near the ecliptic.[18] It follows from Vogt's investigations that none of Ptolemy's h for a star of first magnitude was greater than 12;30°.[19] The mean value of Γ and Ω is about 11°, for Θ_1 and Θ_2 about 7°.[20] The statement in the "Planetary Hypotheses" not only contradicts the whole methodology of the "Phaseis" but also of the treatment of the planetary phases in the Almagest.[21] It seems only a weak excuse to postulate that a star of the first magnitude is of the same brightness as Mars which has an arcus visionis of 14;30°.[22] I do not have enough confidence in the rules of literary criticism to conclude from such contradictions that the "Phaseis" were written after the "Planetary Hypotheses" (and thus a fortiori after the Almagest).

2. Astronomy in the "Harmonics"

The works of an author which determine his importance for us are those which have added to our own understanding and thereby to the development of his field. The Almagest, the Geography and the Tetrabiblos are undoubtedly the writings of Ptolemy which have exercised the greatest influence, each in its specific way, to the history of civilization. We do not know how Ptolemy himself would have judged his own writings, but there can be little doubt that the three books of the "Harmonics" record thoughts for us which he must have felt to be central to his whole life's work. Here he revealed what he must have considered to be the deepest insights into the structure of the cosmos, based on a systematic analysis of the harmonies of audible music and its replica in the harmonies of the planetary system.

It is a strange analogy which links Kepler, the founder of the Astronomia Nova, to the greatest astronomer of antiquity. Kepler also considered his "*Harmonice Mundi*" (1619) as his most significant discovery through which he had penetrated into the most august secrets of the creation.[1] It was a conscious revision of Ptolemy's "Harmonics" which Kepler undertook in his work, resting on heliocentric distances which are fitted into the pattern of spheres, circumscribed and inscribed the five regular polyhedra, an idea originally conceived in the

[17] Cf. Vogt [1920], p. 47f. Cf. also Alm. VIII, 3 for the construction of a globe (above p. 890).
[18] Plan. Hyp. I, 6, Goldstein [1967], p. 9.
[19] E.g. $h \approx 11;50°$ for Γ of Regulus.
[20] Vogt [1920], p. 16.
[21] Cf. above I C 8, 2 and below V C 4, 5 C.
[22] Cf. above p. 261.
[1] Kepler, Werke VI (1940) and German translation (1939) by M. Caspar, both with very valuable commentaries by Caspar.

"*Mysterium Cosmographicum*" (1596)[2] but now refined by the newly obtained scale of distances, known as Kepler's third law.[3]

Kepler had read Ptolemy's "Harmonics" in a manuscript.[4] Originally he had planned to publish with his Harmonice Mundi a Latin translation of the astronomical sections in his predecessor's work but the outbreak of war made the project impracticable. Consequently he limited himself to a critical summary, printed as an appendix to Book V of the Harmonice Mundi. Ptolemy's Harmonics consist of three books, the last of which concerns the astronomical harmonies; but the end (Chap. 14–16) is lost or only fragmentarily preserved. Characteristically Kepler restored the lost sections, in order to refute them. Doubtlessly he would have done the same with the original version since the shift to heliocentric distances made all Ptolemaic models obsolete. On the other hand Kepler was now compelled to explain the influences of the planetary aspects although the earth was no longer near the center of the planetary orbits. He ascribed a sensitive soul to the earth which feels the influences of the rays emanating from the planets and measures their directional angles. For the earth it is much the same as it is with the influence of the planetary rays on the human soul at a nativity, a theory which he passionately defended against the arguments of such antagonists of astrology as Pico della Mirandola.[5] Some uncomfortable feeling seemed to have remained, however, since the heliocentric trend of the theory obliged Kepler not to leave the sun without some sort of a soul and he therefore condemned (tentatively) the *Νοῦς* to be associated with the central body of the whole system. He was quite ready to deduce from the rotation of the sun similar data for all planets and their volumes. Catholic dogma as well as doctrines of Plato, Aristotle, Proclus and others are adduced to convince the reader.

One has to know about such outbursts of mediaeval madness if one wants to appreciate Kepler's work against the background of his own time. Such brilliant mathematical investigations (in Book I of the Harmonice Mundi) as his discussion of regular and semiregular polyhedra and of polygons,[6] his penetrating study of Euclid's Book X – all this obtained for him its significance only as foundation for his discoveries of cosmic symmetry and harmony.

While Kepler's unbridled imagination fascinates the reader by the scale and perverse methodology of its constructions one is tempted to describe Ptolemy's pedestrian description of planetary harmonies as simple nonsense, nonsense of the same dreary character as is displayed in the Tetrabiblos and in the whole astrological literature. Such a description seems quite adequate for the "astronomical" part of Book III but does not apply at all to the bulk of the Harmonics. Ptolemy repeatedly refers to experimental arrangements for the empirical testing

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[2] Kepler, Werke I.

[3] First announced in the Harmonice Mundi V, 3 (Werke VI, p. 302, 21, transl. p. 291), its origin explained in "Epitome Astronomiae Copernicanae" IV, 2 (1620) Werke VII, p. 291, 11 ff and p. 306, 33 ff.

[4] Also the commentary by Porphyry, which reaches, however, only to II, 7 of Ptolemy's text.

[5] Kepler, Werke VI, p. 257, 9 et passim. It is, of course, ridiculous when modern historians want to absolve Kepler from adherence to astrology. It was only his very peculiar personal doctrine which made him argue against the conventional brand of astrology. Its basic principles he fully accepted.

[6] He came, e.g., close to the principle of duality in the discussion of plane configurations.

of harmonies[7] in healthy opposition to purely numerical speculations, a fact which makes the Harmonics a work of outstanding importance within the Greek literature on the theory of music.[8] This part is marred by the sterile tendency to formal classifications. One simply must accept it as a fact that neither ancient philosophers nor Ptolemy found it absurd to coordinate harmonies and virtues, and to neatly classify them into seven categories.[9]

The only natural transition from harmonics to astronomy is suggested by the harmonic divisions of a chord corresponding to the divisions of the circumference of the ecliptic by the astrological aspects. The octave, e.g., is represented by the opposition (division 2:1) and in the same fashion fifth, fourth, etc., correspond to ratios also found in the aspects trine, quadrature, etc.[10] From now on, however, the parallelism becomes purely speculative. The chapters III 10 to 12 postulate harmonic counterparts to the variations of the three main parameters of planetary motion, *μῆκος, βάθος, πλάτος*. The first, the motion in longitude, is assimilated to the pitch, bounded by silence corresponding to the interval of invisibility between heliacal setting and rising. In the same time highest pitch and culmination are paralleled in spite of the fact that culmination has nothing to do with the synodic period of the preceding association.

The variation in "depth," *βάθος* (i.e. anomaly), is associated with the "genera" of musical theory. The third variable, *πλάτος*, is not the "latitude" as one would normally understand this term. Obviously the planetary latitudes behave much too irregularly to be representative for harmonies. Hence Ptolemy denotes now as *πλάτος* deviations from the equatorial plane, i.e. declinations. By using the declinations which correspond to the longitudes $\lambda = k \cdot 30^\circ$ ($k = 1, 2, \ldots, 12$) he obtains 7 parallels (cf. Fig. 96) which then supposedly correspond to the 7 transpositions of the scales.

Having disposed (in a purely qualitative fashion) of the planetary motions in λ, α, and δ the tetrachords of the "*σύστημα τέλειον*" are set into relation to the planetary and lunar phases. As this requires again a simple numerical pattern Ptolemy disregards all his refined theory of planetary visibility[11] and operates with the archaic concept of a fixed arc of invisibility of 30° in the midpoint of which the sun is located.[12]

For the next two chapters we have only the titles: III, 14 concerns the numerical data for the planetary orbits (*ταῖς πρώταις σφαίραις*), III, 15 the ratios of the mean (?) motions (*οἱ τῶν οἰκείων κινήσεων*); finally III, 16 dealt with the astrological qualities of the planets and was hardly of astronomical interest. The chapters 15 and 16, however, seem to have contained numerical data for the distances and velocities of the planets and one would like to know whether or not Ptolemy

[7] This is very much reminiscent of Ptolemy's attitude in the case of geometric and physiological optics; cf. above p. 893 f.

[8] Cf., e.g., Düring, Harm. I, p. LXX and III, p. 140. I have to admit my own total incompetence in all matters of musical theory. For the literature cf. also van der Waerden in R.E. 23, 2 col. 1840–1847; for the position of the Harmonics within Greek philosophy cf. Boll [1894], p. 93–111. For the terminology of Greek music in general consult, e.g., The Oxford Classical Dictionary s.v. Music.

[9] Harm. III, 9 (Düring I. p. 97, 16–20; III. p. 120).

[10] Harm. III, 10.

[11] Cf. above I C 8, 2.

[12] Cf. above IV D 3, 4.

had made use in this context of the nested spheres from Book I of his "Planetary Hypotheses."[13] On the other hand, the "Canobic Inscription" contains a list of musical scales and numbers in arithmetic progression for nine "spheres" (fixed stars, seven planets, and the four elements); the numbers run from 36 down to 4 (difference 4) and have no astronomical meaning.[14] It is futile to search for the origin of such "dreams" as Kepler rightly calls these speculations.

3. The "Geography"

Conventionally one calls "Geography"[1] a work (in eight books) entitled γεωγραφικὴ ὑφήγησις, i.e. "Geographical Directory," a title which implies that a description of individual countries is not intended, but instead a list of localities with their coordinates, encompassing the whole known (or "inhabited") part of the globe.[2] The construction of a grid for a world map is explained in Book I[3]; for smaller regional maps a simple rectangular grid is considered sufficiently accurate.[4] Given the list of coordinates every reader can make his own maps as he sees fit.

Few books have exercised such a profound influence on human thought and civilization as Ptolemy's Geography. In its theoretical as well as practical consequences it far exceeded the importance of the astronomical "Ptolemaic System" and its Copernican modification, of interest and accessible to only a handful of men.

It is in the Geography that for the first time a mathematically clear theory of geographical mapping was presented and with it went the creation of a consistent grid of coordinates, reckoned in degrees: longitudes from the westernmost point of the known world, the "Fortunate Islands," and latitudes, replacing the traditional time coordinates, local time (Δt) to the west and east of Alexandria, combined with the hours of longest daylight (M). Astronomy after the "Almagest," written at the beginning of Ptolemy's career, and geography after the "Geography," probably one of his latest works[5], took an entirely new character and was to influence scientific thought from then until modern times.

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[13] Cf. above V B 7, 5.

[14] Ptolemy, Opera II, p. 154, 1–10; also Halma, Hypoth., p. 61 f. Obviously the original sequence was

fixed stars	♄	♃	♂	☉	♀	☿	☾	4 elements
36	32	28	24	20	16	12.	8	4.

The extant manuscripts give instead the following numbers

36	32	24	21 1/3	18	16	12	9	8.

The fraction 1/3 is a misinterpretation of the 20 as a sexagesimal fraction. Düring, Harm. III, p. 280f., doubts the authorship of Ptolemy for this section because its data do not agree with Ptolemy's system of harmonies. I have no judgement in this matter.

[1] To mediaevalists also known as the "Cosmography" (e.g. Thorndike, Magic III, p. 589).

[2] Ptolemy in his introduction (Book I, Chap. 1) stresses the mathematical character of his approach; for the terminology cf. Mžik, p. 13–16, notes.

[3] For the theory of map projections cf. above V B 4.

[4] Cf. below p. 935f.

[5] Cf. above p. 835.

The editio princeps of the "Geography" is due to Erasmus, printed by Froben in Basle in 1533. Erasmus, himself one of the greatest scholars of the Renaissance, contributed much to the understanding of the text. It took more than three centuries before the first "modern" edition by C.F.A. Nobbe was printed (in Leipzig, 1843, 1845 [6]). An English translation by E.L. Stevenson, incompetent in technical matters, was published in New York in 1932. Four centuries during which classical scholars kept talking about the cultural heritage from antiquity have not yet produced a reliable edition of the Greek text. Instead an enormous literature grew up, most of which revealing a remarkable ignorance of elementary astronomy, ancient as well as modern. [7]

A. Spherical Astronomy

As boundaries of the "inhabited world" Ptolemy accepted in the north the Island of Thule at $\varphi = 63°$, in the south the parallel south of the equator at which $M = 13^h$, i.e. the parallel $\varphi = -16;25°$. [1] The longitudinal extension is assumed to be 180°, from the Fortunate Islands to China. Ptolemy reckons 500 stades to 1° on the equator and thus gives the following distances s for 180° of longitude [2]:

at $\varphi = -16;25°$	$s = 86\,330\ 1/3$ stades
equator	90000
Syene ($\varphi = \varepsilon = 23;50$)	82336
Rhodes ($\varphi = 36$)	72000
Thule ($\varphi = 63$)	40000.

Obviously one should have $\cos\varphi = s/90000$ and indeed $8.63303/9 = .959226 = \cos 16;25°$ and similarly $8.2336/9 \approx \cos 23;49°$. The two last values, however, are only crude roundings. [3] The latitudinal width is taken to be $79;25° \approx 80° = 40000$ stades.

Fortunately it is not necessary here to continue the hotly debated question of why Marinus and, following him, Ptolemy changed Eratosthenes' norm of 700 stades for the equatorial degree to 500 stades, and what actual distance was meant by one stade. [3a] The latter problem is of particularly little interest since it is clear that any agreement or disagreement with modern data for the size of the earth must be purely accidental.

Book VIII deals with regional maps, 26 in number, which cover areas small enough to allow for a simple grid of orthogonal straight lines for meridians and parallels. The scale in these two directions is chosen in such a fashion that it

[6] Reprinted 1898 and 1966 (with bibliographical additions by A. Diller).

[7] The article on Ptolemy as geographer by E. Polaschek in Pauly-Wissowa, Suppl. 10 (1965) col. 680 to 833 is an example. A bibliography of almost 1500 titles concerning Ptolemy's Geography was compiled by W.H. Stahl (New York 1953), and is also of very little use because of its technical incompetence. A brilliant exception is the book by H. v. Mžik, Des Klaudios Ptolemaios Einführung in die darstellende Erdkunde (1938).

[1] This is a value from the list of latitudes which are rounded to the nearest 0;5° given in Geogr. I 23, 5 (Nobbe, p. 46, Mžik, p. 66). Alm. II, 6 gives $\varphi = 16;27°$ (Heiberg, p. 106, 7); cf. above p. 43 and Table 2 there.

[2] Geogr. VII 5, 15 (Nobbe, p. 180).

[3] Corresponding to $\varphi = 36;52°$ and $63;37°$, respectively.

[3a] Cf. above p. 653.

corresponds approximately to the ratio of the length of one degree of longitude on the middle parallel of the map to one degree on the meridian. For example Map 3 of Europe ("Gallia") is bounded by the parallels of 42° and 54° (Geogr. VIII, 5). Hence the above mentioned ratio should be about $\cos 48° = 0;40,9$. Ptolemy prescribes the ratio 2:3; similarly, though not always that good, in all other cases.[4]

Table 27

Months		Sun				Sun	Months	
Alex.	Julian	in	$\Delta\delta$	$[\delta]$	$\Delta\delta$	in	Julian	Alex.
X	June	♋		+23;50°	+3;20			
XI	July	♌	−3;20°	+20;30	+8;50	♊	May	IX
XII	Aug.	♍	−8;50	+11;40	+11;40	♉	Apr.	VIII
I	Sept.	♎	−11;40	0	+11;40	♈	March	VII
II	Oct.	♏	−11;40	−11;40	+8;50	♓	Febr.	VI
III	Nov.	♐	−8;50	−20;30	+3;20	♒	Jan.	V
			−3;20	−23;50		♑	Dec.	IV

Relations of spherical astronomy are, of course, repeatedly at the basis of data found in the Geography. A small schematic table of the monthly differences of the solar declinations ($\Delta\delta$) is found in some manuscripts of Book VIII.[5] Emending some trivial scribal errors the text can be described by a pattern as shown in our Table 27.[6] The declinations themselves (not given in the extant text, thus denoted by $[\delta]$ in Table 27) are roundings to 10 minutes, obtainable from any table of solar declinations.[7] The associations Thoth (I)-September-Libra, etc., represent the conventional schematization of the relations between the alexandrian and julian calendar and solar longitudes.[8]

The reason for including such a table in the Geography lies in its connection with the problem of zenith positions of the sun for localities between the equator and latitudes $|\varphi| \leq \varepsilon$. For such places the sun reaches the zenith twice every year when its declination δ satisfies the condition $\delta(\lambda_\odot) = \varphi$, i.e. at a distance $\Delta\lambda$ from the summer solstice with $\lambda_\odot = 90 \pm \Delta\lambda$. In the right spherical triangle with the arc from the vernal point to the zenith as hypotenuse we have therefore

$$\sin \varepsilon = \sin \varphi / \sin \lambda_\odot .$$

[4] A list of the boundaries of the 26 regional maps is given (with variants) in Schnabel, T. u. K., p. 96–99. The ratios assumed by Ptolemy must be taken from Nobbe's edition (section 1 of each chapter). The resulting values of φ agree reasonably well with the middle parallels of the single maps. Obviously one is dealing here only with conveniently rounded scales.

[5] So in one of the most important manuscripts of the "Geography," the Cod. Urb. gr. 82, shown in facsimile in Fischer's edition (II, 1) on F. 110 (109)r, end of column I. Nobbe ignored this table which would fall in his edition between p. 260 line 5 and the "scholion" (the latter occupies in the Cod. Urb. the top of column II). Transcription in Kubitschek [1935], p. 110; other versions given in Schnabel, T. u. K., p. 64f. (with doubtful readings).

[6] The correct interpretation of this table was given by Ferrari d'Ochieppo (R.E. Suppl. 10 col. 749, 14ff.) though he did not realize its simple schematic character. The comments by Polaschek are totally misleading.

[7] E.g. Alm. I, 15 or Handy Tables (Halma I, p. 144/5).

[8] In the same fashion, e.g., in the lunarium Vat. gr. 1291 fol. 9v.

Since one knows the values of φ which correspond to a longest daylight M (e.g. from Alm. II, 6 or from the Handy Tables[9]) one can easily compute $\Delta\lambda$ to given M. Table 28 shows the values derived in this fashion from Alm. II, 6. In Book VIII of the Geography Ptolemy mentions[10] $\Delta\lambda$ for all localities[11] for which M lies between 12^h and $13;30^h$ (which corresponds to $\varphi = \varepsilon$). This would give a good opportunity for checking the underlying table for φ as function of M, were it not for the abominable condition in which such numerical data are given in Nobbe's edition which reproduces even glaring scribal errors without any critical apparatus.

Table 28

Alm. II, 6		computed		Alm. II, 6
M	φ	$\lambda_\odot$	$\Delta\lambda$	
$12;15^h$	4;15	10;34	79;26	79;30
12;30	8;25	21;14	68;46	69; 0
12;45	12;30	32;23	57;37	57;40
13	16;27	44;30	45;30	45; 0
13;15	20;14	58;52	31; 8	31; 0

The zenith positions of the sun have also found pictorial representation on some of the world maps, e.g. in the famous Cod. Urb. 82.[12] There one sees depicted on the margin beside the latitudes $|\varphi| \leq \varepsilon$ the rays of the sun, shining from the proper zodiacal sign in the proper month as listed in the above mentioned little table.[13] The zenith positions of the sun and the corresponding changes of the shadow of the gnomon must have been an important argument in the demonstration of the sphericity of the earth.

B. Historical Remarks

The novelty of Ptolemy's approach in geography, expressed in his mathematical theory of mapping and in the consistent use of spherical coordinates probably suggested to him that he make the transition from the traditional attitude less abrupt by appending at the end (Book VIII) the list of the most "important cities" with their conventional time coordinates, longest daylight M and Δt with respect to the meridian of Alexandria. In other words Ptolemy did the same thing in the Geography as he did in the Almagest which he also concluded with an antiquated but traditional topic, the planetary phases.[1]

How the longest daylight M ever could have become a major geographical coordinate is difficult to understand. In Babylonian astronomy, long before the discovery of the sphericity of the earth, the norm $M:m = 3:2$ for the ratio of

[9] Cf. above p. 44 and below p. 980f.

[10] Geogr. VIII 16, 22, 26 to 28.

[11] Called localities ὧν ὁ ζῳδιακὸς ὑπέρκειται (Geogr. VIII, 2, Nobbe, p. 195, 18).

[12] Cod. Urb. gr. 82, Fischer II. 2 Pl. I; also Pl. XXVIII.

[13] This has first been remarked by Kubitschek [1935], p. 110.

[1] Cf. above I C 8, 2.

longest to shortest daylight is not a geographical element but is needed for the time reckoning with evening epoch,[2] being not more than a convenient crude rounding.[3] There is no good reason, however, to use such a parameter for variable localities. Since only very few places on earth have a horizon even remotely corresponding to ideal conditions, M is hardly ever observable directly, not to mention that it is an extremely ill defined quantity.[4] In contrast φ can be found in many ways, throughout the year, in daytime or at night, by observations in the meridian, thus with a minimum of (unknown) optical disturbances. In view of all this it seems incredible that M should ever have been a primary parameter and one is almost forced to the conclusion that φ was actually known first, either from direct observations or estimated from itineraries, while M was determined from it. A simple analemma construction allows one to find M from φ.[5] Unfortunately all this remains conjectural since we have no textual evidence which motivates the preference for M over φ. Only a very small number of values of φ is based on actual observations. This is a priori to be expected and can be explicitly confirmed by comparison with the known modern values.

The situation for the longitudes is still worse. In principle, it should have been easy enough to establish reliable data since lunar eclipses are very frequent and their occurrence easily predictable. Nevertheless Ptolemy had only one such time signal at this disposal, the lunar eclipse of -330 Sept. 20 preceding the battle of Arbela, recorded in Carthage as well, supposedly with a difference of 3 hours in local time[6]; actually the difference should be only $2;15^h$. The corresponding longitudinal distortion affected all of mediaeval geography which depended on Ptolemy. In spite of complete theoretical insight into the astronomical requirements for the determination of accurate geographical coordinates the absence of any scientific organization in antiquity reduced geography to the unsatisfactory game of constructing compromise solutions from accidental records of travelers and merchants, from the writings of historians, or, perhaps, some Roman itineraries and army records. Not before the time of Bīrūnī (i.e. around A.D. 1000) was there a more systematic approach possible.[7]

The comparatively great antiquity of describing geographical latitudes by means of hours of longest daylight is shown by the concept of "climata" which goes back to Eratosthenes, i.e. the third century B.C.[8] The definition of longitudinal distances by means of hours of local time is certainly of equal antiquity and in fact quite natural and useful. We know that some of the Handy Tables[9] contain lists of localities arranged by Δt with respect to Alexandria and in seven climata. The comparative antiquity of this table is secured by its occurrence in a papyrus

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[2] Cf. above II 1, 4.
[3] Ptolemy replaced the Babylonian value $M = 14;24^h$ by $14;25^h$ because he used 1/12 as smallest unit fraction (Geogr. VIII, 27, Nobbe, p. 235).
[4] Modern literature discusses the influence of refraction although it is of no possible interest for extremely crude clocks and obstructed horizons.
[5] Cf. Fig. 21 (p. 1378).
[6] Ptolemy, Geogr. I, 4 (Nobbe, p. 11, 19f.; Mžik, p. 21). For this eclipse cf. Ginzel, Kanon, p. 184f. (No. 18) and P.V. Neugebauer, Kanon d. Mondf., p. 42.
[7] Cf. Bīrūnī, Taḥdīd.
[8] Cf. Honigmann, S.K., p. 10ff. and above p. 334, n. 8.
[9] Cf. Halma H.T. III, p. 34/35 (with the addition of an unrelated table of solar parallaxes).

Table 29

No.	Meridian of	Δt H.T.	Geogr. VIII	Δt		Geogr.	long.	[Δt]	Δt modern
1	Tangier	$+3;36^h$	13,3:	$+3\ 1/2\ 1/12^h$	$(=3;35^h)$	IV 1,5:	6;30°	$+3;36^h$	$+2;23^h$
2	Narbonne	2;36	5,7:	2 1/2 1/12	(=2;35)	II 10,9:	21;30	2;36	1;48
3	Rome	1;36	8,3:	1 1/2 1/8	(=1;37,30)	III 1,61:	36;40	1;35,20	1;10
4	διὰ ταναίδος	0;48							
5	Alexandria	0		0		IV 5,9:	60;30	0	0
6	Babylon	−1;12	20,27:	−1 1/4	(= −1;15)	V 20,6:	79; 0	−1;14	−0;58

of the second century A.D.[10] Table 29 shows the six meridians thus selected.[11] The modern values illustrate again how much the whole map was stretched in a longitudinal direction.

The "Geography" is not Ptolemy's only geographical work. It is preceded by several excurses into geographical topics, beginning with Alm. II, 6 and II, 8 (terrestrial zones, length of daylight, shadow lengths, etc.)[12]; geographical divisions appear in the Tetrabiblos,[13] possibly followed by an independent shorter treatise ἔκθεσις τῶν πινάκων τῆς οἰκουμένης ("Layout for the maps of the inhabited earth") eventually incorporated as Book VIII in the Geography.[14] Finally the long list of "important cities" with their longitudes and latitudes became part of the Handy Tables.[15] The "Geography" itself is then the crowning work of Ptolemy's writings.[16]

The geographical data accepted by Ptolemy during the different stages of this development vary greatly. In the Almagest he still follows Hipparchus in such basic assumptions as placing Rhodes and Alexandria on the same meridian.[17] In the Geography Rhodes is located $1/8^h$ ($\approx 2°$) west of Alexandria (in fact 1;55°). Other changes of geographical data are probably due to Marinus of Tyre whose work incorporated data until about A.D. 114.[18] To him is also due the shift from the older norm of 700 stades[19] for the equatorial degree to 500 stades, a norm accepted by Ptolemy first in the Handy Tables.[20] The extension of the southern

[10] P. Lond. 1278, Neugebauer [1958, 1], p. 95 f.

[11] From Halma, H.T. III, p. 34/35 and Vat. gr. 208 fol. 125r and Marc. gr. 314 fol. 218r (both unpublished). P. Lond. 1278 contains fragments for the meridians $+1;36^h$ and $+0;48^h$ and climata II, III, IV, with II above III, III above IV (i.e. south on top). For No. 4 Halma gives δια Ταναιδος δια Τουλιδα (in the translation only Tanais!); Vat. gr. 208 has ἐπὶ τοῦ διὰ ταναίδος, Marc. gr. 314 ἡλίδες ὦ. I do not know which part of the Tanais was meant. The longitudes in the fourth column are also found in Vat. gr. 1291 (cf. Honigmann S.K., p. 194 ff., excepting Rome with 36;20°).

[12] Cf. above p. 43.

[13] Cf. Schnabel [1930], p. 220 f.

[14] This is Schnabel's hypothesis ([1930], p. 222 and T. u. K., p. 33). The ἔκθεσις is available only in Halma H.T. I, p. 109–131, a late and uncritically edited version.

[15] Cf. below p. 1025.

[16] This is the now generally accepted sequence. Fortunately little depends on its correctness in details.

[17] Cf. Almagest V, 3 (Manitius I, p. 266, 25).

[18] Honigmann in R.E. 14, 2 col. 1768, 66. Cf. also Mžik, p. 25, note.

[19] So with Heron (cf. above p. 848). Hipparchus and Eratosthenes (Schnabel [1930], p. 226, p. 228 f.). Cf. also above p. 935.

[20] Schnabel [1930], p. 219.

limit of the world maps to a latitude of 16;25° south of the equator is only a step in the latest phase of Ptolemy's work.[21]

Fortunately we do not need to discuss the intricate problems of the manuscript tradition of the Geography. Schnabel enumerates 52 manuscripts,[22] unfortunately none older than about A.D. 1200, i.e. copied more than a millenium after the original composition. We may only mention that one distinguishes, following Fischer, two major classes: one called "version A" with 26 maps as suggested by the headings in the Books II to VII and in Book VIII (and the ἔκϑεσις); the other "version B" which gives 64 smaller maps.[23] The rather fruitless controversy about the origin of these maps does not concern us here.

Bibliography

For the text of the "Geography":
Book I: German translation: Mžik
Book II 1, 1 to 9: German translation: Mžik
Book II 7 to III 1: Greek text: Cuntz
Book IV and V: Greek text and Latin translation: Müller
Book VII 1 to 4, 13: Greek text and French translation: Renou
Book VII, 6 and 7: English translation: Neugebauer [1959, 1]
For all the rest: Greek text: Nobbe; English translation: Stevenson
Facsimile of the Greek text of Cod. Urb. gr. 82 and maps: Fischer
Main articles in Pauly-Wissowa:
Erdmessung (Kubitschek): Supp. 6, col. 31–54
Geographie (Gisinger): Suppl. 4, col. 521–685
Karten (Kubitschek): 10, col. 2022–2149
Klima (Kubitschek): 11, col. 838–843
Marinos (Honigmann): 14, col. 1767–1796[24]
Ptolemaios (Polaschek): Suppl. 10, col. 680–833

4. Philosophy; Fragments

Boll, in his excellent "Studien über Claudius Ptolemäus" (1894), investigated the philosophical background of Ptolemy's work, revealed through occasional remarks in the Almagest, amplified in the "Harmonics," and made the subject of a special monograph "On judgment and guiding reason" (*Περὶ κριτηρίου καὶ ἡγεμονικοῦ*). The latter work has been edited by Lammert in Ptolemy's Opera III, 2 (p. I–XVI and p. 3–36; 1952) and discussed in RE 23, 2 col. 1854–1858 (1959). It is not surprising that Ptolemy followed peripatetic or stoic doctrines but with a certain eclectic attitude. Fortunately these philosophical theories are without importance for his actual astronomical work.

Fragments from other writings are only preserved through quotations in much later authors.[1] A purely mathematical fragment contains an attempt at proving

[21] Schnabel [1930], p. 225. Cf. above p. 935, note 1.
[22] Schnabel, T. u. K., p. 121 and p. 128.
[23] For the interrelations between these versions and the extant text see Bagrow [1946].
[24] The article in Suppl. XII (1970) col. 791–838 by Photinos is valueless.
[1] Collected by Heiberg in Ptol. Opera II, p. 263–270; cf. also R.E. 23, 2 col. 1839/40.

the parallel postulate.[2] A book "On dimensionality" (*Περὶ διαστάσεως*) intended to show that no more than three dimensions are possible. Other fragments on "Weights"[3] and on "Elements" are perhaps taken from a work in three books on "Mechanics."[4] In the latter circular motion is considered the alternative to being at rest "in the proper place."[5] This law of inertia is, of course, basic for all pre-Newtonian astronomy.

Finally, Pappus in his commentary to Book V of the Almagest quotes Ptolemy on the construction of an instrument called "Meteoroscopeion" (similar to the armillary sphere described in Alm. V, 1), a passage not found in any of Ptolemy's preserved works.[6]

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[2] Euclid I, Post. 5. For Ptolemy's "proof" cf., e.g., Heath, Euclid I, p. 204–206 or Heath GM II, p. 295–297.
[3] Translated in Cohn-Drabkin, Source Book, p. 247 f.
[4] Cf. Heiberg, p. 264, note*.
[5] Translated in Festugière, Proclus Comm. Tim., IV, p. 150.
[6] Rome [1927], p. 102 and Rome, CA I, p. 4, note (1) considers therefore the possibility of another lost work of the type of the "Planisphaerium" or the "Analemma".

C. The Time from Ptolemy to the Seventh Century

§1. Introduction

To continue Ptolemy's work and to improve on it would not have been an easy task. Today it would seem obvious that Ptolemy's observations should have been repeated and refined by comparing them with the predictions from the theory. This would have resulted inevitably in the recognition of systematic deviations and thus in corrections for the basic parameters of the models. In short, Tycho Brahe could have been Ptolemy's successor in place of Pappus or Theon.

We know that this did not happen, but to seek the causes lies outside the topic of the present work. It will suffice to remark that there were many external and internal events in the time of the later Roman Empire which created a cultural climate unfavorable to scientific research.[1] One should not forget, however, that Kepler did his work at the time of the Counter-Reformation and of the Thirty-Years-War.

In our context it suffices to recall that we have ample evidence that didactic trends increasingly obscured the achievements of the age of Marinus, Menelaus, and Ptolemy. Topics which Ptolemy never found necessary to discuss, like sexagesimal division or linear interpolation, became the subject of commentaries, and this happened in a period which was still in full possession of the great classical works from the preceding era.[2]

In this same period of declining astronomical work, admiration for the philosophical tradition (factual or of recent invention) began to create an increasing opposition to the method and the spirit of scientific astronomy.[3] In antiquity this trend had little practical consequences but during the Middle Ages it produced some attempts to return to models based on homocentric spheres. More than anything else the philosophical discussion made the return to empirical procedures difficult for many centuries to come.

While the scientific astronomical literature became increasingly sterile the astrological interest remained active as ever. For astronomy proper this had

[1] Many attempts have been made to describe this period of transition to the Middle Ages, e.g. in the work by Ernest Stein, Histoire de Bas-Empire (2 vols., 1959–1968) or in Pierre Courcelle, Les lettres grecques en occident de Macrobe à Cassiodore (Paris 1943; English translation by Harry E. Wedeck, Harvard Un. Pr., Cambridge 1969). Particularly concerned with the sciences is W. H. Stahl [1959] and his book "Roman Science" (1962), a work unfortunately marred by a lack of technical competence (cf. my review in AJP 85 (1964), p. 418–423).

[2] Cf., e.g., Pappus on division (Mogenet [1951]).

[3] For a typical example see, e.g., the concluding section in Proclus' "Hypotyposis" (Chap. VII, 50–58, Manitius, p. 236–239).

no beneficial effect. Astrology is a dogmatic discipline, following a strict ritual in combining certain data without worrying how reliable these data were. This attitude is reflected in the fact that astrologers for centuries used arithmetical methods, e.g. for planetary positions or for determining the length of daylight, which were long superseded by more accurate procedures. No astrologer cared about the reliability of the basic parameters of his planetary tables. Astrologers eventually turned to the use of the "Handy Tables", not out of interest in their theoretical basis but because they were, indeed, very "handy". Hence one may well say that at no stage in the development of astronomy did astrology have any direct influence, beneficial or otherwise, on astronomy beyond the fact that it provided a secure market for treatises and tables and thus contributed to the survival of works which otherwise would hardly have reached us.

§ 2. The Time from Ptolemy to Theon

1. Chronological Summary

Continued from p. 780; for continuation cf. below p. 1028. Most dates are only approximately known, even if not expressly queried.

Sosigenes	≈A.D. 170	PSI Inv. 515 (treatise, moon)	200(?)
Hyginus	170(?)	PSI 1491 (treatise, planets)	200(?)
Carpus	≈200(?)	P. Lond. 1278 ("Handy Tables")	200
Artemidorus	170 to 213		
Hippolytus	230	P. Heid. Inv. 4144 + P. Mich. 151 (table for ♂)	250(?)
Porphyry	234 to 304	P. Ryl. 27 (treatise, moon et al.)	250
Censorinus	238		
Achilles	250		
Solinus	3rd cent. (?)		
Pancharius	3rd cent. (?)		
Anatolius	280		
Pappus	320	P. Antin. 141 (treatise, moon)	300
Chalcidius	300 to 350		
Ausonius	309 to 394		
Firmicus Maternus	350		
Anonymus	354	P. Heid. Inv. 34 (ephemeris)	345 to 349
Julian, emperor	361 to 363	P. Vindob. 29370 (ephemeris)	348
Cleomedes	370		
Paulus Alex.	378		
Anonymus	379		
Hephaistio	born 380		
Theon Alex.	≈330 to 405	Vat. gr. 1291 ("Handy Tables")	written 813/820
Synesius	410		
Hypatia	died 415		

Notes

Sosigenes: not to be confused with his namesake of the first century B.C. (cf. above p. 575).

Carpus: between Geminus and Pappus; cf. also Rome [1934] and Proclus, Comm. Rep., trsl. Festugière III, p. 171 (note 1).

Achilles: often referred to as Achilles Tatius; cf., however, RE Suppl. 1 col. 7, 34f.

Hippolytus: it is generally agreed that the same person wrote the "Refutation of all Heresies" who promoted a 16-year Easter cycle which is also engraved on his statue found 1882 at the Via Tiburtina in Rome. The "Refutation" is securely dated by its contents to about A.D. 230; in it is also found the list of cosmic distances ascribed to Archimedes (cf. above IV B 3, 2 B). Fortunately we do not need to be concerned about other writings of Hippolytus and with the controversy whether or not our "Hippolytus" actually should be called Josippus (as suggested by P. Nautin, Études et Textes pour l'Histoire du Dogme de la Trinité, 1 and 2, Paris 1947, 1949).

Porphyry: in all probability part of his horoscope is what is mentioned by Hephaistio (II, 10, ed. Pingree I, p. 112, 16): Jupiter in Cancer, Mars in ♌8, Moon in ♍8. The corresponding date is 234 Oct. 5.

Pappus: 320 is the plausible date for the commentary on the Almagest (cf. below p. 966) while the "Collection" was written around 340 (cf. Rome, CA I, p. 255, note).

Firmicus Maternus: cf. below p. .

Cleomedes: cf. for his date below V C 2, 5 A.

Theon: he probably died between 402 and 415; cf. above p. 873.

Vat. gr. 1291: cf. for the Helios miniature (for 753 or 830) below p. 978, n. 3.

2. Papyri and Ostraca

The papyri, however fragmentary, contribute significantly to our knowledge of ancient astronomy. Without them we would know nothing about the planetary tables of the Roman imperial period, we would see much less clearly the hellenistic influences on India, and many details of astronomical and astrological practices would escape us. Yet, one must remember that it is exclusively Egypt that is represented in these sources and that their number can only be a minute fraction of what once existed. We have about 50 fragments of astronomical papyri of the time from Augustus to the fifth century (and 4 from the preceding centuries). Among these fifty texts ten are demotic,[1] probably all from the first or second century of the Roman rule in Egypt.[2] Also about 50 astrological treatises have reached us in fragments (less than 10 demotic), to which the horoscopes, almost exclusively Greek, may be added, the earliest from the first century B.C., rapidly increasing in number toward the first and second century A.D.[3]

The little we know for the time between Ptolemy and Theon does not indicate drastic changes. The naive but often used argument that a text must be earlier than Ptolemy when it shows no influence of the Almagest is discredited by such

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[1] Cf. above III 4 B.
[2] For the Greek texts of the period before Ptolemy see above p. 787f.
[3] Cf. the statistics above Fig. 1, p. 1371.

treatises as P. Ryl. 27 which, a century after Ptolemy still operate with arithmetical methods.[4] Vettius Valens, Ptolemy's younger contemporary, also takes no notice of the Almagest.[5] On the other hand Ptolemy's "Handy Tables" had come into existence and we have at least two fragments related to these new tables.[6] We also know of a different type of tables through some introductory passages that have survived.[7] As far as we can say, however, it is only in the time after Theon that "ephemerides" in the strict sense of the term make their appearance.[8]

The following is a descriptive summary of the material that seems to belong to the period before Theon. For additional bibliographical references cf. Neugebauer [1962, 1] and Pack, Texts[(2)] (1965).

1. P. Iand. 84

Small fragment from a treatise on spherical astronomy, paleographically dated to the second half of the second century; cf. Neugebauer [1962, 1], p. 386, No. 15. Lack of indices makes it difficult to exclude a connection with any of the extant works but I did not find a configuration which would lead to exactly the sequence of letters found in our fragment.

2. PSI Inv. 515

Preliminary publication: Manfredi [1966], p. 237–243. Recto: paleographically second century; verso: traces of receipt, third century. Fragment from a column of 46 lines of a treatise concerning the use of some lunar tables, reminiscent of Ptolemy's or Theon's introductions to the Handy Tables.

A specific example is discussed for which the following parameters are given:

(mean?) longitude:	156;28,[..,..,]23
anomaly (*βάθος*):	51;14,7,44
argument of latitude:	238;48,[..

Then the second line of a "thirty-day table" is mentioned and a "four-year" interval. A number 294;35,[.. is probably another argument of latitude, about 4 days later. Finally the solar anomaly is mentioned and a number 2;24,4[0 which could be a (maximum?) equation. Unfortunately no larger context can be established, but it is clear that we are dealing with an advanced form of lunar theory.

3. P. Mich. 150

Fragmentary list of full moons, 3rd or 4th cent. Latest discussion: Neugebauer [1942, 2], p. 251 f. Only one section is preserved, giving hours ("day" or "night") and longitudes for 13 consecutive oppositions during one year; this column is followed by the column of 13 month names for a later year. The longitudes found in the text can be fitted very well to actual full moon positions[9] (e.g. from

[4] Cf. above V A 2, 1.

[5] Cf. above V A 1, 3 and V A 2, 2.

[6] P. Lond. 1278 and P. Ryl. 522+523; cf. above p. 938 f. and below p. 974.

[7] PSI 1491; cf. below p. 946, No. 5.

[8] Cf. below V C 5, 3.

[9] Using the tables for New and Full Moons by H. H. Goldstine.

288 Aug. 29 to 289 Aug. 18 or 383 Aug. 29 to 384 Aug. 17) but the hours do not agree throughout and I did not arrive at a secure date.

4. O. Bodl. 2176

Ostracon, published in Tait-Préaux, Greek Ostraca in the Bodleian Library, II (1955), p. 388 f. The text concerns a 40-year period of new moons in Epiphi (month XI, Alexandrian calendar) that precede the rising of Sirius. An example, unfortunately not fully preserved, mentions the year 4 to 5 of Valerian and Gallienus and a conjunction on Epiphi 5 — which is indeed correct for the new moon of 257 June 29. It is also correct that this year is two years away from an intercalary year (i.e. a year with a 6th epagomenal day, here 255 Aug. 29). Apparently Epiphi 24 (July 18) is here considered to be the normal date for the rising of the star.[10] What a period of 40 Alexandrian years has to do with syzygies is not clear since this interval does not contain an integer number of lunations ($\approx 494;34^m$).

5. PSI 1491

To be published in PSI, Vol. XV; paleographically 2nd cent. Introduction to planetary tables arranged in (at least) 7 columns, probably one for each planet; this arrangement excludes the Handy Tables. The extant fragment of 20 lines seems to deal with apsidal lines and with the anomalistic motion.

6. P. Ryl. 464

Small fragment, 3rd cent., from treatise or introduction to tables. Cf. Pack, Texts[(2)] No. 1604.

7. P. Heid. Inv. 4144 + P. Mich. 151

Published Neugebauer [1960, 2]; 3rd cent. The preserved part consists of fragments of six consecutive tables each of which carried a heading and 32 lines. Each one of the four partially preserved columns (A to D) had 46 lines; the first table began in line 14 of column A, the sixth began in line 40 of column D. Hence it is clear that the text had additional material before and after the extant columns.[11]

All tables are of exactly the same type. The heading of Table 6 is ♉♊ and it is therefore practically certain that the preceding tables were headed ♋♌, ♍♎, etc. Each table gives a strictly linear sequence of numbers, arranged like a Babylonian multiplication table. As an example may serve Table 4[12]:

0;30,46	0;20
1; 1,32	0;40
1;32,18	1
3; 4,36	2
4;36,54	3
etc.	
46; 9	30

[10] The conventional date is Epiphi 25; cf., e.g., Hephaestio I, 23, ed. Pingree. I p. 66, 4.
[11] Cf. Fig. 1 in Neugebauer [1960, 2].
[12] The position of the semicolon is mine but the zeros are in the text.

We have here a table for $k \cdot a$ with $k = 1/3, 2/3, 1, 2, \ldots, 30$ and $a = 1;32,18$. In this way the context of the text can be described as a set of tables $a \cdot k$ with

$$\begin{array}{llll} a = 1;54 & \text{in } [♍♎] & 1;32,18 & \text{in } [♑♒] \\ \quad 1;21,30 & [♏♐] & 1;48,27 & [♓♈] \\ & & 1;29,48 & ♉♊ \end{array} \tag{1}$$

Though the arithmetical structure of these columns is simple it is not easy to penetrate to their astronomical purpose. The only clue is the division of the ecliptic in six consecutive pairs of signs. This arrangement has only one parallel in our material: the six zones for the synodic arcs of Mars in Babylonian (and Indian) astronomy.[13] I think it is certain that also the present tables concern this planet though much remains inexplicable. In the Babylonian theory we have the following synodic arcs for the six segments, beginning with ♉ and ♊:

$$\begin{array}{ll} w_1 = 45° & w_4 = 1,0 \\ w_2 = 30 & w_5 = 1,30 \\ w_3 = 40 & w_6 = 1,7;30 \end{array} \tag{2}$$

with a mean synodic arc[14]

$$\Delta\lambda = 48;43,18,30°. \tag{3}$$

Since the w_i represent the influence of a sign as a whole we must also in our tables turn to the entries for 30°. Hence we take from (1) the values

$$30 a_i = w_i. \tag{4}$$

beginning with ♉ and ♊ ($i = 1$). This gives us the following set

$$\begin{array}{ll} w_1 = 44;54 & w_4 = 40;45 \\ w_2 = [\quad] & w_5 = 46;\ 9 \\ w_3 = 57;\ 0 & w_6 = 54;13,30. \end{array} \tag{5}$$

The arithmetical mean of the five preserved values is 48;36,18, hence very near to the value (3) of $\Delta\lambda$. Assuming that our text is based exactly on the value (3)[15] we find

$$w_2 = 49;18,21 \quad \text{thus} \quad a = 1;38,36,42. \tag{6}$$

The traces in column A agree for the multiples from 24 to 30 quite well with the multiples of a number ending in 6. Thus it seems possible that the value for a in (6) was truncated to 1;38,36 in order to agree with the number of digits in the other tables. This would lead to

$$w_2 = 49;18 \quad \text{thus} \quad \Delta\lambda = 48;43,15. \tag{7}$$

In spite of a small margin of uncertainty in the restoration (6) or (7) it seems virtually certain that the numbers w_i represent the synodic arcs for the six segments of the ecliptic, well-known from the Babylonian theory of Mars. Beyond this, however, it seems impossible to proceed any farther in the interpretation of our

[13] Cf. above II A 5, 1 B; for India see Neugebauer-Pingree, Pañcasiddhāntikā II, p. 120.
[14] Cf. above p. 409 (14) or ACT, p. 302 (2b).
[15] There is very little leeway in the choice of this basic parameter because it is a direct consequence of the fundamental period relations for the planet; cf. above p. 391(11).

text. Not only can we not explain why the Babylonian parameters (2) should have been changed to (5) and (6) or (7) but this change makes it also impossible to use the new parameters in the same fashion as in the Babylonian theory. In order to cross from one zone to the next one needs transition coefficients

$$c_i = (w_{i+1} - w_i)/w_i \tag{8}$$

which are small convenient fractions when computed from (2). The new parameters w_i, however, do not produce a single coefficient c_i which is a finite sexagesimal fraction. Hence the whole Babylonian technique is inapplicable to the new set of synodic arcs. This seems to be clear evidence for the existence of a greatly modified Babylonoid theory of Mars although we can only guess at its working.[16]

3. Second and Third Century

By a fortunate accident some works were preserved which illuminate the background of the period in which Ptolemy worked and of the century after it when the impact on practical astronomy of such new tools as the "Handy Tables" must have become felt.

The latter fact is easily established[1]: we have a fragment from a commentary (by one Artemidoros) which compares the methods of the Almagest with the Handy Tables, dated by an example to 211/213. From the time shortly before Ptolemy much of a large work by Theon of Smyrna survived (preserving material from another native of Asia Minor, Adrastus of Aphrodisias[2]). Probably to the next century belongs the introductory chapter on the "universe" in an elementary treatise by an Achilles, perhaps only secondarily adapted to serve as a commentary to Aratus. Both these works are certainly products of the schools, run by the "philosophers" for the education of the sons of well-to-do families. Here we see how much of medieval intellectual life is foreshadowed in pagan antiquity.

A. Artemidoros

A. Rome pointed out[1] that we know from Theon's Commentary to the Almagest that similar commentaries had been written before his time. One of these predecessors is, of course, Pappus whose commentaries to Alm. V and VI have survived.[2] But Theon's formulation implies the existence of several earlier commentaries and Rome suggested considering a short chapter in a treatise of an otherwise unknown Artemidoros as a fragment from such a commentary.[3] This short text is embedded in an anonymous treatise,[4] preserved in Cod. Par.

[16] It should be noted, however, that also in the Babylonian solar theory a variant with irregular transition coefficient has been found (Aaboe [1966]); cf. above p. 532.
[1] See also the papyri: above V C 2, 2.
[2] Aphrodisias in Caria, south of the upper Maeander.
[1] Rome [1931, 2].
[2] Edited by Rome CA I.
[3] Perhaps some scraps of papyrus belong to similar commentaries; cf. above V C 2, 2.
[4] Cf. above p. 321, n. 3.

gr. 2841 and published in CCAG 8, 2, p. 129, 3–130, 18 with corrections by Rome [1931, 2]. The text pretends to illustrate the difference between Almagest and Handy Tables by computing the true longitudes of sun and moon for Nabonassar 958 X 28 (=A.D. 211 Apr. 25). This is indeed the date for which the solar position was computed but the lunar longitude corresponds to a moment two years later (Nab. 960 X 28 = A.D. 213 Apr. 24). For both luminaries the results obtained by the Almagest and by the Handy Tables differ: for the sun the Handy Tables are 0;2° ahead, for the moon 0;17° behind the Almagest. Without giving further details the text explains these discrepancies as caused by the equation of time.

This explanation is correct for the moon since the lunar epoch values in the Handy Tables include the effects of the equation of time for the interval from Nabonassar 1 to Philip 1, corresponding to a time correction of about $-0;32^h$. Thus computing, as in our example, without considering the equation of time must result for the tables of the Almagest in a lunar longitude about 0;17,34° greater than obtained with the epoch values given in the Handy Tables.[5]

For the sun, however, the text is obviously wrong since the equation of time cannot produce corrections of opposite signs for sun and moon; furthermore the equation of time is always applied only to the moon since it has only a negligible effect on the solar longitude. Indeed, computing accurately, one finds that the difference of 0;2° for the sun is only caused by sloppy roundings.[6] Obviously one cannot have much confidence in the competence of Artemidoros (or the anonymous redactor) since he was not disturbed by the sign of the solar deviation and since he entered the tables with 2 years difference for sun and moon.

B. Theon of Smyrna and Adrastus

The date of Theon of Smyrna was established through the late Hadrianic style of his bust,[1] dedicated by his son "Theon the priest"; hence Theon. "the platonic philosopher," had died in the years 130/140. He repeatedly quotes Tiberius' astrologer Thrasyllus who died A.D. 36 and the philosopher Adrastus of Aphrodisias who seems to have been Theon's elder contemporary.[2]

All we know about Theon's astronomy comes from a section in his work on "Mathematics useful for the study of Plato". Much of it, if not all, is taken from Adrastus as Theon himself tells us, and therefore it is of little interest to distinguish between the two authors, particularly since what they have to say is very elementary.

Theon of Smyrna has often been identified with the Theon whom Ptolemy mentions in the Almagest[3] for his observations of the maximum elongations of

[5] Cf. below V C 4, 4 B. Rome ([1931, 2], p. 112, n. 1) incorrectly said that the equation of time does not explain the stated discrepancy. He should have computed the equation of time not only for the era Nabonassar but also for the era Philip.

[6] For Nab. 958 = Phil. 534 X 28 one finds from the Almagest $\bar{\lambda}_{\odot} = 30;48,50°$ while the Handy Tables give $\bar{\lambda}_{\odot} = 30;49°$. The equation of time would lead to a correction of $-0;1,19°$.

[1] Cf. RE 5 A, 2 col. 2067, 30–42. Cf. also K. Schefold, Die Bildnisse der antiken Dichter, Redner und Denker, Basel 1943, p. 180, No. 3. It is, of course, ridiculous to use Theon's ignorance of the Almagest as a chronological criterium (which would apply to many modern platonists).

[2] Cf. RE 1, 1 col. 416, No. 7.

[3] Cf. above p. 158 and p. 162.

Mercury (in A.D. 130) and of Venus (in 127, 129, and 132). Ptolemy does not say explicitly that these observations were made in Alexandria, but they were on a professional level and the records were left to Ptolemy.[4] He deduced from them a maximum elongation of 26;15° for Mercury whereas Theon of Smyrna mentions only a traditional round value of 20°.[5] Theon of Alexandria calls Ptolemy's Theon "the old Theon,"[6] a fact that has been taken as evidence that he knew only of one Theon — an argument of remarkable naiveté. At any rate the whole question is without much interest.[7]

It is clear that Theon's treatise does not pretend to make original contributions to astronomy. Unfortunately it is also clear that Theon had not fully digested the material he is presenting to his readers. For example he says correctly that the inequality of the seasons (Hipparchus' parameters) leads to ♊5;30° for the solar apogee.[8] He does not realize, however, that these data also imply the identity of tropical and anomalistic year and happily reports in another context the differences between three solar periods: tropical, anomalistic, and latitudinal.[9]

He gives two proofs for the equivalence of epicyclic and eccentric motion; one he ascribes to Adrastus which only shows the equivalence of motions through whole quadrants; the second is based on a parallelogram in general position, exactly as in Alm. III, 3[10] and certainly common knowledge for some three centuries. What he has to say about the volume of the sun, supposedly assumed by Hipparchus, inspires little confidence.[11]

The only interesting remark about planetary theory is a reference (without naming an author) to a heliocentric arrangement for the inner planets,[12] although it was considered only as an alternative to the assumption of independent epicycles of identical mean longitudes. There are some statements made about mutual occultations of Mercury and Venus, or about Mars eclipsing Jupiter or Saturn, etc., all, of course, without factual evidence. Nothing is said about transits of the inner planets below the sun. A lengthy discussion of the causes of solar and lunar eclipses touches only upon the simplest facts.

C. Achilles

The date of this author, "around 250," is very insecure; an upper limit could be a reference by Firmicus Maternus[1] in the fourth century to a "prudentissimus Achilles" in astrological matters. No such data appear, however, in the extant (fragmentary) work. In the modern literature Achilles is often called Tatius

[4] Ptolemy, Opera I, 2, p. 296, 14f.; p. 299, 12 (ed. Heiberg).
[5] Cf. above p. 804.
[6] Theon's Commentary to the Almagest, Basel edition (1538), pp. 390, 395, 396 (quoting from Theon Smyrn., ed. Martin, p. 8, n. 3).
[7] Equally inconsequential is the identification of Theon of Smyrna with one Theon mentioned in Plutarch's "Face on the Moon" (cf. Loeb, Moralia XII, p. 7, p. 171).
[8] Hiller, p. 157, 5; Dupuis, p. 254, 21.
[9] Cf. above p. 630.
[10] Cf. above p. 57.
[11] Cf. above p. 326.
[12] Cf. above p. 694f.
[1] Math. IV, 17 (ed. Kroll-Skutsch I, p. 238, 18).

because Suidas[2] confused him with this better known erotic author.[3] Suidas mentions a work "On the sphere"; the extant treatise,[4] usually referred to as the "Isagoge," has the title "Introduction to the Phaenomena of Aratus, excerpts from Achilles."[5] The preserved part is probably only the first chapter "On the universe." Pasquali suggested, rightly, it seems to me, that one should consider the references to Aratus only as secondary additions, made in order to change an elementary introduction to astronomy to a "Commentary" on Aratus.

The book is divided in 40 sections of very unequal length, often dealing with unrelated topics. No. 21, e.g., "On the moon" ends with a discussion of the cosmos, the nature and composition of stars, of the central position of the earth, and of the distinction between "stars" and "constellations" – all topics also discussed separately in other sections, and some of them repeatedly.[6] The whole treatise is heavily laced with quotations from poets and philosophers but almost no astronomers (the names of Ptolemy and Hipparchus appear only once). For the modern collector this is a valuable source of "fragments" but for the student a useless accumulation of names.

There is only one astrologically influenced section in the whole work as we have it. Operating with the archaic sexagesimal division of the circle[7] Achilles discusses (in No. 26 and again in 29) the five zones on the earth which are bounded by the parallels of latitude $\pm(90-\varphi)$ and $\pm\varepsilon$. In No. 29 these five zones are associated with the five planets, beginning with Saturn for the arctic zone and ending with Mercury for the antarctic. I do not know of any parallel to this peculiar association.

In the same section Achilles also gives the widths of these five zones in stades such that the total corresponds to Eratosthenes' 252000 stades for the earth's circumference. On the other hand the division in seven "climata" and their characterization by the ratio between longest and shortest daylight[8] is known to Achilles.[9] His basic area is always the "Hellespont"[10] with $M:m=5:3=15^h:9^h$ (i.e. $\varphi \approx 41°$) but the division in five zones assumes $\varphi=36°$ or $\varphi=38°$ as the somewhat garbled section No. 26 shows.[11] The obliquity of the ecliptic is always taken to be 24°.

In No. 18 periods for the outer planets are listed. First the crude ones: 30 years for Saturn, 12 for Jupiter, 2 for Mars. Besides these ordinary periods, however, we are told that each planet returns in a certain huge number of

[2] Cf. RE I. 1 col. 247. Suidas, ed. A. Adler I. 1, p. 439. 22–27.
[3] Who, says Suidas, "finally became a christian and a bishop."
[4] Greek text published in Maass, Comm. Ar. rel., p. 27–75; Greek with Latin translation in Petavius, Uranol. (1630), p. 121–164.
[5] This is the interpretation suggested by G. Pasquali for the subscript Maass, Comm. Ar. rel., p. 75, 22 f. (Nachr. d. Ges. d. Wiss. zu Göttingen 1910, p. 223–225).
[6] Achilles seems to be interested in terminology; cf. No. 2 ("mathematics"), Nos. 14, 21, 23 ("stars"), No. 25 ("parallels"), Nos. 38, 39 ("risings" and "settings"), No. 32 (μετέωρα).
[7] Cf. above p. 590.
[8] Maass, p. 70, 11.
[9] No. 19, Maass, p. 47, 13. Honigmann SK. p. 2 accuses Achilles of confusing geographical longitude and latitude; but one has only to delete the insertion "and Alexandria" between "Syene and Elephantine" (p. 47, 11; cf. also p. 67, 5) to obtain a correct text.
[10] Cf., e.g., above p. 729.
[11] Cf. above p. 733.

years[12] "from point to point" which apparently represent a sidereal period. These numbers are

Saturn:	350635 years	(= 1,37,23,55)
Jupiter:	170620	(= 47,23,40)
Mars:	120000	(= 33,20, 0)

as compared with "365 days and a fraction" for the "solar year." I do not know what speculation lies behind these numbers which show no relation to any known set of planetary periods.

The astronomical contents of the rest does not go beyond the most elementary concepts of spherical astronomy.

4. Fourth Century

A. Astrology

The fourth century sees the victory of Christianity but it is by no means the century which saw the end of belief in astrology by large sections of the population, christian as well as pagan. Although it is clear that by now many high ranking persons found it advisable to profess the new religious faith of the rulers it is not surprising to see that they nevertheless retained much of the traditional concepts of pagan civilization.

A characteristic example of the naive combination of pagan and christian elements is preserved in the famous "Calendar of 354," a work presented as a new-year's gift to a Valentinus, obviously a man of importance in Rome and a Christian.[1] Nevertheless, the calendar proper, decorated by the famous calligrapher Philocalus, is exclusively based on astrological and mythological concepts of the late imperial period, to which is simply added a second part that concerns the christian festivals and the chronology of the dignitaries of the church.[2]

This combination of seemingly contradictory elements is not restricted to the capital. We find exactly the same adherence to astrological doctrine and christian faith among men from the provinces. Ausonius (about 310 to 394) was born in Gaul, became grammarian and rhetor, was then appointed tutor to Gratian who finally made him consul in 379. While he emphatically professes Christianity when addressing the emperor he clearly accepts astrological doctrine in his poems and in his correspondence with his friends.[3]

It is of interest to see that Ausonius could assume a certain level of astrological and astronomical knowledge on the part of his readers. In his "Riddle of the Number Three" one of his examples concerns (besides the Holy Trinity) the

[12] Called "great year" in the heading of the section; it has nothing to do with any common return.
[1] Cf. for all details the excellent monograph by H. Stern, Cal. (1953).
[2] One could mention as a curious parallel for the combination of christian piety and astrological tradition the calendars in the French "Books of Hours" of the 14th and 15th century.
[3] Cf. the introduction by H. G. White to his edition of Ausonius (Loeb I, p. XII ff.) and H. de la Ville de Mirmont, L'astrologie chez les Gallo-Romains (Bibliothèque des Universités du Midi 7, Bordeaux 1904). For the general background cf. A. Momigliano (ed.), The Conflict between Paganism and Christianity in the Fourth Century, Oxford 1963.

stars: "*triplex sideribus positus, distantia, forma.*"[4] Here the three "positions" clearly refer to the three spatial coordinates in planetary theory: longitude, latitude, and depth (*βάθος*)[5]; for the three "distances" one may take apogee, mean distance, and perigee; finally "*forma*" corresponds to Greek *σχῆμα* and represents the three astrologically important aspects: trine, quartile, and opposition.[6]

Concerning the lengths of the seasons from vernal to autumnal equinox four ancient sources agree on 94 1/2 and 92 1/2 days, respectively: Almagest III, 4 (based on observations by Hipparchus),[7] Theon of Smyrna,[8] Ausonius,[9] and Cleomedes.[10] For the two remaining seasons Theon follows the computations in the Almagest by assuming durations of 88 1/8 and 90 1/8 days, respectively. Cleomedes, however, gives 88 and 90 1/4 whereas Ausonius assumes 88 1/4 and 90 days for unknown reasons.

Another person of high social rank, simultaneously involved in astrological doctrine and imperial anti-pagan policy is the "*vir clarissimus*" (i.e. "senator") Firmicus Maternus. Born in Sicily, educated as a lawyer, he compiled a huge astrological treatise, called "Mathesis," in the years between about 335 and 355.[11] At the same time, probably between 345 and 348, he wrote a very polemical pamphlet[12] "On the error of pagan religions", without reference of course, to astrology which is not a "religion", in which he suggested to the emperors Constans and Constantius, the sons of Constantine, the secularization of the property and treasures of the pagan temples.

The "Mathesis" is one of the largest treatises on astrology (about three times the size of the Tetrabiblos) that has come down to us from antiquity. It claims to make available in Latin for the first time the astrological wisdom of the Egyptians and Babylonians and their Greek pupils.[13] A factual indebtedness to Manilius is not mentioned.[14]

There is very little of astronomical interest in the Mathesis and even this is of a low level. We already mentioned the garbled statements of Firmicus concerning climata and rising times.[15] Equally bad is a short paragraph in Book VIII on the actual size of the zodiacal circle.[16] Firmicus gives the length, measured in stades, of 1° and of 30° on the zodiac; unfortunately the text is corrupt. However there

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[4] Loeb I, p. 366 v. 75 translates "triple classification of stars, according to their station, distance, and their magnitude"(!).

[5] Cf., e.g., Theon of Smyrna, ed. Hiller, p. 172, 18–20; Dupuis, p. 278/9. Scaliger suggested declination as third coordinate (cf. Ville de Mirmont, above note 3, p. 36) but declination is not independent of λ and β and plays no role in ancient planetary theory.

[6] Cf., e.g., Geminus, Isag. II, 13 (Manitius, p. 22, 16): *κατὰ τρεῖς τρόπους αἱ συμπάθειαι γίνονται.*

[7] Cf. above I B 1, 3 B.

[8] Hiller, p. 152/3; Dupuis, p. 248/9.

[9] Eclogues XV and XVII (Loeb I, p. 188/193).

[10] Cleomedes I, 6 (ed. Ziegler, p. 52, 21–27).

[11] Dedicated to Lollianus, consul in 355.

[12] Text and French translation: G. Heuten, Bruxelles 1938; text and German translation: K. Ziegler, München 1953 (Das Wort der Antike 3, in two parts).

[13] Mathesis II, Introd. (Kroll-Skutsch I, p. 40f.).

[14] Cf. Boll, RE 6, 2 col. 2372, 37ff.

[15] Above p. 729.

[16] Mathesis VIII 4, 15 (Kroll-Skutsch-Ziegler II, p. 293, 27–294, 10).

exists one pair among the preserved numbers that shows at least the proper ratio: $1^\circ = 21400^{st}$ and $30^\circ = 642000^{st}$, respectively.[17] If we assume with Ptolemy that 1° on a terrestrial great circle has the length of 500 stades[18] we would have to place the fixed stars at a distance of $\approx 43\, r_e$, an obviously absurd result.[19]

Firmicus does not discuss computational methods at all. His only references to Ptolemy and Hipparchus concern the astrological doctrine of "*antiscia*".[20] There is no way of knowing who determined the planetary positions in the horoscope of Ceionius Rufius Albinus (303 March 14),[21] *praefectus urbi* in 336/7.

By a fortunate accident we have one more document from the same period which was written in Rome (in Greek) in the year of the consuls Olybrius and Ausonius, i.e. in A.D. 379[22] that demonstrates the continued interest in astrological theory. This short treatise[23] concerns the astrological significance of 30 fixed stars of first and second magnitude; their coordinates are derived from Ptolemy's catalogue, using his constant of precession for the longitudes, leaving latitudes[24] unchanged. The astrological characterization of these stars is expressed by associating them with the planets and thus with their "temperaments."[25] As has been shown by Boll[26] similarity in color lies at the basis of these associations, probably taken from some treatise under the name of Hermes Trismegistos.[27] This tradition has been followed down to the "Syntaxis" of Georgios Chrysokokkes.[28]

The extent to which the astrology of this late period is only a compilation of earlier treatises (which are not much better) is shown by the "Apotelesmatica" of Hephaistio[29] which are based in a large part on the Tetrabiblos and on the astrological poem of Dorotheus of Sidon[30] (first cent. A.D.). It is not surprising to find in a treatise of this type a mixture of contradictory technical methods, e.g. in II, 10 rising times "of the old ones" (i.e. System A) for Alexandria,[31] while II, 11 is based on the Handy Tables for the clima of the Hellespont or for Lower Egypt.[32]

The treatise of Hephaistio provides us with a good example for the creation of a wealth of meaningless planetary parameters by means of purely numerological

[17] I do not know why Boll emended the first number to 21 040 (Sphaera, p. 395, note 2).
[18] Cf. above IV B 3, 3 A.
[19] In Ptolemy's model of nested planetary spheres the fixed stars are at a distance of at least 5,31,5 r_e (cf. Goldstein [1967], p. 7).
[20] Cf. above, p. 331.
[21] Mathesis II, 29 (Kroll-Skutsch I, p. 81, 9ff.); cf. for the horoscope Neugebauer AJP 74 (1953), p. 418–420.
[22] Cf. above p. 952.
[23] Published in CCAG 5, 1, p. 194–226. The anonymous author is supposedly of Egyptian descent.
[24] Characteristically called πλάτος ἤτοι ἄνεμος; cf. above p. 670.
[25] Called κρᾶσις.
[26] Boll [1916]; for the "Anonymous of 379" cf. in particular, p. 71–82.
[27] Cf. Gundel, HT, p. 146.
[28] Kunitzsch [1964], p. 406ff. A slightly different branch of this tradition still reached the "Uranometria" of Bayer (1603) who assigned to each of the extra-zodiacal constellations the temperament of two planets, while the stars in the zodiac are treated individually.
[29] He gives us his horoscope for Sunday, Athyr 30 Diocletian 97, i.e. A.D. 380 Nov. 26 (actually a Thursday). Engelbrecht gives incorrectly A.D. 381, a date repeated in the R.E. For the text of the horoscope cf. Hephaistio, ed. Pingree I, p. 87, 3; p. 91, 27; p. 117, 11 and Neugebauer-Van Hoesen, Gr. Hor., p. 131f.
[30] Cf. the list of parallels given by Pingree I. p. VI–XI.
[31] Pingree I, p. 115, 25.
[32] Pingree I, p. 124 and p. 133f., respectively. probably taken from Pancharius.

speculations.[33] The starting points are periods and cycles which make sense, at least individually, though they are now counted as "months" instead of years[34]:

♄ 30	☉ 19	☾ 25	
♃ 12	♀ 8		(1)
♂ 15	☿ 20		

Consequently the total 129 of these numbers is now taken to mean 10 years and 9 months and is thus applicable to predictions concerning the length of life, etc.[35] But this is only the beginning of numerical patterns. The number of each planet is changed into a new one and all remaining parameters are modified in the same ratio such that we obtain a total of 49 new values. For example the period for Jupiter is changed from 12 to 34 and the remaining numbers are also multiplied with the ratio 34/12 = 2;50. Thus one obtains for Saturn 85, for Mars 42 (actually 42;30), etc. Hence the total changes from 129 to 365 which probably motivated the initial ratio. Similarly, if we change the period of Mercury from 20 to 93[36] we obtain a total of 600, and so forth.[37] It is remarkable to see how arbitrarily constructed new doctrines are accepted by a willing audience.

B. Astronomical Considerations

Paulus of Alexandria in his "Isagoge," written in A.D. 378[1], has a reasonably good grasp on the astronomical elements that underly astrological practices and he also presents a comparatively clear discussion of the individual topics of astrological lore. It is perhaps for this reason that Olympiodorus,[2] almost two centuries later, made the work of Paulus the subject of a series of lectures, apparently close to similar lectures on Aristotle's "Meteorologica." Rhetorius, in the early sixth century, reckons Paulus to be among "the old ones";[3] yet, still in the seventh century, he found in Ananias of Shirak a translator,[4] perhaps on the basis of material brought to Armenia by David, a pupil of Olympiodorus and also a commentator of Aristotle.[5]

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[33] Hephaistio II, 29, ed. Pingree I, p. 200f.

[34] For a similar list cf. above p. 899 or p. 606; also below p. 958 (from Paulus Alexandrinus).

[35] Cf. for similar tricks, e.g., Vettius Valens VI, 5 (ed. Kroll, p. 251 ff.).

[36] The text has 96 (Pingree I, p. 210, 10).

[37] None of the other totals allow of a ready interpretation. The basic new periods are in the order of (1): 210, 34, 52, 83, 15, 93, 148.

[1] As an example he determines the weekday for Diocletian 94 Mekheir 20 (= 378 Febr. 14) which he calls "the present day" (ed. Boer, p. 40, 3; p. 41, 3–16; p. 135f.). Gundel, RE 18, 4 col. 2376, 34 incorrectly writes Febr. 20. The day is a Wednesday; cf., e.g., Ginzel, Hdb. I, p. 231.

[2] For Olympiodorus as the author of the commentary, hesitatingly attributed to Heliodorus by its editor E. Boer (1962), cf. below V C 5, 2 B 4. For bibliographical reasons references will be given here to "Heliodorus," ed. Boer.

[3] CCAG I, p. 154, 12f.

[4] This translation concerns, however, only the first two chapters (corresponding to p. 1–10 of the ed. Boer). I owe this information to my colleague D. Pingree who has at his disposal a collation of the Armenian text made by Prof. R. Thomson of Harvard University. R. H. Hewsen referred to the Armenian version in Isis 59 (1968), p. 42, No. 6.

[5] Cf. Ernst Günther Schmidt, Die altarmenische "Zenon"-Schrift. Abh. d. Deutschen Akad. d. Wiss. zu Berlin, Kl. f. Sprachen, Lit. u. Kunst 1960, 2. Cf. also Westerink [1971], p. 19 and Anon., p. XXIII.

Bīrūnī in a frequently quoted passage of his "India"[6] assumes a Paulus of Alexandria as the author of the "Pauliśasiddhānta." This widely accepted identification with the author of the "Isogoge" has been challenged, however, by D. Pingree[7] who not only pointed out the multiplicity of works which are called Pauliśasiddhānta but who also underlined the difference in contents between the Greek astrological treatise and the relevant Sanscrit works. We know important sections of this work through the summary given by Varāhamihira in Book III of his Pañcasiddhāntikā. None of the astronomical procedures described in the Indian work are found in the Greek treatise. For the only overlapping topic, the variation of the solar velocity, the Indian treatise uses an arithmetical pattern (of System B type)[8] whereas Paulus refers to the Handy Tables,[9] hence to a trigonometric variation. In other words there is no visible connection between the extant Greek text and the Sanscrit work.

The rule which Paulus gives (in Chap. 28) for finding "roughly" the longitude of the sun for a given day has nothing to do with the Handy Tables. If d is the number of days after Thoth 1, m the corresponding number of months, Paulus considers

$$\lambda = \lambda_0 + d - \frac{m}{2} \tag{1}$$

an estimate for the (mean) longitude of the sun. Obviously this is very nearly the case since the solar motion for one year would be $365 - 12/2 = 359^\circ$. As epoch position of the sun on Thoth 1 is assumed either[10] $\lambda_0 = 156^\circ$ or 158°; the first value is also used by Olympiodorus,[11] the second is supported by a statement[11a] that the interval from the vernal equinox to September 1 amounts to 158 days; it also could be related to the epoch value ♍8° used by Vettius Valens[12].

Having thus found the longitude of the sun Paulus proceeds (in Chap. 29) to the determination of the position of the ascendant H for a given hour h after sunrise.[13] If the sun is located in the sign s at g degrees we are told that the longitude λ_H of the rising point is located at $g + 15 \cdot h$ degrees counted in the sign s, or in modern notation

$$\lambda_H = \lambda_\odot + 15 \cdot h. \tag{2}$$

This is, of course, only approximately correct since $15 \cdot h$ is the distance of the sun from H in right ascension, assuming for h equinoctial hours. Paulus also discusses seasonal hours h', as determined by the use of a (plane) astrolabe[14]. He then

[6] Translation Sachau, I, p. 153.

[7] Pingree [1963, 1], p. 237, n. 63; [1969], p. 172.

[8] As described in the Pañcasiddhāntikā III, 18 (Neugebauer-Pingree, Pc.-Sk. II, p. 31 f. and Fig. 7).

[9] Chap. 28, ed. Boer, p. 79. The extrema are 0;57°/d and 1;1°/d in the Indian model, whereas Paulus gives 0;57°/d and 1;2°/d "roughly"; cf. also Heliodorus Comm. ed. Boer, p. 93, 2–7.

[10] Cf. the apparatus to Paulus, p. 79, 15.

[11] Heliodorus, ed. Boer, p. 89, 16; p. 90, 11.

[11a] Cod. Scor. II Ψ 17, fol. 49r, 12f. (unpublished).

[12] Cf. above V A 1, 3 A. The true longitude of the sun for Thoth 1 (= August 29) increases between 150 and 550 A.D. from about ♍4;45 to ♍7;45.

[13] Paulus also covers the case of an hour after sunset by considering the diametrically opposite positions.

[14] Boer, p. 80, 12f.

refers to the Handy Tables[15] to find for a given climate and given solar longitude the length $c(\lambda_{\odot})$ of the corresponding seasonal hour of daytime, expressed in equatorial degrees[16]. In the same tables one finds the rising time $\rho(\lambda_{\odot})$; hence

$$\rho(\lambda_{\odot})+c(\lambda_{\odot})\cdot h'=\alpha(\mathrm{H}),$$

the right ascension of the ascendant, is known. One should now use the table for sphaera recta and find to $\alpha(\mathrm{H})$ the corresponding longitude $\lambda(\mathrm{H})$, but nothing is said about this step in the text[16a]; also Olympiodorus' commentary ignores this point, explaining instead[16b] that $15\cdot 2=10\cdot 2+5\cdot 2$.

Finally, having found H, Paulus determines the culminating degree M, based on arithmetical methods (System A), following a procedure we have described before (p. 720). Olympiodorus comments on this topic[17] but replaces the linear sequence of rising times by the correct trigonometric one (for Alexandria).[18] Instead of introducing the correct relation[19]

$$\alpha(\mathrm{M})=\rho(\mathrm{H})-90^{\circ} \tag{3}$$

he follows Paulus slavishly. Because Paulus enumerates the rising times explicitly for the consecutive signs up to H, Olympiodorus feels obliged to do the same. Hence, for his example H = ♋ 20°, he computes from the tables step by step the rising times for the single signs, only to add them up again to the starting value ρ(♋ 20) = 97;46°, although two scribal errors produce 96;48° instead. He should have used (3) now which would have given him

$$\alpha(\mathrm{M})=97;46-90=7;46^{\circ}\quad\text{thus}\quad \mathrm{M}=\text{♈}\,8;28^{\circ}.$$

Olympiodorus, however, thinks that the 90° in (3) are the length of the arc ♑ + ♒ + ♓ and that the difference 96;48° − 90° = ♈ 6;48 is the longitude of M, obviously analogous to the primitive procedure followed by Paulus. This shows that Olympiodorus did not understand the geometrical reasoning that underlies (3).

As usual, astronomical parameters are occasionally found between the astrological material; e.g., the round value of 15° elongation from the sun is mentioned as boundary for lunar visibility[20] and for planetary phases.[21] Chap. 15, on planetary stations, mentions for the inner planets also the extremal elongations of 48° and 22°, respectively.[22] For Jupiter and Saturn ±120° are declared to be the elongations from the sun when the planet becomes stationary[23]; in fact these numbers correspond approximately to the positions of the planets on the epi-

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[15] He calls them "Handy Tables of Claudius Ptolemaeus" (p. 79, 9f); perhaps Theon's version did not yet exist at that time.

[16] Called ὡριαῖοι χρόνοι as in the Handy Tables (Halma II, p. 1 ff.).

[16a] Boer. p. 80, 18-22 does not belong here; the remaining lines concern again the case of night time.

[16b] Heliodorus, ed. Boer, p. 91, 3.

[17] Boer, Heliodorus, p. 81 f. Note the incorrect division of sentences, p. 81, 19/82, 1.

[18] His numbers may have been taken either from the Almagest (II, 8) or from the Handy Tables (Halma II, p. 20/21and I. p. 150/151).

[19] Cf. above p. 42.

[20] P. 33, 23; p. 35, 2. ed. Boer.

[21] P. 29, 6; also scholion 22 (p. 111). Cf. for this traditional norm. e.g., above p. 762.

[22] P. 32f.; also scholion 24 (p. 111); cf. also above p. 804.

[23] P. 31, 4f.; also scholion 29 (p. 112).

cycle,[24] not to the elongations from the sun. For Mars, however, the elongation is said to be 82° at which point the planet should also make its morning and evening rising. Paulus obviously no longer understood the relation of his parameter to Pliny's "*nonagenarius*," the generalized "stationary" interval, and the visibility limit of 8° known from Porphyry.[25]

The concluding chapter[26] again contains play on numbers from planetary periods and astrological doctrines. One set of numbers is the same as found in Hephaistio (above p. 955 (1)); the other comes from the astrological concept of "terms," i.e. sections in each zodiacal sign associated with one of the five planets.[27] The total of these domains amounts to

$$\begin{array}{llll} ♄ & 57° & ♀ & 82° \\ ♃ & 79 & ☿ & 76 \\ ♂ & 66 & \text{total} & 360 \end{array} \tag{4}$$

originally counted in degrees but then again taken to represent time intervals. To these parameters are finally added for sun and moon the numbers

$$☉\,120 \qquad ☾\,108 \tag{5}$$

in order to obtain again a set of seven parameters which can be combined with the original seven periods of (1), p. 955.[28] The total $120+108=228=19\cdot 12$ evidently represents the 19-year luni-solar cycle which contains in 228 calendaric "months" $228+7=235$ lunations.[29]

This juggling with numbers is typical of the whole astrological literature. It is clear that its authors no longer had any interest in the factual basis of their parameters. In this respect the philosophical treatises represent a higher level. Of course, also their real motive is by no means a genuine interest in astronomy. If one were bent on showing that Plato was the fountainhead of all later knowledge one ought to acquire some professional competence in order to ascribe it to the venerated master.

To this group of treatises belongs the commentary on the Timaeus, written by Chalcidius in the first half of the fourth century,[30] which contains two large astronomical sections.[31] As was established long ago[32] Chalcidius excerpted, just as did Theon of Smyrna two centuries earlier,[33] a Timaeus commentary by Adrastus.[34] At any rate, Chalcidius has nothing new to say. He discusses the

[24] Cf., e.g., p. 1271 Fig. 203.
[25] Cf. above p. 792.
[26] Chap. 36 (Boer, p. 97). The final chapter (37) on the "thema mundi" is obviously a later addition; it is ignored in Olympiodorus' commentary.
[27] Cf. above p. 690. The numbers in (4) are based on the "Egyptian" variety of the terms.
[28] Cf. for these combinations Paulus, p. 12f., p. 97; Heliodorus, p. 143, 10, ed. Boer. Also Bouché-Leclercq, AG, p. 410; Kennedy-Pingree, Māshā'allāh (8th cent.), p. 132; Bīrūnī, Astrol. (written 1029), ed. Wright, p. 255; CCAG 5, 3, p. 132, 26–34; etc.
[29] Cf., e.g., Pañcasiddhāntikā I, 9 (Neugebauer-Pingree II, p. 8).
[30] Cf. RE 3, 2 col. 2042f. and Stahl [1959], p. 121–123.
[31] Chap. 56 to 97 and 98 to 118 (ed. Wrobel, p. 123–184).
[32] Hiller in Rheinisches Museum 26 (1871), p. 582–589. Cf. also B. W. Switalski, Des Chalcidius Kommentar (Beiträge zur Gesch. d. Philos. d. Mittelalters III, 6, Münster, 1902).
[33] Cf. above p. 630. n. 4.
[34] We know of this commentary through a remark by Porphyry in his commentary on Ptolemy's Harmonics (ed. Düring, Göteborgs Högskolas Arsskrift 38, 1932: 2, p. 96, 1f.).

explanation of the planetary motions through eccenter and epicycle models but he leaves the reader with the impression that the tangents to the epicycle from the earth determine the stationary points (Chap. 85). We have had occasion to mention his interest in the maximum elongations for the inferior planets,[35] in particular in the case of Venus[36] (Chap. 110 to 112). In this section we also find the remark that the synodic period of 584 days for Venus is divided by the maximum elongations into two arcs, one of 448 days (containing superior conjunction), and thus another of 136 days (near inferior conjunction). This implies that the arc of the epicycle between maximum elongation and inferior conjunction amounts to $136/584 \cdot 180° = 41;55° \approx 42°$, hence appearing at the observer under an angle of $90-42=48°$. Thus we have here the explicit proof that Chalcidius' reference to a maximum elongation of 50° is only the result of a rounding of 48° to 50°.[37]

The remaining astronomical explanations of Chalcidius are very elementary, except for a reference to solar latitudes and some data for Hipparchus concerning the relative sizes of sun, earth, and moon, both discussed in previous sections.[38]

5. Cleomedes

Cleomedes is known to us exclusively from a little treatise "On the circular motions of the celestial bodies."[1] Since it contains several references to Posidonius it has become a work of much greater renown than it would have won on its own merits — which are very modest indeed. Whatever seems relevant in it for our knowledge of the astronomy of Posidonius has been discussed before (above IV B 3, 3). Nevertheless the treatise was meant to be a composition of its own and it is therefore of some interest as a sample of scientific literature in late antiquity. It is not difficult to realize, however, that Cleomedes compiled his book by excerpting at least two treatises of a very similar elementary character: on planets we read in I, 3 and II, 7; the geographical climata appear in I, 7 and again in II. 1; the solar anomaly features in I, 6 and II, 5; the size of the fixed stars in I, 11 and II. 3; etc.

As we shall see Cleomedes must have written around A.D. 370. This makes him a contemporary of St. Basil who died in 379. A close parallelism between the astronomical remarks in the latter's "Hexaemeron" and Cleomedes has long been observed.[2] The same can be said about another author of this late period, Martianus Capella (probably around A.D. 420) who gives exactly the same numerical data as Cleomedes for the variation of the length of daylight[3]; also a discussion

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[35] Above p. 804.

[36] Cf. above p. 694.

[37] A synodic period of 584 days for Venus is also mentioned in Cleomedes, in the Pañcasiddhāntikā, and by al-Farghānī (cf. above p. 784, n. 24). Farghānī also gives 48° for the maximum elongation (Carmody, p. 30); the same value is also found in P. Mich. 149 (cf. above p. 804).

[38] Above p. 630 and p. 326, n. 7, respectively.

[1] The German translation by Czwalina is unreliable and the explanatory notes are usually valueless if not absurd.

[2] Cf., e.g., Courtonne, St. Basile, p. 96ff.; also Hexaemeron, p. 384–387 (ed. Giet, Sources Chrét.); also Gronau, Poseid. passim.

[3] Cf. above p. 723.

of solar eclipses supposedly providing some information about the absolute size of the luminaries[4] is common to both authors. In short Cleomedes cannot be taken as a source of real independence in the use of much older material.

A. The Date of Cleomedes

Cleomedes states (De motu I, 11 p. 106, 25 to 108, 5 Ziegler) that there exist two bright stars such that the rising of one coincides with the setting of the other: Aldebaran (α Tauri) and Antares (α Scorpii), both being located at the 15th degree of their respective sign. Indeed, according to the Catalogue of Stars in the Almagest the two stars differ in longitude by exactly 180°, each being at 12;40° of its sign.[1] Thus the longitudes given by Cleomedes are 2 1/3° greater than in the Almagest; according to the ancient constant of precession of 1° per century the proper date for these longitudes would be 233 years after the epoch of Ptolemy's catalogue, hence 138+233=371 A.D. If we take Cleomedes' number not too accurately and allow for a rounding of about 1/2° up or down we can say that Cleomedes describes a situation that corresponds to A.D. 370 ± 50 years.[2]

This date finds support in another treatise which goes under the name of the "Anonymous of the Year 379." Its time is accurately fixed by a reference to the Roman consuls Olybrius and Ausonius of the year 379. In this text are mentioned again the diametrically opposite positions of Aldebaran and Antares in ♉15° and ♏15°, respectively,[3] exactly as in Cleomedes.[4]

Three passages in Cleomedes' treatise seem significant for the determination of the locality in which he wrote. One concerns the variation of the length of daylight for which he assumes the extrema 15^h and 9^h i.e. the parameters conventionally associated with the clima of the Hellespont.[5] The second passage, in discussing the size of the earth, adduces data for Lysimachia on the Hellespont.[6] Finally he mentions optical illusions experienced in the area of the Black Sea.[7] All this seems to indicate that he addresses himself to readers in the region of Lysimachia. Since Lysimachia was destroyed about 144 B.C.[8] one might doubt that a fourth century author would refer to it as a contemporary locality. But its existence far beyond

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[4] Cf. below p. 963.

[1] Ptolemy. Opera I, 2, p. 88, 2 and p. 110, 7, Heiberg. The magnitudes are 1 and 2, the latitudes −5;10° and −4°, respectively. Of course, the simultaneity of the rising and setting is only fictitious like so many "observations" in the popular literature, e.g. the crossing of the horizon by the sundisk in 1/720 of one day.

[2] It was Letronne who first (in 1822) recognized the chronological significance of the passage in I, 11 (Mémoires ... acad. des inscr. ... 6 (1822), reprinted in Letronne, Œuvres II, 1, p. 249ff.). He was misled, however, first by the use of the modern constant of precession in computing back from 1786 Aldebaran's position, secondly by a misprint in Halma's edition of the Almagest, and third by some numerical errors of his own. The correct date I gave in AJP 85 (1964), p. 418, n. 1.

[3] CCAG 5, 1, p. 198, 4f. and p. 203, 4 and 16.

[4] Boll [1916], p. 14 and note 5 had seen this parallelism but without drawing the obvious conclusion for the date of Cleomedes. — The diametrical position of the two stars is once more mentioned by Rhetorius, but now for ♉/♏ 16;20°, i.e. for about A.D. 500/510; cf. above p. 258, n. 14.

[5] Cf. above p. 723 and p. 731; also Neugebauer [1941, 2].

[6] Cf. below p. 962.

[7] At the end of II, 6 (Ziegler, p. 224, 8). Similarly Copernicus would mention the atmospheric conditions prevailing in the estuary of the Vistula (De Revol. V, 30).

[8] Cf. R.E. 13, 2 col. 2556, 10–14.

the second century B.C. seems to be secured by its being listed in Ptolemy's Geography[9] as well as among the "important cities" in the "Handy Tables" of Theon's redaction in the fourth century A.D.,[10] contemporary with Cleomedes.

B. Geography and Spherical Astronomy

Cleomedes describes the ordinary pattern for the division of the sphere: five parallel circles, equator, two tropics, arctic and antarctic circle (the boundaries for always or never visible stars[1]) and the corresponding zones; the dependence of the arctic circle on the geographical location is spun out to great length (I, 5 and I, 7). With it goes a discussion of the variation in the length of daylight, a maximum of 18^h being reached in Brittany (I, 7). A short list of localities is characterized in II, 1 by their shortest night – an unusual parameter, instead of longest daylight.

Meroe		11^h
Alexandria		10
Hellespont		9
Rome	less than	9
Massilia		$8\ 1/2^h$
Kelts		8
Maeotis		7
Brittany		6

The three first ones are the canonical climata I, III, and V. Except the Maeotis[2] (to fill in an integer hour) the second group seems to be taken from some treatise on western Europe. Similar dates are found in Geminus.[2a]

The variation of the length of daylight at the clima of the Hellespont is described in detail in I, 6. As we have shown before[3] the pattern given by Cleomedes is the arithmetical scheme following "System B" and surely not his own construction. As an explanation he offers only some generalities about the sphericity of the heavens.[4]

Cleomedes loves to demolish obviously absurd hypotheses. e.g. the assumption of a flat earth (I, 8). It is in this context that he states that Cancer and the Head of Draco lie on the same meridian, Cancer (C) reaching the zenith at Syene (S), the

[9] Geogr. VIII 11, 7 (ed. Nobbe, p. 211, 6) with $M = 15\ 1/12^h$ and $1/3\ 1/15^h$ west of Alexandria and III 11, 13 (Nobbe, p. 190, 27) with $l = 54\ 1/6°$ and $\varphi = 41\ 1/2°$ and the gloss "the present Hexamilion"; it is, of course, impossible to say what "present" means.

[10] Extant, e.g., in Vat. gr. 1291 fol. 18r, a 9th cent. manuscript (cf. below V C 4, 1); Honigmann, SK, p. 196, line 109.

[1] Cleomedes I, 2; cf. above p. 582 and p. 733.

[2] This is again a choice suggesting a region near the Black Sea. Ptolemy, Geogr. VIII 18, 5 (Nobbe, p. 230) gives for the Tanais $M = 17;10^h$, thus $m = 6;50^h$.

[2a] Geminus VI, 8 Manitius, p. 70, 15–20.

[3] Cf. above p. 723 f.

[4] Ziegler, p. 52, 9 prints a text according to which the ecliptic intersects the equator at right angles (πρὸς ὀρθὰς γωνίας, "*ad rectos angulos*"). The obvious meaning, however, is that the ecliptic intersects the equator and the nearby parallels almost "as a straight line," i.e. under a constant angle, in contrast to a changing direction farther away until tangential contact with the tropics. Obviously γωνίας has to be deleted. Cf. also the parallel in Martianus Capella (ed. Dick, p. 463, 15–16); "*zodiacus ... aequinoctialem paene directim secat.*"

Head of Draco (D) at Lysimachia (L). The arc between these constellations is supposedly 1/15 of the circumference, i.e. 24°,[5] while the terrestrial distance SL is assumed to be 20000 stades.[6] Thus on a flat earth (cf. Fig. 97) of diameter $d=AB$ and circumference $c_e \approx 3d$, Cleomedes concludes, $c/15 \approx d/5 \approx 20000$ st. would result in $c_e \approx 300000$ st. instead of the accepted terrestrial circumference $c_e \approx 250000$ st.[7] Hence the earth is not a flat disk. One can only hope that this way of arguing is Cleomedes' own and not Posidonius' or Eratosthenes'.

C. *Moon and Sun*

In II, 3 and 4 Cleomedes describes a variety of hypotheses concerning the physical constitution of the moon. In II, 5 the lunar phases are explained by the changing positions of two small circles on the sphere of the moon: the terminator which separates darkness from light[1] and the circle between visible and invisible area. These circles differ so little from great circles that the phases can be properly described by the relative positions of two great circles.

In question of size and distance of the moon Cleomedes follows Posidonius.[2] An isolated remark concerns Hipparchus who supposedly has shown that the sun is 1050 times larger than the earth[3]; unfortunately this factor seems unrelated to any otherwise known argument.[4]

The lunar orbit is not only eccentric but also inclined to the ecliptic such that the moon can reach the arctic or antarctic circle (II, 5). This makes little sense since it requires a geographical latitude φ such that[5]

$$\bar{\varphi}+\varepsilon+i+2\varphi=180^\circ$$

hence $\varphi \approx 60^\circ$ when $\varepsilon+i \approx 30^\circ$, i being the inclination of the lunar orbit. Probably we have here a residue from some treatise on spherical astronomy in which it was said that for $\varphi \geqq 60^\circ$ (i.e. for $M \geqq 18\ 1/2^h$) the moon can become always (or never) visible.

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[5] In the present context "Cancer" must be taken as the summer solstitial point at which the sun reaches the zenith of Syene; thus $\varphi(S)=\varepsilon\approx 24^\circ$. Since also $\Delta\varphi(S, L)=24^\circ$ one has $\varphi(L)=\varepsilon+\Delta\varphi\approx 48^\circ$, whereas Ptolemy's Geography places Lysimachia at 41;30° (cf. above p. 961, n. 9). If C and D would culminate simultaneously the latitude of D would be $\Delta\varphi=24^\circ$, much too low for any star in Draco. Hence the two culminations must occur 12^h apart which gives for H the latitude $2\varepsilon+\Delta\varphi\approx 72^\circ$ at a longitude near 270°. According to Ptolemy's Catalogue of Stars the star "above the Head" (γ, $m=3$) has the latitude 75;30° but a longitude 233;10 (+2;20 for Cleomedes). A little better is c in the Neck ($m=4$) with $\beta=78;15^\circ$, $\lambda=272;20$ (+2;20). Obviously it is senseless to speak here of "observations." For the "Caput Draconis" cf. also Martianus Capella VIII, 827 (p. 435 Dick).

[6] Hence $1^\circ \approx 833$ st., not 700. In I, 10 he gave 5000 st. each for the distances Syene-Alexandria and Alexandria-Rhodes (cf. Fig. 17, p. 1356); hence Lysimachia-Rhodes would be 10000 st. = Rhodes-Syene. According to II, 3, however, Hellespont-Rhodes = 5000 st. (ed. Ziegler, p. 174, 10f.).

[7] One may add that the discrepancy thus obtained is not very great and that Archimedes considered $c_e \approx 300000$ st. to be a generally accepted estimate (cf. above p. 646).

[1] Cf. Aristarchus' discussion (above IV B 3, 1 D).

[2] Cf. above IV B 3, 3 B. For Cleomedes' report on Posidonius' attempts to estimate size and distance of the sun cf. above IV B 3, 3 C.

[3] Ziegler, p. 152, 5-7. The number is obviously corrupt since the cubic root of 1050 is so near 10, the cubic root of 1000, that no geometric determination of the sun's radius will lead to a volume 1050.

[4] Cf. above p. 327.

[5] Cf., e.g., Fig. 43, p. 1364.

In I, 6 Cleomedes mentions the inequality of the seasons: 94 1/2^d and 92 1/2^d from Vernal Equinox to Summer Solstice and to the Autumnal Equinox. These are the Hipparchian values. For the remaining semicircle he gives 88^d and 90 1/4^d, as compared with 88 1/8 and 90 1/8 in the Almagest.[6] These data are explained in a general fashion by the eccentricity of the solar orbit, a topic once more taken up in II, 5 in connection with the variable length of synodic months. The solar velocity is said to reach a maximum in Sagittarius, traversing this sign in 28 days, and a minimum in Gemini, traversed in 32 days.[7] Also in I, 6 one finds a hazy discussion of the equation of time[8] but only as far as the trigonometric component is concerned although the variation of the solar velocity would have been right at hand.

In I, 11 it is stated[9] correctly that the size of the earth cannot be ignored in relation to the lunar orbit since solar eclipses appear differently from different localities. The solar parallax, however, is taken to be negligible.

The causes and circumstances of lunar eclipses are discussed in II. 6. perhaps the best organized chapter in the whole treatise. The main emphasis is laid on the demonstration of the conical shape of the earth's shadow which reaches beyond the moon's orbit. No numerical data. however, are given. At the end of the chapter a report is mentioned according to which a lunar eclipse has been seen at which both sun and moon were above the horizon. At first this is simply declared an invention to embarrass "astronomers and philosophers."[10] On second thought. however. Cleomedes admits the possibility of refraction which he describes quite correctly.[11]

A book of so composite a character as Cleomedes' treatise cannot be expected to be free of internal contradictions. For example when the conical form of the earth's shadow is demonstrated in II. 2 and II. 6 the moon's shadow is assumed to be 40000 stades wide at a solar eclipse. But 40000 stades is the diameter of the moon (according to II. 1) which shows that one now operates with a cylindrical shadow, perhaps following here Posidonius.[12]

Another difficulty arises in the comparison with terrestrial dimensions. According to Posidonius 40000 stades[13] would also be the earth's radius. When (according to I. 8) the distance Lysimachia-Syene amounts to 20000 stades[14] the whole inhabited world would be in the shadow at a solar eclipse. very much in contrast to the actual experience. It is quite obvious that Cleomedes (and his source) never had a clear concept of the geometrical situation prevailing at a solar eclipse. Referring to the famous eclipse which was total at the Hellespont but only 4/5 at Alexandria,[15] 10000 stades away. Cleomedes says that at six times this distance from the Hellespont the whole sun must have been visible because at

[6] Cf. above p. 58 and Alm. III. 4, Manitius I. p. 170.
[7] Hence the extremal velocities have a ratio $8:7 \approx 1;4{:}0{:}56^{\circ\,d}$.
[8] Cf. above I B 2.
[9] Ziegler, p. 112. 7-23.
[10] Ziegler, p. 222. 6f.
[11] Cf. for Ptolemy's study of refraction above p. 894 ff.
[12] Cf. p. 654.
[13] Above p. 653(1).
[14] Cf. above p. 962.
[15] Cf. for this eclipse p. 316. n. 9.

Alexandria about 1/6 of it could be seen. The idea that obscuration of one sphere by another does not vary linearly with the displacement of an observer on a third sphere does not seem to have occurred to Cleomedes.[16] It is only a weak excuse that this type of arguing is not unique in late antiquity. Martianus Capella declares[17] that often, when a solar eclipse is total at the clima of Meroe (clima I) it is only partial at the clima of Rhodes (IV) while the sun is completely visible at the clima of the Borysthenes (VII). We have here a situation (obviously fictitious) which is roughly the inverse to Cleomedes' example. Somehow one concludes from the climata involved that the moon's shadow is 1/18 (=20°) of the earth's circumference[18] and from the part of the sun left visible at the partial eclipse (presumably at Rhodes) one finds (how?) that the moon's diameter is 3 times the shadow's diameter, thus 1/6 of the earth's circumference (i.e. $d_m = r_e$) and we are back at the assumptions from which Cleomedes started.[19]

D. Planets

The planets are discussed at the beginning in I. 3 and their motion is compared in a conventional fashion with the motion of a person on a moving ship or of ants on a potter's wheel.[1] The periods mentioned are the trivial ones (30 years, etc.) with the exception of 2 years and 5 months for Mars whose motion is described as "more disorderly."[2] In fact, however, this number can hardly be anything but a simple mistake[3] committed by some earlier compilator. Finally we find in I, 4 a loose description of the motion of the planets in latitude at which occasion a terminology is used[4] which is typical for the notation of "steps"[5] although this term itself is not mentioned.

At the very end (in II, 7) a second list of planetary parameters is given which must have been excerpted from a more serious treatise. There we have the following data:

	synodic period	maximum elongation	extremal latitude
Mercury	116^d	20°	±4°
Venus	584	50	±5
Mars	780		±2 1/2
Jupiter	398		±2 1/2
Saturn	378		±1

[16] That the displacement of the observer supposedly takes place along a meridian only complicates the situation. No attention is ever paid to the effect of an oblique position of the shadow cone.
[17] De nuptiis VIII, 859f. (ed. Dick, p. 452f.).
[18] How one reaches this result I do not know; perhaps one uses $\Delta\varphi$ (Meroe-Rhodes) $\approx 20°$. The latitudinal difference Meroe-Borysthenes is about 32°.
[19] Cf. also p. 654 and p. 664.
[1] Cf. above p. 695, n. 13.
[2] Ziegler, p. 30, 24.
[3] The correct value must be very near to 2 years 50 days, the period given in the subsequent list. Cf. also above p. 783.
[4] Ziegler, p. 34, 23–36, 1.
[5] Cf. above p. 671, n. 16.

The two elongations are roundings of little interest.[5a] The latitudes are more specific but do not agree too well with contemporary more accurate parameters.[6] The synodic periods, however, are very accurate as is easy to verify by means of the tables of mean anomaly in Alm. IX, 4, One would like to know where Cleomedes found these numbers which stand out from the mass of commonly given parameters.[7] in particular since they appear also in Varāhamihira's Pañcasiddhānkitā (sixth cent. A.D.) in sections which are of definite Babylonian origin.[8] In India in the case of Jupiter the last digit is not 8 but 9. hence the period 399^d is undoubtedly correct; also Mercury's period of $115;52,30^d$ $(=1/8 \cdot 927)$ is an improvement over 116^d.

In II, 3 Cleomedes states that no fixed star has an apparent diameter less than 1 finger (a rather absurd exaggeration) while the apparent diameter of Venus should be 2 fingers, i.e. 1/6 of the lunar or solar diameter.[9] Of some interest is the remark (also in II, 3) that the absolute size of fixed stars may reach, or even surpass, the sun. a statement which in part duplicates a discussion in I. 11 where it is said that the earth, seen from the sun, would appear at best as a very small star. Hence the stars must be larger than the sun.

§ 3. Pappus and Theon

Suidas says that Theon flourished in the time of Theodosius (379 to 395) and that Pappus was his contemporary.[1] Accurate data for Theon are, however. available from his commentaries to the Almagest and to the Handy Tables. Thus we know that he observed in Alexandria the solar eclipse of 364 June 16[2] and that he also discussed the subsequent lunar eclipse of Nov. 25.[3] It seems certain that he was no longer alive when his daughter Hypatia was murdered in 415. but he lived long enough to enjoy her collaboration in editing the commentary to Book III of the Almagest.[4]

Usener directed attention to a marginal note in a Royal Canon[5] at the entry for Diocletian (284 to 308): "at that time wrote Pappus." Similar marginal notes indicate the time of Ptolemy. Hipparchus, Timocharis. Meton and Euctemon.[6]

[5a] The same values are found in Theon of Smyrna (Hiller, p. 137, 3-5; 187, 10-13; Dupuis. p. 224, 15-17; 302, 5-7); cf. also above p. 804.

[6] Cf., e.g., below p. 1015f.

[7] Cf. above p. 783 f.

[8] Cf. Neugebauer-Pingree, Varāham. II, p. 109; also above p. 784. note 24.

[9] Cf. also above p. 693.

[1] Suidas, ed. Adler I. 2. p. 702, 10-16 and I. 4. p. 26, 3-7; the essential passages are also given in Monumenta 13. 3. p. 361. note 3.

[2] Text: Halma. H.T. I. p. 77-87; Tihon [1971] III, p. 114-130 and p. 165-183. Discussion: Rome [1950]; Tihon [1971] II. p. 346-445; [1973]. p. 49, p. 88.

[3] Cf. RE 5 A col. 2075. 57-60.

[4] Rome [1926]. p. 6. Cf. also above p. 872.

[5] Leidensis 78 fol. 55r: Usener, Kl. Schr. III, p. 21-23; Monumenta 13, 3, p. 360f., p. 449.

[6] Monumenta 13. 3. p. 360. It is an elementary mistake to interpret this note as saying that Pappus extended the Royal Canon "until" Diocletian (an interpretation accepted, e.g., also by Honigmann SK. p. 72ff.).

The only accurate date we have for Pappus was found by A. Rome[7] in Pappus' commentary to Book VI of the Almagest in a discussion of the solar eclipse of 320 Oct. 18, apparently written shortly before the event. Hence one may safely say that Pappus' commentaries to the Almagest precede Theon's by about forty years. For the "Collection" Rome suggested a date around 340.[8]

Pappus probably wrote commentaries on all thirteen books of the Almagest although only the commentaries to V and VI are extant.[9] There exist, however, traces of I and IV[10] and III is quoted in an anonymous introduction to Alm. I.[11] Arabic sources mention a commentary on Ptolemy's "Planisphaerium".[12] For a commentary to the "Analemma" of Diodorus cf. above V B 2, 2 C. Suidas also mentions a work on geography; the original is lost but Armenian versions are extant.[13] A commentary to the Handy Tables and one to Ptolemy's "Harmonics" were ascribed to Pappus on insufficient grounds.[14]

The relative order of Theon's astronomical commentaries can be established from internal references:[15] the commentaries on the Almagest are followed by the "Great Commentary on the Handy Tables" (in five books) and finally by the "Small Commentary" to these tables.[16] A marginal note in this last commentary[17] contains a reference to the year Diocletian 94, i.e. A.D. 377; hence we see that by that time even the latest of these commentaries was in the hand of some user.

Under Theon's name also goes a list of Roman consuls, year after year, from Antoninus (i.e. A.D. 138) to A.D. 372.[18] The initial year was perhaps chosen because it is the epoch year for Ptolemy's catalogue of stars. The list of names is preceded by four columns; the first two give the consecutive years of the eras Philip and Augustus. The third column is headed "epacts or intercalary days"[19] and gives the number of days by which the Egyptian Thoth 1 precedes the Alexandrian Thoth 1. Thus this number increases every four years by 1. The first two years of the list, Augustus 167 and 168, are given 40 as the value of this "epact." This shows that

$$\begin{aligned}&\text{Augustus } 5 \approx \text{epact } 0,\\&\text{Augustus } 9 \approx \text{epact } 1.\end{aligned} \tag{1}$$

[7] Rome, CA I, p. X–XIII, p. 180–183. Cf. also Rome [1939], p. 212 for an example concerning A.D. 323, perhaps taken by Theon from Pappus.
[8] Rome, CA I, p. 255, note.
[9] Edited by Rome, CA I.
[10] Rome, CA I, p. XVIII; p. 255, note.
[11] Cf. Rome, CA I, p. XVI; Mogenet [1951], p. 17; cf. below p. 1043.
[12] Rome, CA I, p. IX; Heath GM II, p. 357.
[13] R. H. Hewsen, The *Geography* of Pappus of Alexandria: a Translation of the Armenian Fragments. Isis 62 (1971), p. 186–207.
[14] Cf. above p. 838, n. 4 and p. 839, n. 10.
[15] Rome [1939], p. 213; Tihon [1971] I, p. VI f.
[16] This "Small Commentary" contains an example for Diocletian 77 Thoth 22 (= A.D. 360 Sept. 19); cf. Halma H. T. I, p. 31, 27; Tihon [1971] III, p. 12, 3.
[17] Halma H. T. I, p. 74; Tihon [1971] III, p. 107. The words "as is seen to happen in the year 94 of Diocletian at the conjunction in the Egyptian (month) Phamenoth" i.e. Nov. 377) cannot belong to the original text which describes only in general terms the procedure of finding the date of the mean conjunction. The above note slipped into the text at the end of the section on mean syzygies.
[18] Published as "Fasti Theonis Alexandrini" by Usener in Monumenta 13, 3, p. 375–381 from the only extant manuscript (Laur. 28, 26 fol. 40ʳ–43ᵛ).
[19] Two equivalent terms: ἐπακταὶ ἤτοι ἐμβόλιμοι.

Indeed in the year Augustus 9 the Alexandrian Thoth 1 corresponds for the first time to the Egyptian Thoth 2 (-21 Aug. 30). The fourth column repeats the sequence 1, 2, 3, 4 such that the "epact" always increases by 1 in a year associated with the number 4. Thus Augustus 6 is given the number 1.

Exactly the same term "epact or intercalary days" is found in Theon's Small Commentary[20] though used in a very different meaning. While the "epacts" in the "Fasti" represent a monotonically increasing sequence of numbers of purely calendaric significance the "epact" in the commentary is a basically astronomical concept; it measures the interval between the solar year and 12 lunar months.

Theon gives three rules for the determination of this epact, depending on the era in which the years are counted. Let N be the years of the era Philip, N_2 the corresponding years of Diocletian, N_3 of Augustus. Then Theon defines a residue r modulo 19 by

$$r \equiv \begin{cases} N \\ N_2 - 1 \\ N_3 + 9 \end{cases} \text{mod. } 19 \qquad (0 \leqq r < 19) \tag{1}$$

and with it the "epact" e by

$$e \equiv 11 \cdot r \text{ mod. } 30 \qquad (0 \leqq e < 30). \tag{2}$$

These rules give the same value for e for all three eras as follows from the intervals[21]

$$N_2 = N - 607, \qquad N_3 = N - 294. \tag{3}$$

Therefore, if $N \equiv r$, then $N_2 \equiv r + 1$ and $N_3 \equiv r - 9$ (mod. 19). q.e.d.

It follows from (1) and (2) that $r = 0$, thus also $e = 0$, for the years Augustus 10 and Diocletian 1. Now we have for the Alexandrian Thoth 1:

Augustus 10: -20 Aug. 29
Diocletian 1: 284 Aug. 29.

Both of these dates can be accepted as (mean) new moon dates since the first coincides with a true new moon, while the second is only one day late.[22] If, therefore, a near N has the epact e we may say that e gives the age of the moon on Alexandrian Thoth 1, and in particular that $e = 0$ for Diocletian 1 I 1.

If $e = 0$ characterizes new moons then $e = 15$ will indicate full moons. Since the whole method is based on assuming mean synodic months of 29 1/2 days each we can say that each additional month lowers the date of a syzygy by 1/2 day. Thus the new moon of months m $(m < 12)$ falls $e + \frac{m}{2}$ days before Thoth 1.[23] We shall meet this rule again in the 7th century in the writings of Stephanus.[24] Finally, the concept "epact" is taken over into the christian easter computus,[25] apparently in direct continuation of a tradition which goes back at least to the time of Theon.

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[20] Halma, H.T. I, p. 70f.; Tihon [1971], p. 97f.
[21] Cf. Halma, H.T. I, p. 31 or below VI A 2, 3.
[22] Cf. Goldstine, New and Full Moons.
[23] Cf. Halma, H.T. I, p. 71.
[24] Cf. below p. 1047.
[25] Cf. Rome [1950], p. 214; Ginzel, Hdb. III, p. 140ff., p. 302ff.

It seems to be a generally accepted assumption that the "Handy Tables" which are still at our disposal represent a version edited by Theon. As far as I know the only basis for this is a reference by Delambre to a heading found in Par. gr. 2399 and the title printed by Halma H.T. II, p. 1.[26] In fact, however, this manuscript does not contain the Handy Tables and the heading in question belongs to Theon's "Small Commentary." Hence nothing definitive can be said about a Theonic edition of the Handy Tables without an investigation of all extant manuscripts. It may be significant, however, that the late Neoplatonists, Proclus and his followers, never refer to Theon in connection with tables.[27]

Of Theon's introductions to the Handy Tables we now have a modern edition of the "Small Commentary" by A. Tihon [1971].[28] An edition of the "Large Commentary" is planned by J. Mogenet.[29] Of the commentaries to the Almagest the first four books are available in an edition by A. Rome.[30]

In general, neither Pappus' nor Theon's commentaries show any independence of thought in relation to the basic text. As A. Rome formulated it: "si les cours au Musée avaiant l'allure des commentaires de Pappus et de Theon, nous plaignons les étudiants."[31] Indeed, the dullness and pomposity of these school treatises is only too evident. When, e.g., Ptolemy in the chapter on the apparent diameter of sun, moon, and shadow (Alm. V, 14) simply remarks that the tangential cones in question contact the spheres within a negligible error in great circles, then Pappus refers[32] to Euclid's "Optics" to show that the circle of contact has a smaller diameter than the sphere, only to add a lengthy argument to demonstrate that the error committed in Ptolemy's construction is nevertheless negligible. Or, when Ptolemy says that some phenomenon cannot take place, neither for the same clima nor for different geographical latitudes, Pappus feels obliged to explain "same clima" by "either in clima 3, or in 4, or in any other clima," and to illustrate "different" by referring to "Rome or Alexandria."[33]

Occasionally Pappus gives interpretations which are outright wrong. For example he explains[34] the simple rounding of 0;25,15 to 0;25 as the result of replacing the value for the zenith distance $\varepsilon=24^\circ$ by the more accurate value $\varepsilon=23;50$, thus postulating a totally meaningless step which has no basis whatsoever in Ptolemy's text. Similarly a number is said to be the result of tabular interpolation (which it is not), only in order to obtain exactly Ptolemy's rounded number.[35]

It is not surprising to see that instructions in elementary mathematical procedures were compiled for the students. Thus we have both from Pappus and from Theon rules, e.g., for sexagesimal divisions. One of these procedures

[26] Delambre HAA II, p. 617; cf. also Halma H.T. I, p. 157.

[27] Cf. below p. 1044, n. 14 and 15; p. 1045.

[28] The earlier edition by Halma (1822) is unreliable but gives a French translation. For scholia cf. Tihon [1973].

[29] Tihon [1971] I, p. VI.

[30] Rome CA II and III; for the remaining books cf. Rome [1953]. The first two books were edited, with a French translation, by Halma (Paris 1821).

[31] Rome [1927], p. 77; similar [1939], p. 211.

[32] Rome CA I, p. 103, 14–105, 22. Cf. also above p. 640 for Pappus' discussion of a similar problem in Aristarchus concerning the moon's terminator.

[33] To Alm. VI, 6; Rome CA I, p. 234, 13 to 18.

[34] Rome CA I, p. 238, 1–4.

[35] Rome CA I, p. 236, 14. Cf. in general Rome CA I, p. X.

is based on the construction of a multiplication table for each denominator b in quotients a/b which one wishes to express sexagesimally.[36] Thus one prepares a table of multiples of b with factors from 2 to 10 and 20, 30, 40, 50. This is a revival of the Old-Babylonian tables of multiplication which operate with exactly the same multiples.

§ 4. The Handy Tables

οὐρανίων ἄστρων πορείην καὶ κύκλα σελήνης
ἐξεθέμην σελίδεσσι πολύφρονα δάκτυλα κάμπτων

Ptolemy, Opera II. p. CXLVIII Heib.
(also Vat. gr. 1291 fol. 47ᵛ)

1. Introduction

The "Handy Tables" represent one of the most important astronomical documents of antiquity. Here one finds all the tabular material which is needed, e.g., for the computation of ephemerides or of eclipses, in a form more convenient in arrangement and numerical detail than Ptolemy's earlier tables which are dispersed through the text of the Almagest. Consequently the influence of this new collection of tables was felt during the whole Byzantine and Islamic period, i.e. everywhere that Ptolemaic astronomy was kept alive. Through Islamic works sections of the Handy Tables eventually reached even the European West, e.g. in the Alfonsine Tables.[1]

Ptolemy's authorship of the Handy Tables is secured since we still have his own introduction[2] in which he gives instructions on how to use the tables. These instructions fit the actually extant tables, supposedly edited by Theon of Alexandria in the fourth century A.D., in all essential points[3]. Fortunately Theon was a competent but completely unoriginal scholar; hence he tampered only little with Ptolemy's material. Nevertheless the history of the Handy Tables" for the two centuries between Ptolemy and Theon is not without problems which require discussion.[4]

Unfortunately no critical edition of these tables exists and hence we shall most of the time refer (by [H]) to the text as published by Halma[5] although he utilized only few and late manuscripts.[6] Among the manuscripts not seen by

[36] Cf. for Theon: Rome [1926], p. 13f.; CA II, p. 451 ff.; for Pappus: Mogenet [1951].
[1] Cf., e.g., Nallino, Batt. II, p. 231 (ad p. 89).
[2] Greek text in Ptolemy, Opera II, p. 159 to 185, ed. Heiberg; cf. also p. X, p. CLXXV, p. CXC. Greek text and French translation (both unreliable) in Halma, H. T. I, p. 1 to 26.
[3] Cf. above p. 968.
[4] Below p. 973 ff. and p. 1044, n. 15.
[5] Halma, H.T. I, p. 109 to 155; II; III, p. 1 to 37.
[6] Halma's publication contains many more or less obvious errors (whether they are ancient or modern is not always clear) and one can never be sure whether titles of tables or headings of columns are Halma's invention or not, e.g. the "eighth clima" in H.T. II, p. 58 to 65; cf. also H.T. II, p. 2 with a Greek heading which translates "Manuscrit 2399"; etc.

Halma are the three earliest ones which date from the reigns of Leo V (813–820) and Leo VI (886–912), all unpublished (Laurent. XXVIII, 26; Leidensis gr. 78; Vat. gr. 1291). I made extensive use of photographs of one of them, the Vat. gr. 1291, henceforth called [V]. This famous uncial manuscript[7] of beautiful execution, written under Leo V, shares, however, some drastic errors with [H],[8] suggesting that all extant manuscripts depend on some already corrupt archetype.

Theon wrote two commentaries to the Handy Tables, probably designed to replace Ptolemy's concise introduction. The first the "Great Commentary" in five books, still unpublished[9]; later on[10] he wrote a "Little Commentary," published and translated by Halma, reedited by A. Tihon.[11] Another commentary was written by Stephanus[12] in the seventh century and as late as in the 14th century Theodoros Meliteniotes[13] felt the need for a new commentary which now also had to include "Persian Handy Tables."[14]

The earliest extant manuscripts of the Handy Tables contain tables specifically computed for the use in Byzantium. In [V], e.g., one finds four tables which supplement the corresponding tables for the seven climates[15]:

oblique ascensions[16]
parallaxes
planetary phases[17]
circular diagram for ortive amplitudes.

Excerpts made in the 6th century from these tables and commentaries are preserved in a late Latin version under the title "*preceptum canonis Ptolomei*," a work popular in the west during the whole Middle Ages as Honigmann has shown.[18]

The close relationship between the Handy Tables and the Almagest does not exclude important modifications in theoretical aspects as well as in computational details. We shall see, e.g., that in the planetary theory both latitudes and phases were handled in a new fashion.[19] In general a more uniform and denser distribution of the entries facilitates greatly interpolation, while chronological data can

[7] Cf. Nolhac [1887], p. 168f., Boll [1899], p. 112ff., Kubitschek, Kalenderb., p. 66–70, Schnabel [1930], p. 222f., and below p. 977f.

[8] Examples: [H] and [V] show the same interchange of headings which makes the tables for the phases of the inner planets almost useless (cf. above p. 260) and the same transposition of numbers for phases of Mars (below p. 1022) which definitely goes back to a common source. On the other hand [V] omitted 18 lines in the catalogue of stars (below p. 1027) extant in [H] (Halma H.T. III, p. 44 last 5 lines to p. 46, line 13); etc.

[9] Extant in Par. gr. 2450 and Vat. gr. 190; cf. Rome [1930], p. 211, n. 2; Mogenet [1962], p. 206, n. 19.

[10] Reference from the "Little Commentary" to the "Great" one: Halma, H.T. I, p. 27.

[11] Halma H.T. I, p. 27 to 105; Tihon [1971] I.

[12] Usener, Kl. Schr. III, p. 295ff.; cf. below V C 5, 2 B 5.

[13] Theodoros died around 1390 (cf. Beck, Lit., p. 792); cf. also Ševčenko, Métoch., p. 114f.

[14] Cf. Boll [1894], p. 54, note 3. Text: Migne PG 149 col. 965/966.

[15] [V] fol. 5^r–6^v, 7, 8^r, and 3^r, respectively, out of place and out of order in the present manuscript which begins with fol. 9^r; cf. below p. 978.

[16] Not contained in Par. gr. 2399 (cf. Halma, H.T. II, p. 58/59).

[17] Cf. below p. 1024.

[18] Honigmann, S.K., p. 102–107. The date is taken from an example for A.D. 534; the original authorship is shown by a "*sic theon docet.*" Cf. also van de Vyver [1936], p. 687.

[19] Cf. below p. 1006ff. and p. 1017ff.

be used directly as entries instead of the "completed" intervals in the Almagest.[20] The Era Nabonassar (−746) was useful in the Almagest for the discussion of basic early observations; the Handy Tables are based on the Era Philip (−323) but extend the tables for mean motions in 25-year steps over one complete Sothic period,[21] i.e. to the year Philip 1476 (Thoth 1 = A.D. 1151 Nov. 9). This extension far into the future no doubt contributed to the usefulness of the Handy Tables during the Middle Ages, if only as a ready means for comparing computed or observed data with the Ptolemaic theory.

In most of the tables numbers are given to minutes only. It is clear that the mean motion tables of the Almagest with six sexagesimal digits were never used to that extent. The Handy Tables moved to the opposite extreme and thus unnecessarily reduced the accuracy of the tables. In order to provide results with an accuracy to minutes the constituent tables should have included seconds. But even for the moon we find only minutes. This makes no sense if at the same time the effects of precession or of the equation of time are taken into consideration. In this context it is of interest that we have fragments of planetary tables from the time before Theon which give mean motions down to seconds.[22] Is the reduction to minutes an innovation made by Theon?

In comparison with the Almagest one can observe in the Handy Tables an increasing interest in geographical problems. Not only was a list of nearly 400 "Important Cities" with their coordinates added; but the table of parallaxes and the tables of planetary phases were now enlarged to tables for the seven canonical climata while another new table allows the quick shift from longest daylight to geographical latitude.[23] All this agrees with the general sequence of Ptolemy's work.[24]

A. Arrangement

A list of topics covered by the Handy Tables has been compiled by Heiberg on the basis of Ptolemy's introduction and numbered from 1 to 21.[1] Unfortunately this numbering is not concordant with the counting from 1 to 21 of the sections of Ptolemy's text, also introduced by Heiberg. The first of these two

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[20] Cf. for an example below p. 984. Only the tables for mean syzygies (Alm. VI, 3) are constructed for entries with the given date (cf. above p. 118).

[21] Ideler, as far back as 1806, realized (Astron. Beob., p. 296, note 1) that the covering of one Sothic period determined the size of the tables of mean motions; he, in turn, refers to Syncellus, Chronogr. (p. 503, 5–8 Dindorf) ≈ A.D. 800. This also leads to the explanation of the 25-year steps which have nothing to do with the synodic lunar period of 25 years since the planetary tables also are arranged in the same way. We know from Ptolemy's own statement that he had chosen the 18-year steps in the Almagest in order to fit his tables into columns of about 45 lines (cf. above p. 55). For the Handy Tables he must have decided on the common format of about 30 lines per column; this is convenient for single degrees in zodiacal signs, for 12 + 12 months, for 30 days, and 24 hours. If one then wishes to tabulate one Sothic period in two columns one needs steps of 1461/60 ≈ 25 years, and again one column will accommodate the 25 lines for the single years. Hence it is only a matter of convenient arrangement that determined the 25-year steps.

[22] Cf. below p. 975.

[23] Cf. below p. 980f.

[24] Cf. above p. 834f.

[1] Ptolemy, Opera II, p. CXCf., also accepted by Rome, CA I, p. LIff.

ways of counting was accepted by van der Waerden and coordinated with Halma's edition.[2]

At present it is impossible to establish the original arrangement of the tables. A future edition will probably have to adopt some arbitrary order of the tables and assign them numbers in order to constitute at least a definite system of references. Ptolemy's introduction does not establish an accurate sequence of the tables since he discusses the methods of computation in a different order than the tables, e.g. lunar latitudes followed by planetary latitudes. For us it is probably best to give a survey of the contents of the tables according to subject matter in an order which may not deviate too far from the original arrangement.

Auxiliary tables:
  geographical coordinates of important cities[3]: [V] fol. 17^{v}–21^{v}
  royal canon: [H] I, p. 139–143, [V] 16^{v}, 17^{r}
  stellar coordinates: [H] III 44–58, [V] 90^{v}–94^{v}
Spherical astronomy:
  right ascensions: [H] I 148–155, [V] 22^{r}–23^{v}
  oblique ascensions for 7 climata: [H] II 2–65, [V] 24^{r}–37^{v}
  solar declinations
  lunar latitudes } [H] I 144, 145, [V] 44^{r}, 45^{v}, 46^{r}, 47^{v}
  length of daylight and geographical latitude: [H] I 132, 133, II 96, 97, [V] 44^{v}
  diagram for ortive amplitudes for 7 climata: [H] III 43, [V] 45^{r}
  equation of time: [H] I 148–155, [V] 22^{r}–23^{v}
Mean motions and equations:
  sun, mean motions: [H] II 66–77, [V] 38^{v}–40^{v}
    equations: [H] II 78–89, [V] 41^{r}–43^{v}
  moon, mean motions: [H] II 66–77, [V] 38^{v}–40^{v}
    equations: [H] II 78–89, [V] 41^{r}–43^{v}
  planets, mean motions: [H] II 112–133, [V] 57^{r}–62^{v}
    equations: [H] II 134–193, [V] 63^{r}–77^{v}
    latitudes: [H] III 1–10, [V] 78^{r}–82^{v}
Planetary phenomena:
  stations: [H] III 11–15, [V] 83^{r}–85^{r}
  max. elongations: [H] III 32, [V] 90^{r}
  phases for 7 climata: [H] III 16–29, [V] 85^{v}–88^{v}
Eclipses:
  parallax for 7 climata: [H] II 98–111, [V] 50^{r}–56^{v}
  correction for parallax: [H] I 146, 147, [V] 48^{v}
  inclinations: [H] I 146, 147, II 94, [V] 48^{v}
  eclipse magnitudes: [H] II 90, 91; 94, 95, [V] 49^{r}, 49^{v}

The material enumerated here certainly belongs to the "Handy Tables" as designed by Ptolemy. Except for minor details in arrangement all of it is necessary for dealing with the astronomical problems discussed in the Almagest. Some tables, however, found in several of our manuscripts, are probably not

[2] Van der Waerden [1958, 1], p. 55.
[3] Not the "ἔκθεσις" given in Halma H.T. I, p. 109–131 (cf. above p. 939), but the "Important Cities" (Honigmann S.K., p. 193–208).

part of the original plan. For example both [H] and [V] give also the tables of planetary phases for the latitude of Phoenicia, known from Alm. XIII, 10 (excepting Venus).[4] Similary both texts contain a table of right ascensions as function of λ.[5] A "lunarium",[6] i.e. a schematic table to estimate the position of the moon in the signs of the zodiac during the single days of the year, is certainly not of Ptolemaic origin. This primitive pattern is far below the level of Greek or Babylonian mathematical astronomy and it is hard to see how such a device could get incorporated with the Handy Tables.

The three earliest manuscripts[7] share large comparative calendars which permit the transposition of dates from local calendars of the eastern Roman provinces to the Alexandrian or Roman calendar. Kubitschek who edited these three calendar tables assumed that tables of the same type were already joined with the Handy Tables by Ptolemy[8] from similar lists at his disposal. This, however, seems rather unlikely since Ptolemy makes no mention of such tables (which are also absent from [H]) in his introduction.

It should be noted that the Handy Tables do not contain any specific astrological material, in marked contrast to common practice in mediaeval works.[9]

B. Variants in the Handy Tables

When we speak about "Handy Tables" we mean a collection of tables based on the methods developed in the Almagest and arranged according to the principles described in Ptolemy's introduction and explicitly known from the "Theonic" version. Hence we exclude from our present discussion tables (or perhaps even collection of tables) based on other, earlier, methods, i.e. fragments of which have come down to us on papyri[1], as well as tables computed for specific dates, e.g. the "Almanacs."[2] In spite of this restriction to authentic Ptolemaic material it is by no means evident that Ptolemy's "Handy Tables" did not circulate in different versions. We know from Ptolemy's extant writings that he was continuously at work, changing parameters, varying methods, and expanding his field of interest. The two centuries between Ptolemy and Theon may also have left their imprint on a collection of tables compiled for the convenient use of the practitioner.

Our material is much too fragmentary to reveal details of such processes. But there are enough traces, however isolated, of essential variants to shake our confidence in the assumption that we have in the extant version "the" Handy Tables of Ptolemy. The instances enumerated in the following deserve at least some attention as witnesses for some changes in the structure of the Handy Tables.

Geographical Data. We know that Ptolemy inaugurated the change from time coordinates (local time in relation to Alexandria and longest daylight) to geographical longitudes (reckoned from the Fortunate Islands) and latitudes.

[4] Cf. above p. 258.
[5] Cf. below p. 978 ([H] II. p. 92, 93. [V] fol. 48'); p. 981.
[6] [H] III. p. 36, 37; [V] fol. 9'; also Vat. gr. 208 fol. 124' (unpublished).
[7] Mentioned above p. 970.
[8] Kubitschek, Kalenderb., p. 79 and p. 75.
[9] Cf., e.g. Kennedy, Survey, p. 144 f.
[1] Cf. above V C 2, 2.
[2] Cf. below V C 5, 3.

Both types of coordinates appear in "Handy Tables" but not combined, at least as far as I know. And a fragment from a papyrus codex of the 3rd century (P. Ryl. 522 + 523 [3]) shows that the list of "Important Cities" known from [V] admits of earlier variants. Similarly for the time coordinates in [H]: the arrangement in the earlier papyrus version (P. Lond. 1278 [4]) is slightly different from [H] (cf. above p. 939 and note 11 there). For the intricate problem concerning the different versions of the "Important Cities" cf. Honigmann, S.K., p. 71.

Oblique Ascensions. Again papyri demonstrate the existence of variants which differ from the Theonic version (cf. below p. 980) both with respect to numerical data and arrangement (cf. below p. 980), e.g. by omitting the columns for seasonal hours, thus following the Almagest rather than the Theonic version.

Equation of Time. The fact that P. Lond. 1278 uses for the equation of time the Era Nabonassar (cf. below p. 985) and not the Era Philip which is otherwise typical for the Handy Tables may perhaps indicate the existence of an earlier type of tables which are still based on the Almagest itself.

Lunar Velocity. In [V] (fol. 46ʳ) one finds a table for the hourly lunar velocity as function of the anomaly α, computed by means of coefficients $c(\alpha)$ listed in a special table (fol. 48ᵛ) [5] headed *προκανόνιον* rounded from a table called in Alm. VI, 8 *διορθώσεως κανόνιον*. [6] In [H] (I, p. 146/147) the table of $c(\alpha)$ is combined under the common title *προκανόνιον* with a table which is again preserved by itself in [V] (fol. 48ᵛ) [7] under the heading *κανόνιον διορθώσεως*, but concerns parallaxes.

Parallax. The tables for parallax in the Handy Tables are based on the difference between lunar and solar parallax and a special rule is given for how to increase the tabulated amount if the lunar parallax alone is desired. [8] Some manuscripts nevertheless contain a table for solar parallaxes alone [9] and could again be a residue from a version still closer to the Almagest.

The tables for correcting the parallax corresponding to arbitrary values of the anomaly α and the elongation $\bar{\eta}$ [10] seem to be contracted in all our manuscripts into one single table under the above-mentioned title *κανόνιον διορθώσεως*. Apparently it was no longer understood that there were actually two functions tabulated. We do not know when these tables were combined with the tables of the lunar velocity, but it was obviously done in order to save the repetition of the columns for the argument.

Planetary Mean Motions. Fragment 4 of P. Lond. 1278 [11] has preserved some tables of mean motions for Saturn which show interesting variants. Table 30

[3] Ryl. Pap. III. p. 142–146. The geographical table is followed by a table of oblique ascensions for which cf. below p. .

[4] Cf. Neugebauer [1958. 1], p. 94–97.

[5] Also in Vat. gr. 208 fol. 111ʳ.

[6] Heiberg I, p. 522.

[7] Also in Vat. gr. 208 fol. 109ʳ.

[8] Cf. below p. 990.

[9] Cf. below p. 999 f.

[10] Cf. below p. 994 ff.

[11] Neugebauer [1958. 1], p. 106–109.

Table 30

A	B	C	D	E	F
[1]	[211. 1.46]	[79.25]	[1]	[347.32, 0]	[155.24]
[26]	[156,21.]	[25, 0]	[2]	[334, 4, 1]	[310.47]
[51]	[101,41.]	[330,35]	[3]	[322,36, 2]	[106.11]
[76]	[47. 1,]	[276,10]	[4]	[310, 8, 3]	[261.34]
[101]	[352.21.]	[221.45]	[5]	[297,40, 4]	[56.58]
	etc.			etc.	
626	284,21,10	159, 0	[21]	98,12,17	2[3. 16]
651	229,41, 9	104,3[5]	[22]	85,44,17	178.[39]
[676]	[175,]1, 8	[50, 10]	[23]	73.16,18	334. 3
[701]	[120.21, 6]	[5, 45]	[24]	60,48,19	129.[26]
[726]	[65,41, 5]	[310, 20]	[25]	48.20.20	284, 50

gives beginnings and ends of six columns from this text. Column A shows the 25-year steps characteristic for the Handy Tables, 30 lines per page; obviously one may restore a second set of the columns A, B and C to complete the usual 1476 years.[12] Column B gives the mean motion of the center of the epicycle of Saturn with respect to the sidereally fixed apogee. The epoch value 211;1,46° agrees with the corresponding value 211;2° for the year Philip 1 found in [H] and also the subsequent values agree in essence though the papyrus gives minutes and seconds against minutes only in [H] which remains in its roundings consistently by about 0;0,15° above the papyrus text. Column C is not found in [H]; it gives the longitudes, now reckoned from ♈0°, of the center of the epicycle, beginning for Philip 1 with ♊19;25 in good agreement with the position resulting from Theon's version, i.e. ♊19;26. It is interesting to note that we unquestionably have here "Handy Tables" which are neither in their parameters nor in their arrangement identical with the Theonic version.

Column D gives the single years from 1 to 25, E the mean anomaly of Saturn in agreement with [H], but again tabulated to seconds. The yearly increment of 155;23,36° in column F is a complete mystery attested in no other text and unrelated to any mean motion which concerns Saturn (or any other planet).

Planetary Phases. In I C 8, 5 we described the far reaching differences between the Handy Tables for the planetary phases (in the seven climata) and the tables in Alm. XIII, 10. Nevertheless we know of Islamic tables which seem to be based on the methods of the Almagest but extended to the seven climata (cf. p. 260). Once again one might see here an indication for the existence of Handy Tables nearer to the Almagest than the extant Theonic version.

The latter incorporated the tables for Phoenicia from the Almagest (cf. p. 258) but replaced the table for the phases of Venus with a table which seems to be computed for Alexandria. All this shows that it is not justified to assume a priori stability in the tradition of the Handy Tables, because even the fragmentary character of our sources suffices to demonstrate the existence of variants which are not explicable as ordinary scribal modifications.

[12] Cf. above p. 971.

C. Bibliography

To the best of my knowledge Henry Dodwell was the first to draw attention to the Handy Tables as an independent work (in his Dissertationes Cyprianicae, Oxford 1684[1]). It was, however, mainly the Royal Canon which interested Dodwell in connection with his chronological studies. This remained the situation until Ideler.[2]

The astronomical contents of the Handy Tables was made known and competently discussed by Delambre at the end of his Histoire de l'Astronomie Ancienne[3] (1817), to be followed soon by the first (and still only) edition of text and tables by Halma (1822–1825).

The next hundred years brought little, if any progress. Boll, in his study of 1899 on the "Ueberlieferungsgeschichte der griechischen Astrologie und Astronomie" added the Vat. gr. 1291 (our [V]) to the discussion[4] but without going into astronomical details and introducing a much too early date for the Helios miniature.[5] In 1915 Kubitschek published a learned and voluminous study on the calendaric concordances which are found in some MSS,[6] while Schnabel in 1930 described the earliest extant MSS[7] and discussed their list of "Important Cities"[8] in relation to Ptolemy's "Geography." Schnabel had planned an edition of the Handy Tables but died before the completion of a work for which he in any case lacked competence.

The astronomical and historical significance of the Handy Tables was brought into proper perspective by the excellent notes of A. Rome in his edition of the Commentaries by Pappus and Theon to the Almagest.[9] Based on Rome's work are the detailed studies on the theory and the use of the tables by van der Waerden.[10] Finally A. Aaboe opened the way to the understanding of the tables for the planetary phases.[11] As usual Nallino's penetrating investigations of Islamic astronomy contributed to the clarification of the structure of the Handy Tables.

D. Appendix. Notes on Manuscripts

The following is only a preliminary collection of information which may serve as a lead to sources concerning the Handy Tables, used to some extent in the last 150 years.

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[1] Scaliger knew at least the Royal Canon (Isag. Can. III, 1606); cf. Fabricius, Bibl. Gr. (1796), p. 289. Also Bainbridge, Ptol., p. 47ff.

[2] Ideler, Astr. Beob. (1806), p. 37–64; he discussed also the tables of mean motions (p. 293–299) but had no more material at his disposal.

[3] HAA II, p. 616–631.

[4] Boll [1899], p. 110–138.

[5] Cf. below p. 978, note 3. Boll even seems to have accepted the star maps, later added on fol. 2ᵛ and 4ᵛ, as belonging to the original manuscript.

[6] Cf. above p. 973. Kubitschek invented for these tables the term "Kalenderbücher" as if they had some independent existence.

[7] Schnabel [1930], p. 10ff.

[8] These lists were published by Honigmann S. K., p. 193ff.

[9] In particular Rome CA III (1943).

[10] Van der Waerden [1953], [1958, 1] and his article on Ptolemy in RE 23, 2 col. 1823–1827 (1952).

[11] Cf. below V C 4, 5 C.

1. **General.** For the manuscripts which contain Ptolemy's "*Introduction*" to the Handy Tables cf. Heiberg in Ptolemy. Opera II. p. X f. and the edition p. 159–185.

Numerous manuscripts of the "*Handy Tables*" are extant, of course all in the "Theonic" version, though often indiscriminately called in the modern literature "Ptolemy's Handy Tables." The four oldest one. from the 9th century, are described by Schnabel [1930], p. 222 f.; one of them is Vat. gr. 1291 for which cf. below. Schnabel states that he knows of 40, more recent, manuscripts but gives no details.[1] Halma based his publication on Par. gr. 2394. 2399, and 2493.[2]

Fragments of papyrus codices from the time between Ptolemy and Theon: P. Lond. 1278 and P. Ryl. 522+523.[3]

The last mentioned three codices from Paris also contain the "*Little Commentary*" by Theon, published by Halma H.T. I. p. 27–105. The same work is also found, e.g., in Vat. gr. 175[4] and Vat. gr. 208.[5] Cf. Tihon [1971].

Theon's "*Great Commentary*" (unpublished) is extant in Par. gr. 2450 and Vat. gr. 190.[6]

Schnabel claimed that he had discovered four manuscripts of a Latin translation of the Handy Tables "die schon aus dem 6. Jahrhundert stammt" but he was unable to identify these manuscripts some years later.[7] Obviously he had discovered the "*preceptum canonis Ptolomei*" which contains an example for the year 534.[8] For the Handy Tables these late excerpts are of little interest.

2. **Vat. gr. 1291** (= [V]). Because I have made extensive use of this unpublished manuscript I shall at least give a table of contents, following the present order of the folios which is, however, certainly not the original one.[1] At present the beginning is made with the tables which concern the latitude of Byzantium:

fol. 3ʳ:	ortive amplitudes
5ʳ–6ᵛ:	oblique ascensions [2]
7:	parallaxes
8:	planetary phases.

[1] For Vat. gr. 208 which contains on foll. 23 to 132 the Handy Tables cf. Cod. Vat. gr. I. p. 255 f.; also Neugebauer [1958, 2].

[2] Halma H.T. I, p. XII ff.; cf. Omont BN. p. 252 f., p. 270. Par. gr. 2399 was used for Delambre's discussion (HAA II. p. 616). Halma's references H.T. II, p. 58/59 to Par. gr. 2463 and 3493 are misprints for 2493.

[3] Cf. above p. 974.

[4] Cod. Vat. gr. I. p. 199. One page (Halma H.T. I. p. 59/60) is reproduced by Turyn, Cod. gr. Vat. Pl. 97.

[5] Cf. above note 1.

[6] Cf. Rome [1939]. p. 211, note 2; Mogenet [1962], p. 206, note 19.

[7] Schnabel. T. u. K.. p. 5 and Nachtrag. p. 128.

[8] Cf. above p. 970, note 18.

[1] In the following I refer only to those pages which belong to the original uncial manuscript of the 10th century. There are later additions which have no relation to the "Handy Tables," e.g. the star maps on fol. 2ᵛ and fol. 4ᵛ. A comparatively complete table of contents was first given by Boll [1899], p. 112–115, with remarks on the history of the manuscript. Schnabel [1930], p. 222 f. discussed (not without mistakes) the present arrangement of the folios. For the contents of three additional pages (fol. 1 and 95ᵛ) cf. Mogenet [1969].

[2] Fol. 5ʳ shows the code of arms of Domenico Domenici, bishop of Breschia in 1465. This folio must have been the beginning of the manuscript in the 15th century. Cf. also Nolhac [1887], p. 168. Fol. 3 is probably out of place and belongs after fol. 8. On fol. 4ᵛ one finds a note, written by the hand of Domenico Domenici "Tabule in astronomiam persice sive caldaice."

The original beginning was undoubtedly fol. 9^r with the famous Helios miniature,[3] Fol. 9^v is a lunarium,[4] 10^r to 15^v give a concordance for a variety of lunar calendars with the Roman and the Alexandrian calendar.[5] What follows on fol. 16^v to 46^r is basically material from the Handy Tables of Ptolemy and Theon:

fol. 16^v, 17^r: royal canon
17^v–21^v: important cities
22^r–23^v: right ascensions (α'); equation of time
24^r–37^v: oblique ascensions for climata I to VII
38^r–43^v: mean motions and equations for sun and moon
44^r: solar declination, lunar latitude (cf. 45^v)
44^v: excess of longest daylight over 12^h as function of φ
45^r: diagram of ortive amplitudes for the seven climata
45^v, 46^r: lunar latitude (cf. 44^r); hourly lunar velocity.

What follows can hardly be part of the original (Theonic) tables:

46^v: circular diagram of unknown purpose (36 decans?), incomplete
47^r: circular diagram for "epacts" from Diocletian 30 to 257 (A.D. 313/4 to 540/1) with an example for Diocl. 239 (A.D. 522/3). The same diagram is found in several manuscripts, following notes on the Handy Tables[6].
47^v: circular diagram for "steps", supposedly for Mercury. Cf. above p. 672.

This last page contains the epigram which is heading our present chapter (above p. 969). It marks, most likely, the final page of the original manuscript. What constitutes today folios 48^r to 94^v is, however, again genuine material of the Handy Tables and should perhaps follow directly fol. 44^v.

48^r: correction of length of daylight, α and $\sin\alpha$ as function of λ
48^v: κανόνιον προσνεύσεων; correction for parallax
49 : eclipse magnitudes
50^r–56^v: parallaxes for climata I to VII
57^r–88^v: planets (cf. above p. 972)
89 : planetary phases, latitude of Phoenicia (=Alm. XIII, 10)
90^r: maximum elongations of Venus and Mercury
90^v–94^v: catalogue of brighter stars near the ecliptic.

[3] Reproduced in color in the Enciclopedia dell'arte antica, classica e orientale, Vol. IV facing p. 1046. For the date, 8th or 9th century, cf. van der Waerden [1954, 1], correcting Boll [1899].
[4] Cf. above p. 973.
[5] Cf. above p. 973.
[6] Cf. Tihon [1973], p. 5, note 2; e.g. Marc. gr. 314 fol. 223^r.

2. Spherical Astronomy

The tables for solar declinations[1] ($\varepsilon = 23;51°$) are not only less accurate (two digits against three) than the corresponding tables in the Almagest (I, 15) but also less convenient because of their 3°-steps instead of the single degrees of the argument in the Almagest. This is a curious and inexplicable reversal of the ordinary relation between Almagest and Handy Tables.

The arguments are reckoned from solstice to solstice, not from an equinox as in the Almagest. Combined with the table of solar declinations is a table for lunar latitudes,[2] beginning, as in Alm. V, 8 column 7, with the northernmost point of the lunar orbit[3] ($i = 5;0°$).

A. Rising Times

In discussing the computation in the Almagest of right ascensions we have seen[4] that the right ascension of the culminating point M of the ecliptic is related to the oblique ascension $\rho(\mathrm{H})$ of the ascendant by

$$\rho(\mathrm{H}) = \alpha(\mathrm{M}) + 90°.$$

It is obviously for this reason that the Handy Tables directly tabulate[5] the "*normed right ascension*"

$$\alpha'(\lambda) = \alpha(\lambda) + 90°,$$

beginning with $\alpha' = 0°$ at $\lambda = ♑ 0°$.

Example: Given the ascendant H = ♒ 28° in climate III; find the culminating point M. From the tables one obtains

$$\rho(\mathrm{H}) = 337;37°$$

to which value corresponds in the table for sphaera recta (by interpolation)

$$337;37° = \alpha'(♐ 9;21,34).$$

Hence M ≈ ♐ 9;22.

In one of the fragments of P. Lond. 1278 is found a table of right ascensions α themselves (i.e. not α'), as function of λ; combined with it is a table which is, in effect, a table of $\sin \alpha$.[6] This latter table is also found in [V][7] and, with one sexagesimal digit more, in [H].[8] We shall see[9] that such tables are convenient for the determination of the "ascensional differences."

In [H] and [V] the tables of the (normed) right ascensions α' are followed by the tables of oblique ascensions,[10] tabulated for single degrees of λ for each

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[1] [H] I, p. 144/5; [V] fol. 44. Ptolemy, Introd. No. 11 (Heiberg II, p. 170, Halma H.T. I, p. 12).

[2] Ptolemy, Introd. No. 12 (Heiberg II, p. 171, Halma H.T. I, p. 13).

[3] In [H] (not in [V]) are also given the numbers for the "steps" of 15° each (cf. above p. 669), not understood by Halma and therefore placed almost at random.

[4] Cf. p. 42.

[5] [H] I, p. 148 to 155; [V] fol. 22/23.

[6] Cf. Neugebauer [1958, 1], p. 103.

[7] [V] fol. 48r.

[8] [H] II, p. 92/93.

[9] Below p. 981.

[10] [H] II, p. 2–57; [V] fol. 24–37.

one of the seven climata, i.e. from $\varphi=16;27^\circ$ $M=13^h$ (Meroe) to $\varphi=48^\circ$ $M=16^h$ (Borysthenes).[11]

Another fragment from a papyrus codex, P. Ryl. 522+523,[12] preserves parts of the tables of oblique ascensions for the climata IV, V, and VI. In about half of the cases the minutes differ from the values in [V] and [H] by 1 to 3 units, consistently toward smaller values.

Fragments of the rising times for the climata I and II are found in P. Lond. 1278,[13] tabulated for intervals of 10°. The preserved parts agree exactly with the tables in Alm. II, 8, even in so far as the totals are listed, a feature not found in the Theonic version of the Handy Tables.

B. Seasonal Hours; Ascensional Differences

Neither one of the above mentioned papyri combines the length of seasonal hours with the oblique ascensions as is done in the extant Handy Tables.[14] There the columns for $\rho(\lambda)$ are followed by a column which gives, for the same λ, the length (in degrees) of one seasonal hour of daylight. For climate III, e.g., we find for $\lambda=\text{♊}\,30^\circ(=\text{♋}\,0^\circ)$ an hour of 17;30°, hence as greatest length of daylight $M=12\cdot 17;30^\circ=3,30^\circ=14^h$ as it should be.

This table is also useful for converting seasonal in equinoctial hours and vice versa. Assume, e.g., a solar longitude $\lambda_\odot=\text{♎}\,8^\circ$ when $\mathrm{H}=\text{♒}\,28^\circ$ is rising (cf. p. 979). Then the tables give for climate III

$$\rho(\lambda_\odot)=189;12^\circ, \qquad \rho(\mathrm{H})=337;37^\circ$$

hence a right ascensional difference $\Delta\alpha=\rho(\mathrm{H})-\rho(\lambda_\odot)=148;25^\circ$. The same tables give for our $\lambda_\odot$ as length of one seasonal hour 14;41°. Consequently the given moment is $148;25/14;41\approx 10;6$ seasonal hours after sunrise, or $148;25/15\approx 9;54$ equinoctial hours.[15]

A table,[16] not found of such length in the Almagest, relates the "altitude of the pole," i.e. the geographical latitude φ, to the "excess of hours" ΔM of the longest daylight over 12^h:

	φ	ΔM		φ	ΔM
beginning:			end:		
	1; 8°	0; 4^h		56;57°	5;44^h
	2;16	0; 8		57;13	5;48
	3;24	0;12		57;29	5;52
	4;32	0;16		57;45	5;56
	5;39	0;20		58; 0	6; 0.

Obviously this table is constructed for a linear increase of ΔM by $0;4^h$ per line. The limits are $M=12+0=12^h$ (equator) and $M=12+6=18^h$ (climate VII).

[11] The corresponding table for Byzantium ($\varphi=43;35^\circ$ $M=15\,1/4^h$) is given in [H] II, p. 58-65, [V] fol. 5, 6. The heading "eighth climate" is Halma's invention.
[12] Ryl. Pap. III. p. 147-150. For the preceding geographical tables cf. above p. 974.
[13] Neugebauer [1958, 1], p. 105.
[14] [H] II, p. 2-57 (Byzantium: p. 58-65); [V] fol. 22-37 (Byz.: fol. 5, 6).
[15] This example is taken from a horoscope for A.D. 260 (Neugebauer-Van Hoesen, Gr. Hor., p. 60f.).
[16] [H] I, p. 132/3 and again II, p. 96/7; [V] fol. 44^v. Error common to [H] and [V]: between 9;4° and 25;6° all values of φ are 1° too low.

The method of deriving φ from M must be the same as in the Almagest since the values in the new table agree (with a few minor and irregular exceptions) with the data in Alm. II, 6.[17]

A piece of P. Lond. 1278 [18] brings to light a variant of the above table for φ and ΔM but there are added two more columns, both of constant difference and beginning with 0 at $\varphi = 0°$. The following sample will suffice to show the arrangement:

φ	ΔM	(a)	(b)
51;19°	4;28^h	5;35°	33;30°
51;41	4;32	5;40	34; 0
52; 3	4;36	5;45	34;30.

The difference of 0;5° in column (a) can be interpreted as $0;5° = 1/12 \cdot 15 \cdot 0;4^h$. Hence (a) gives the excess in degrees of the longest seasonal hour (at the latitude φ) over $15° = 1$ equinoctial hours. Similarly we can write the difference in (b) as $0;30° = 1/2 \cdot 15 \cdot 0;4^h$. Hence (b) gives the excess over 90° of the longest day-arc of the sun, i.e.

$$n_0 = 1/2\,M - 90°, \tag{1}$$

a quantity which is the "ascensional difference" for the longest daylight (represented in Fig. 30, p. 1215 by EG [19]).

On p. 36 we have shown how Ptolemy determined in Alm. II, 7 the ascensional differences $n(\lambda)$ for any point of longitude λ of the ecliptic by means of a relation which is the equivalent of

$$\sin n = \sin n_0 \cot \varepsilon \tan \delta \tag{2}$$

where $\delta(\lambda)$ is the declination of the point λ. In the right triangle formed by the ecliptic arc λ the corresponding right ascension α and the declination δ, holds

$$\cot \varepsilon \cdot \tan \delta = \sin \alpha. \tag{3}$$

Consequently with (2)

$$\sin n = \sin n_0 \sin \alpha. \tag{4}$$

Our papyrus shows that $n_0(\varphi)$ had been tabulated (column (b)). It is therefore not surprising that the same text also contained tables for α and $\sin \alpha$, although it is evident that Ptolemy did not realize that he could have taken the coefficients

$$\sin \alpha = \frac{\sin n}{\sin n_0} = \frac{\operatorname{crd} 2n}{\operatorname{crd} 2n_0} = 1/2 \operatorname{crd} 2\alpha \tag{5}$$

directly from his table of chords instead of performing the divisions indicated in (5).[20]

The values resulting from (5) are tabulated for each single degree of longitude for one quadrant,[21] not only in the London papyrus but also in [H] and [V].[22]

[17] Cf. above Table 2 p. 44 and p. 43, note 3.
[18] Neugebauer [1958, 1], p. 95ff.
[19] We now denote by n_0 the total EG (called $n = n_1 + n_2$ in I A 4, 3) and by $n(\lambda)$ the quantity previously called $n_2 = EQ$ in Fig. 30. Cf. also p. 865.
[20] Alm. II, 7 (Manitius I, p. 90).
[21] From $\lambda = 0°$ to 90°, hence headed "degrees of the sun from (the nearest) equinox."
[22] [H] II, p. 92f.; [V] fol. 48^r.

Table 31

1	2	3	4	5	6	7	8	9	1
	Alm. II, 8					P. Lond.	Battānī		
λ	α	sin α	Alm. II, 7	[H]	[V]	1278	α	sin α	λ
0	0	0	0	0	0		0	0	0
10	9;10	0; 9,33,30	0; 9,33	0; 9, 0	0; 9		9;10,46	0; 9,34,17	10
20	18;25	18,57,20	18,57	18,48	18	18	18;26,51	18,57,10	20
30	27;50	28, 0,51	28, 1	26,48	27		27;53, 4	28, 3,41	30
40	37;30	36,31,32	36,33 !	34,48	35	35	37;33,40	36,34,36	40
50	47;28	44,12,47	44,12 !	42,28	43	43	47;31,25	44,15,12	50
60	57;44	50,44, 4	50,44	49,28	50		57;47,26	50,45,56	60
70	68;18	55,44,53	55,45	55,32	55	55	68;20,24	55,45,50	70
80	79; 5	58,54,51	58,55	58,48	59	59	79; 6,34	58,55,12	80
90	90	1; 0	1; 0	1; 0	1; 0		90; 0	1; 0	90

The values in [V] and in the papyrus, given to one digit only, are exactly the same; [H] includes also minutes (cf. Table 31) which are, however, not always the basis for the roundings in [V]. The values in Alm. II, 7 which agree within one minute with the exact values of sin α (cf. cols. 3 and 4 in Table 31) are much more accurate.

The insight that the coefficients which transform sin n_0 into sin n are simply sin $\alpha(\lambda)$ has been reached in Islamic astronomy, though the essential relations were probably first discovered in India where the ascensional differences were systematically used.[23] In al-Battānī's zīj one finds a table[24] in which α and sin α are tabulated as function of λ (cf. our Table 31 cols. 8 and 9).

The Handy Tables in all their stages of development contain a variety of geographical tables, of which the tables of the "important cities" are the best known ones. We have discussed this material in connection with Ptolemy's Geography, above V B 8, 3 B.

C. *Ortive Amplitudes*

The determination of the ortive amplitudes of the endpoints of the zodiacal signs is, of course, a problem of spherical trigonometry.[25] These data find their use, however, in connection with the "inclinations" of eclipses[26] and for this reason the corresponding circular diagram[27] is found with the tables which concern eclipses.[28] In the extant manuscripts of the Handy Tables[29] this diagram is essentially the same as in the Almagest,[30] based on 16 radii for the 12 zodiacal signs and the four cardinal directions; each radius carries the name of a wind.[31]

[23] Cf., e.g., Pc.-Sk. IV 26 and IV 34.
[24] Nallino, Batt. II, p. 58.
[25] Cf. the discussion above p. 37.
[26] Cf. p. 997ff.
[27] Cf. Fig. 32, p. 1216.
[28] Cf. p. 970 and p. 1000.
[29] [H] III, p. 43; [V] fol. 45'.
[30] Alm. VI, 11 (Heiberg I, p. 543, 24) and folding plate at the end of the volume.
[31] Omitted on Manitius' plate.

Again we find in [V][32] a specific Byzantine addition in the form of a circular diagram with 12 equidistant radii which give the names of the winds and the ortive amplitudes of the zodiacal signs at the horizon of Byzantium. The following table

	♈♎	♍♓♉♏	♌♒♊♐	♑♋
climate VI	0;0	16;38	29;42	34;53
Byzantium	0;0	16; 7	28;43	33;41
climate V	0;0	15;32	27;38	32;22

shows that these values had not been obtained by linear interpolation in the diagram of the Almagest but by independent computation.[33]

3. Theory of the Sun

The solar theory in the Handy Tables is based on the parameters of the Almagest,[1] i.e. the same mean motion, a tropically fixed apogee at

$$\lambda_A = \text{♊}\,5;30$$

and identical eccentricity $e = 2;30$. The arrangement of the tables. however. is more convenient than in the Almagest.

A. Solar Longitude

The mean longitudes of the sun are reckoned from the apogee ($\bar{\kappa}$ in Fig. 55, p. 1221). The new epoch, Philip 1 Thoth 1 (= −323 Nov. 12) is 424 Egyptian years later than Nabonassar 1 Thoth (= −746 Febr. 26), an interval for which one finds from Alm. III. 2 a solar mean motion of 256;54.56°. The distance of the mean sun from the apogee at Nabonassar 1 was $\bar{\kappa}_0 = 265;15°$.[2] Hence at the new epoch the sun had the distance 256;55 + 265;15 = 162;10° from ♊ 5;30. This is indeed the first entry in the tables of mean motions.[3] The rest of the tables progress in steps of 25 years of the Era Philip, followed by tables for single years, months, days, and hours, always referring to the given moment, not to the completed intervals as in the Almagest.

The tables for the equation of center[4] $c(\bar{\kappa})$ agree with the table in Alm. III. 6, excepting the tabulation for single degrees of the argument.

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[32] [V] fol. 3r; not in [H].

[33] A complete diagram which incorporates as the third ring from the outside the numbers for Byzantium (characterized by $M = 15\ 1/4^h$, $\varphi = 43;5°$) is given in Laur. gr. 28, 7 fol. 62v. The values for the amplitudes are, however, simply the mean values between the two adjacent climata.

[1] Cf. p. 58.

[2] Cf. p. 60.

[3] Halma H. T. II. p. 66/67 column 2.

[4] Halma H. T. II. p. 78–89. column 3. [V] counts the double column of the arguments as 1 and 2, the column of the solar anomaly as 3, followed by four columns for the lunar anomaly, numbered from 3 to 6.

Example. Find the solar longitude for Diocletian 77 Thoth 22 (Alexandrian), 11^h of daytime.[5] This moment corresponds to Philip 684=Nab. 1108 IV 28 (Egyptian) 5^h after noon.[6]

Alm. III, 2 and III, 6		*H.T.*	
810^y	163; 4,12	Phil. 676	358; 4
288^y	289;58,50	8^y	358; 3
9^y	357;48,43	month IV	88;42
3^m	88;42,26	day 28	26;37
27^d	26;36,44	hour 5	0;12
4^h	0;12,19	$\bar{\kappa}$	111;38
$\Delta\bar{\lambda}$	206;23,14	$c(\bar{\kappa})$	−2;15
epoch $\bar{\kappa}_0$	265;15	κ	109;23
$\bar{\kappa}$	111;38	λ_A	♊ 5;30
$c(\bar{\kappa})$	−2;15,22	$\lambda_\odot$	♊ 114;53 = ♍ 24;53.
κ	109;23		
λ_A	♊ 5;30		
$\lambda_\odot$	♊ 114;53 = ♍ 24;53.		

It is interesting that Ptolemy in his introduction to the Handy Tables[7] describes a graphical determination of the true position of the sun by assuming a degree-division drawn on the eccenter (of center M) and on the ecliptic (with center O) such that $\lambda_\odot$ can be read directly on the latter circle (cf. Fig. 55, p. 1221). We shall come back to similar graphical methods for the moon[8] and for the planets.[9]

For the tables of declination of the sun cf. above p. 979; for solar parallax and eclipses below p. 999 f.

B. *Equation of Time*

In the Almagest the "equation of time" concerns the time difference Δt between the given moment t and the epoch t_0=Nab. 1 Thoth 1 when the mean solar longitude was $\bar{\lambda}_0$ = ♓ 0;45°.[1] By shifting the epoch 424 years to Philip 1 Thoth 1 the longitude of the mean sun becomes for the Handy Tables $\bar{\lambda}_0$ = 227;40 = ♏ 17;40° or $\bar{\kappa}_0$ = 162;10° distant from A = ♊ 5;30.[2]

These two epoch dates accidentally stand in a very peculiar relation to the equation of time. As was stated correctly by Ptolemy the equation of time between two given moments reaches its maximum if the sun at the endpoints of these intervals was near the beginning of Scorpio and the middle of Aquarius, respectively,[3] i.e. at points which differ only by about 15° from the above-mentioned epoch values. Hence equations of time in reference to these two epoch dates differ by the total amplitude, i.e. by about $0;32^h$ or about 8^o.[4]

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[5] From Theon's (little) commentary to the Handy Tables (Halma H. T. I, p. 33 f.); cf. Delambre HAA II, p. 619; Rome CA III, p. CXXXV.
[6] Julian day 1852.810 = A.D. 360 Sept. 19.
[7] Heiberg, p. 165 (No. 5); Halma H. T. I, p. 7 f.
[8] Below p. 990.
[9] Below p. 1004.
[1] Above p. 63.
[2] Cf. above p. 63.
[3] Cf. p. 62.
[4] Computing the equation of time for the Era Philip with respect to the Era Nabonassar one finds with Ptolemy's elements a correction of about $7;38^o = 0;30.32^h$, an amount which differs by only a negligible amount from the maximum.

A definite advantage of the Handy Tables over the Almagest is that the equation of time had been tabulated[5] (as function of the solar longitude) instead of being computed independently for each individual case as in the Almagest: Fig. 98, p. 1406 shows the resulting values. A comparison with our earlier graph derived from the Almagest (p. 1222, Fig. 57) shows that the present graph is the mirror image of the previous one. This is exactly what one has to expect when the difference between the two functions amounts to the maximum amplitude.

One can formulate this situation in a slightly different fashion. According to Ptolemy's definition the "accurate" time $\Delta\bar{t}$ since epoch is obtained from the "simply" reckoned interval Δt by subtracting ΔE from Δt:

$$\Delta\bar{t} = \Delta t - \Delta E, \qquad \Delta E = \bar{\lambda} - \bar{\lambda}_0 - (\alpha - \alpha_0).$$

Since at the Almagest's epoch $\bar{\lambda}_0$ is near the minimum of the curve in Fig. 57 we can say that for this epoch the equation of time is always subtractive. The shift to the epoch Philip 1 brings $\bar{\lambda}_0$ near to the maximum of the curve, hence for this new epoch the equation of time will always be additive. No such simple rule can exist, e.g., for the epoch of the Canobic inscription (Augustus 1) since for it the sun is in Virgo for Thoth 1.[6]

In other words: tables which use the Era Nabonassar will show the value zero for the equation of time in Aquarius, whereas the Era Philip results in zero for Scorpio. This simple criterium gives us important information about the tables in P. Lond. 1278: its equation of time is zero in Aquarius,[7] hence its epoch is the epoch of the Era Nabonassar. We have already discussed the implications of this fact for the early history of the Handy Tables.[8]

Another consequence of this situation is the fact that the absolute values of the equation of time for the same solar longitude, but referred to one of the two different epochs, must always add up to the same total, i.e. to the maximum:

$$|\Delta E\,(\text{Nab.})| + |\Delta E\,(\text{Phil.})| \approx 0;32^h.$$

Consequently one can use the Handy Tables which tabulate ΔE (Phil.) to find the equation of time as well for problems in the Almagest. A small inaccuracy of one or two minutes of time has, of course, no importance for lunar positions. Example: for $\lambda_\odot = ♑23$ one finds $|\Delta E\,(\text{Phil.})| = 0;28,42^h$, thus $|\Delta E\,(\text{Nab.})| \approx 0;3^h$. Accurate computation[9] gives about $0;30^\circ = 0;2^h$.

The effect of the equation of time is taken into account only for the moon. This is evident from the epoch values given for Philip 1. For the sun one obtains for the 424 years since Nabonassar 1 from Alm. III, 2:

414^y	259;20,49
10^y	357;34, 7
epoch	330;45
	227;40

[5] As third column in the table of right ascensions. [H] I, p. 148–155. [V] fol. 22r–23v. The units are minutes and seconds of equinoctial hours.

[6] Cf. p. 902.

[7] Cf. Neugebauer [1958. 1], p. 98, Fig. 2 and p. 101.

[8] Above p. 974.

[9] Cf. above p. 64. For another example cf. below p. 988.

which is exactly the value found with the Handy Tables for Philip 1:

$$162;10+65;30=227;40.$$

The equation of time would lower this longitude by 0;1,19°. For the moon, however, the motion during $0;32^h$ is no longer negligible and it is therefore included in the epoch values (cf. below p. 987).

C. Precession

Exactly as in the Almagest[1] the constant of precession is assumed to be one degree per century (of Egyptian years). This follows, e.g., from the tables of planetary mean motions where one also finds a column for the longitudes λ_R of Regulus, e.g.[2]

$$\begin{array}{lll} \text{Philip} & 1 & \lambda_R = 117;54^\circ \\ & 1001 & 127;54. \end{array}$$

This parameter became increasingly important in Ptolemy's later works. For example the longitudes of the bright stars near the ecliptic are given in the Handy Tables[3] with respect to Regulus (hence independent of time) in contrast to the tropical longitudes in the catalogue of stars in the Almagest. Similar changes are made for the parameters of the planetary theory, to be discussed presently.[4] Since Ptolemy's constant of precession produces a deficit of almost 1° in two centuries, noticeable discrepancies result comparatively soon.

4. Theory of the Moon

The Handy Tables for the moon are based on the cinematic model developed in the Almagest with its movable eccenter and all the other features of Ptolemy's refined lunar theory.[1] Only in the arrangement of the tables one finds considerable differences.

A. The Tables for Mean Motions

In contrast to the Almagest the Handy Tables do not contain tables of mean motions by themselves but start with mean positions at moments equidistant from epoch, in steps of 25 Egyptian years.[2]

The tables for the mean positions of the moon[3] are combined with the corresponding solar tables which we ignore here in counting the columns.[4] Reckoning the common arguments as column 1 the lunar columns are counted from 2 to 5. In these columns one finds the following elements:

Col. 2. "Apogee of the eccenter": if $\bar{\lambda}$ represents the mean longitude of the moon, i.e. the longitude of the center C of the epicycle, $2\bar{\eta}$ the double elongation,

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[1] Cf. above p. 54.
[2] [H] II, p. 112/113 (giving incorrectly 117;51 — correct in [V] fol. 57r) and p. 116/117.
[3] Cf. V C 4, 6 B.
[4] Cf. p. 1002.
[1] Cf. above p. 84 ff.
[2] For the origin of this arrangement cf. above p. 971, n. 21.
[3] [H] II, p. 66–77; [V] fol. 38r–40v.
[4] Cf. for this type of norm above p. 983, n. 4.

i.e. the angle between C and the center M of the eccenter (cf. below p. 1407, Fig. 99), then the tabulated angle is the angle

$$\mu = -\lambda_M = -(\bar{\lambda} - 2\bar{\eta}). \qquad (1)$$

Col. 3. "Center of the epicycle." Tabulated is the double elongation $2\bar{\eta}$.

Col. 4. "Center of (the moon) itself." This is the mean anomaly $\bar{\alpha}$ which locates the moon P on the epicycle with respect to the mean apogee $\bar{A}$.

Col. 5. "Northern limit," i.e. $-\lambda_N$, the negative longitude of the northernmost point N of the inclined plane of the deferent.

In contrast hereto the Almagest (IV, 4) tabulates the increments of

$$\bar{\lambda} \quad \bar{\alpha} \quad \bar{\omega}' \quad \bar{\eta}$$

where $\bar{\omega}'$ is the argument of latitude of the mean moon C, reckoned from N. Consequently

$$\lambda_N + \bar{\omega}' = \bar{\lambda} \quad \text{or also} \quad \bar{\omega}' = \bar{\lambda} - \lambda_N \qquad (2)$$

and

$$\bar{\lambda} = 2\bar{\eta} - \mu. \qquad (3)$$

As is the case for all tables of mean motions in the Handy Tables the entries represent current dates, not completed intervals. This is also visible in the computing of these tables. It suffices to cite a few lines in the tables for single days concerning the mean anomaly $\bar{\alpha}$ (col. 4):

Alm. IV, 4: days	1	13; 3,54	H.T.: day	1	0; 0
	2	26; 7,48		2	13; 4
	3	39;11,42		3	26; 8
				4	39;12

etc., and similarly in all other cases.

B. Epoch Values

In order to obtain the parameters of mean motion for the epoch Philip 1, one has not only to add the mean motions during the 424 years which separate Philip 1 from Nabonassar 1, but one must also take into account the equation of time which amounts to about $0;32^h$ and is subtractive.[1] Using (1) and (2) one should obtain the initial values in the Handy Tables. Indeed:

Alm. IV, 4	$\bar{\lambda}$	$\bar{\alpha}$	$\bar{\omega}'$	$\bar{\eta}$
414^y	283; 7, 0	9;33,36	7;13,46	23;46,11
10^y	213;47,42	167;11,15	47; 7,52	216;13,35
$-0;32^h$	−0;17,34	−0;17,25	−0;17,38	−0;16,15
epoch	♉ 11;22	268;49	354;15	70;37
	♉ 147;59, 8	85;16,26	48;19,0	310;20,31
	$\bar{\lambda}$ = 177;59		$-\lambda_N = \bar{\omega}' - \bar{\lambda}$	
	$\mu = -(\bar{\lambda} - 2\bar{\eta})$		= −129;40	$2\bar{\eta}$
	= 82;42		= 230;20	= 260;41
H.T.:	82;40	85;17	230;19	260;40

[1] Cf. above p. 984.

The agreement is excellent; the deviations in μ and $\bar{\alpha}$ could be reduced to one minute and exact agreement could be obtained for λ_N and $2\bar{\eta}$ if one took $-0;33^h$ as equation of time, but differences in roundings alone suffice to explain the small discrepancies.

C. *The True Moon*

The true longitude λ of the moon is, of course, derived from the mean position $\bar{\lambda}$ by adding to $\bar{\lambda}$ a correction c which is computed from coefficients k_3 to k_6 tabulated in the tables of "anomaly."[1] These coefficients are identical with the coefficients c_3 to c_6 in Alm. V, 8[2] except for the intervals of tabulation and a difference in arrangement such that

$$\begin{array}{lll} k_3 \approx c_3(2\bar{\eta}) & & \multirow{2}{*}{$k_3 \gtrless 0 \quad \text{for} \begin{cases} 0 \leqq 2\bar{\eta} \leqq 180 \\ 180 \leqq 2\bar{\eta} \leqq 360, \end{cases}$} \\ k_4 \approx c_6(2\bar{\eta}) & & \\ \left.\begin{array}{l} k_5 \approx c_4(\alpha) \\ k_6 \approx c_5(\alpha) \end{array}\right\} & \alpha = \bar{\alpha} + c_3 = \bar{\alpha} + k_3 & k_5 \gtrless 0 \quad \text{for} \begin{cases} 180 \leqq \alpha \leqq 360 \\ 0 \leqq \alpha \leqq 180. \end{cases} \end{array} \tag{1}$$

The final equation is given by

$$c = k_5 + k_4 \cdot k_6 = c_4 + c_5 \cdot c_6. \tag{2}$$

Finally

$$\lambda = \bar{\lambda} + c \quad \text{where} \quad \bar{\lambda} = 2\bar{\eta} - \mu. \tag{3}$$

In computing the mean position of the moon one enters the tables with the given date, except for the hour which has to be modified by the equation of time.[3]

Example. Find the position of the moon for Philip 292 XI 5 Alexandria noon (=Nab. 716 = −31 July 1) using the Almagest as well as the Handy Tables (cf. Table 32, p. 989).

Computing with the tables Alm. III, 2 and 6 one finds for the sun

$$\bar{\lambda}_\odot = 96;33 \quad \text{thus } \bar{\kappa} = 31;3 \;\; c = -1;11 \quad \text{hence } \lambda_\odot = 95;22 \quad \text{and } \alpha_\odot = 95;52$$

for the right ascension (Alm. II, 8). Thus with respect to Nab. 1[4]

$$\Delta\bar{\lambda} = 96;33 - \text{♓}\,0;45 = 125;48$$
$$\Delta\alpha = 95;52 - 335;\,8 = 120;44.$$

Therefore

$$|\Delta E| = 5;4^\circ = 0;20,16^h$$

must be subtracted from the given moment.

Computing with the Handy Tables one finds for the sun

$$\bar{\kappa} = 31;3 \quad \text{thus } c = -1;11 \quad \text{hence } \kappa = 29;52 \quad \text{and } \lambda_\odot = \text{♊}\,5;30 + \kappa = \text{♋}\,5;22$$

and from this directly[5]

$$|\Delta E| = 0;11.34^h$$

which must be added to the given moment.

[1] [H] II, p. 78–89; [V] fol. 41r–43v. For the numbering of the k_i cf. above p. 983, n. 4.

[2] Cf. p. 93ff.

[3] Cf. above p. 984ff.

[4] Following Ptolemy's rules; cf. p. 62.

[5] [H] I, p. 153. Note that the two values of $|\Delta E|$ add up to $0;32^h$; cf. above p. 985.

Table 32

Alm. IV, 4	$\bar{\lambda}$	$\bar{\alpha}$	$\bar{\varpi}'$	$\bar{\eta}$
702^y	104;24,54	0;33,30	356;36,23	275; 5,15
13^y	241;56, 1	73;20,37	133;16,14	245; 5,39
10^m	352;54,53	319;29,41	8;48,19	57;13,27
4^d	52;42,20	52;15,36	52;55, 3	48;45,47
$-0;20,16^h$	−0;11, 8	−0;11, 2	−0;11,10	−0;10, 8
	31;47	85;28	191;25	266; 0
epoch	♉ 11;22	268;49	354;15	70;37
	$\bar{\lambda}$ = ♊ 13; 9	$\bar{\alpha}$ = 354;17	$\bar{\varpi}'$ = 185;40	$\bar{\eta}$ = 336;37
	c = +1; 3	c_3 = −6;48	c = +1; 3	$2\bar{\eta}$ = 313;14
	λ = ♊ 14;12	α = 347;29	ω' 186;43	
				c_3 = −6;48
		c_4 = +0;59		c_6 = 0; 8
		c_5 = 0;29		
			$c = c_4 + c_5 \cdot c_6 = 0;59 + 0;4 = +1;3$	

H. T.	μ	$2\bar{\eta}$	$\bar{\alpha}$	$-\lambda_N$
Phil. 276	155;45	273; 7	2;56	147;3
16^y	277;52	187;56	339;30	309;21
XI	121;32	114;27	319;30	15;53
5	44;49	97;32	52;16	0;13
$+0;11,30^h$	0; 5	0;12	0; 6	0; 0
	240; 3	313;14	354;18	112;30
	$-2\bar{\eta}$ = −313;14		k_3 = −6;48	
	$-c$ = −1; 4	k_3 = −6;48	α = 347;30	
	$-\lambda$ = −74;15	k_4 = 0; 8		
	λ = ♊ 14;15		k_5 = +1; 0	
			k_6 = 0;30	
			$c = k_5 + k_4 \cdot k_6 = 1;0 + 0;4 = +1;4$	

After the accurate mean interval $\Delta\bar{t}$ has now been found the determination of the true longitude of the moon follows practically the same pattern in both methods (cf. Table 32) and yields almost identical results. The small differences (0;3° for the final λ) are due mainly to the unnecessary crudeness of the tabulated coefficients c_i and k_i. The main advantage of the Handy Tables over the Almagest lies in the tabulation of the equation of time and in the uniform spacing of 1° for all arguments.

In computing the lunar latitude in the Almagest one must enter column 7 of Alm. V, 8 with the argument $\omega' = \bar{\varpi}' + c$. In the Handy Tables the same column is combined with the solar declinations,[6] the argument being found from (2), p. 987 as $\omega' = \lambda - \lambda_N$. Hence in our example (cf. Table 32)

$$\omega' = \bar{\varpi}' + c = 185;40 + \quad 1;\ 3 = 186;43$$

or

$$\omega' = \lambda - \lambda_N = \ 74;15 + 112;30 = 186;45,$$

respectively.[7] Hence in both cases $\beta \approx 4;58°$.

[6] [H] I, p. 144/5; cf. above p. 979.

[7] It should be noted that the element $-\lambda_N$ in the Handy Tables should not be modified by the equation c, in contrast to $\bar{\varpi}'$ in the Almagest.

Graphical Methods. It is of great interest to observe that Ptolemy gives instructions[8] for finding the longitude of the moon by graphical constructions.[9] Obviously he assumes (without saying so explicitly) a board on which is permanently drawn a circle with subdivided circumference and center O, much in the same fashion as is known to us from his "Analemma."[10] Dimensions are not given but it is clear that the parameters from the Almagest are used (probably in the equivalent form $e=12;30$ $r=6;20$ $R=60$ from the "Planetary Hypotheses"[11]). The construction follows the pattern prescribed by the theory in drawing angles $\mu=-\lambda_M$, $2\bar{\eta}$, and $\bar{\alpha}$ (in this order) as indicated in our Fig. 99.

The importance of this description lies in the fact that we find here again a Ptolemaic prototype of astronomical instruments ("analogue-computers" one would say today) which were greatly developed in Islamic and finally in western mediaeval astronomy.[12]

D. Parallax and Prosneusis

1. Arrangement of the Tables. The parallax tables of the Handy Tables play an important historical role since they were copied, except for minor modifications, throughout the Middle Ages. These tables give the components in longitude and latitude of the difference between lunar and solar parallax; henceforth called "*adjusted parallax*"[1], and for maximum distance of the moon.[2] The adjusted parallax is obviously of interest only for solar eclipses. If, however, the lunar parallax alone is needed one only has to increase the tabulated value by 1/20 of its amount,[3] corresponding to Ptolemy's value of about 0;3° for the maximum of the solar parallax in comparison with about 1° for the moon.[4]

The tables provide entries with respect to three parameters: geographical latitude φ, longitude λ of the moon, and time $\mp t$ before or after noon. The values chosen for φ correspond to the seven climates from $M=13^h$ to $M=16^h$ in steps of $1/2^h$ of the longest daylight (exactly as, e.g., in Alm. II, 13[5]). The tabulated longitudes are for the beginnings of the signs, the times t integer hours, except for the moments of sunrise and sunset which are determined according to the variation in the length of daylight for the beginning of each sign (again as in Alm. II, 13).[6] Linear interpolation has to be used for all intermediate values of the parameters.

Tabulated are, to minutes of arc, the longitudinal and latitudinal components p'_λ and p'_β, respectively, of the adjusted parallax. The tables in the extant manuscripts contain many scribal errors which, however, usually can be eliminated on the basis of necessary symmetries. Indeed, if (for given φ) $p'(\lambda, t)$ denotes either one of

[8] Ptolemy, Opera II, p. 165, 23 to 166, 18 (No. 6); Halma H.T. I, p. 8.
[9] Similarly for the sun; cf. above p. 984.
[10] Above p. 852.
[11] Above p. 903.
[12] Cf., e.g., Kennedy [1951], [1950/52]; Price, Equatorie.
[1] A term used in Islamic astronomy; cf. Kennedy [1956], p. 35.
[2] These facts were first clearly understood by A. Rome (CA I, p. XLIX ff.).
[3] Cf. Ptolemy's introduction to the Handy Tables, Heiberg II, p. 175, 19–21, and Theon's commentary, Halma HT I, p. 68 (cf. also Rome, CA, I p. L, note 2).
[4] Cf. above p. 112 or below p. 991.
[5] Cf. above p. 50.
[6] These limits confirm that we are dealing with daylight and hence with solar eclipses.

the components of parallax, then, for any angle α

$$p'(90+\alpha, +t)=p'(90-\alpha, -t). \quad (1)$$

Similarly for the last column[7] in which an angle $\theta=\Delta OM'$ in the horizon (cf. Fig. 100, p. 1407) is tabulated:

$$\theta(90+\alpha, +t)+\theta(90-\alpha, -t)=180. \quad (2)$$

2. Computation of the Tables. For the computation of parallaxes only one characteristic parameter needs to be known: the parallax for a position of sun and moon in the horizon, the so-called "*horizontal parallax*." The value of this parameter which has been used in the present tables is easy to determine. In clima II, i.e. at a latitude $\varphi=\varepsilon$, the ecliptic is perpendicular to the horizon when ♎0° and ♈0° rise, respectively set. For this situation the tables give $p'_\lambda=0;51°$ (and, of course, $p'_\beta=0$); this then is the underlying (adjusted) horizontal parallax.[8]

The tables in the Almagest V, 18[9] lead for a zenith distance of 90° to the adjusted parallax $0;53,34-0;2,51=0;50,43\approx 0;51°$. This indicates that the parallaxes in the Handy Tables were derived from the parallaxes of Alm. V, 18. Furthermore it is plausible that the computation of the components p'_λ and p'_β is based on the tables Alm. II, 13 for zenith distances and angles.[10] Indeed, one can easily verify that p'_λ and p'_β are obtainable from $p'\cos\gamma$ and $p'\sin\gamma$, respectively, where γ is the angle between the ecliptic and the circle of altitude found in Alm. II, 13.

Example. Computation of the components of parallax for Rhodes (clima IV) when Scorpio 0° is 5^h before the meridian.

In the tables of Alm. II, 13 we find for this situation the zenith distance

$$\zeta=85;5°. \quad (1)$$

The corresponding parallaxes for the sun and for the moon (at maximum distance) are found in Alm. V, 18 from

$$\begin{array}{lll} \zeta=84 & c_2=0;2,50 & c_3=0;53,21 \\ 86 & 0;2,50 & 0;53,29 \end{array}$$

hence $c_3-c_2=0;50,31$ and $0;50,39$, respectively, and by linear interpolation

$$p'=0;50,35°. \quad (2)$$

Again from Alm. II, 13 we know that at -5^h the "eastern angle" between the ecliptic (at ♏0°) and the vertical is

$$\gamma=162;28° \quad (3)$$

normed as shown in Fig. 38, p. 1217. Hence

$$p'_\lambda=-p'\cos\gamma=0;50.35\cdot 0;57,13=0;48,14\approx0;48° \quad (4a)$$

$$p'_\beta=-p'\sin\gamma=-0;50.35\cdot 0;18,\ 5=-0;15.15\approx -0;15° \quad (4b)$$

in agreement with the Handy Tables.

[7] For its use cf. below p. 997f.

[8] Actually the moon's horizontal parallax is about 0;57°. The solar parallax is slightly less than 0;0,9° and therefore negligible within the accuracy of the present tables.

[9] Cf. p. 112ff.

[10] Cf. p. 50ff.

If one carries out the same operation for all entries in Scorpio, rounding the results to integer minutes, one again obtains agreement except for p'_β at $\pm 3^h$ where a rounding for $\pm 0;0,32°$ is ignored (cf. Table 33).

Table 33

Cl. IV ♏ 0°	ζ	p'	γ	$\cos\gamma$	$\sin\gamma$	$p'_\lambda = -p'\cos\gamma$	$p'_\beta = -p'\sin\gamma$	Handy Tables p'_λ	Handy Tables p'_β
$-5;25^h$	90; 0°	0;50,43°	E. 164; 7°	−0;57,43	0;16,25	+0;48,44°	−0;13,52	0;49°	0;14°
−5	85; 5	50,35	162;28	57.13	18, 5	48,14	15,15	48	15
−4	73;55	48,59	157;51	55,34	22,37	45,22	18,28	45	18
−3	63;48	45,51	150;34	52,15	29,29	39,56	22,32	40	22
−2	55;26	42, 8	140;20	46,11	38,18	32,26	26,54	32	27
−1	49;42	39, 5	126;50	35,58	48, 1	23,25	31,17	23	31
n	47;40	37,54	111; 0	21,30	56, 1	13,35	35.23	14	35
+1	49;42	39, 5	W. 95;10	5.24	59,45	3,31	38,55	4	39
2	55;26	42, 8	81;40	+0; 8,42	59.22	−0; 6, 7	41.41	6	42
3	63;48	45,51	71;26	19, 6	56,53	14,36	43,28	15	44
4	73;55	48,59	64; 9	26,10	54, 0	21,22	44, 5	21	44
5	85; 5	50,35	59;32	30.25	51,43	25,39	43,37	26	44
5;25	90; 0	50.43	57;53	31.54	50,49	26,58	43, 7	27	43

In the case of clima I ($\varphi < \varepsilon$) one has to take into account that Ptolemy had to avoid negative angles between the ecliptic and the circles of altitude and hence adds (or subtracts) 180°. Thus we find in Alm. II, 13 for clima I and ♉0° at 2^h after noon as western angle $\gamma_W = 172;0°$, at 2^h before noon $\gamma_E = 146;0°$ or $\gamma_W + \gamma_E = 318;0°$ whereas at noon $\gamma_0 = 69;0°$ thus $2\gamma_0 = 138° = \gamma_W + \gamma_E - 180°$ against ordinarily $2\gamma_0 = \gamma_W + \gamma_E$ (cf. p. 48). Hence we replace Ptolemy's western angle γ_W by $\gamma = \gamma_W - 180 = -8;0°$. For the adjusted parallax one finds with $\zeta = 29;28$ as zenith distance

$$p' = 0;29,19$$

and thus the components

$$p'_\lambda = -p'\cos\gamma = -0;25,19 \cdot 0;59,25 = -0;25,4 \approx -0;25°$$
$$p'_\beta = -p'\sin\gamma = -0;25,19 \cdot -0;8,21 = +0;3,31 \approx +0;4°$$

in agreement with the tables. For the occurrence of positive latitudinal parallaxes cf. below p. 997 and Fig. 106, p. 1409.

We still have to discuss the computation of the angle θ which is tabulated in the fourth column of the table of parallaxes. This angle is defined as the arc ΔM' of the horizon (cf. Fig. 100) from the setting point Δ of the ecliptic to the point M' where the orthogonal PMM' to the ecliptic meets the horizon.[11] M represents the longitude of the moon at the middle of an eclipse, i.e. one of the points $\lambda = 0°$, 30°, 60°, etc.

[11] Ptolemy in his introduction to the Handy Tables, Opera II, p. 174, 23 to p. 175, 5 or Halma HT I, p. 16. P denotes here the pole of the ecliptic.

How this angle[12] was computed we are not told. Theon in his commentary to Book VI of the Almagest[13] computes θ for the case of clima IV ($\varphi=36°$), ♌ 0°, 3^h before noon and finds by means of his theoretically accurate method $\theta=99;45°$ in comparison with $\theta=100°$ given in the Handy Tables. But Theon's procedure is so complicated that it is evident that it cannot have provided the basis for the (about 500 different) tabulated values. Delambre in his characteristic fashion gave the corresponding modern procedure which "exige 24 logarithmes différens."[14] The problem becomes quite simple, however, when one makes systematically use of data already available in the tables of the Almagest.

Asking for $\bar{\theta}=180-\theta$ instead of θ we are led to consider the right spherical triangle HMM' in which $\bar{\theta}$ is the hypotenuse (Fig. 101). The side $MH=\Delta\lambda$ can easily be determined and the angle ν between ecliptic and horizon can be found by interpolation in the tables Alm. II, 13. Hence one can find $\bar{\theta}$ from

$$\tan\bar{\theta}=\tan\Delta\lambda/\cos\nu, \tag{1}$$

of course for Ptolemy from the equivalent Menelaos configuration.[15] Except for (1) this procedure requires only interpolations in extant tables and will lead to sufficiently accurate results, since θ is at any rate given to integer degrees only.

If we follow this method in the case of Theon's example (clima IV, ♌ 0°, -3^h) we must first determine the rising point H of the ecliptic. According to Alm. II, 13 the half length of daylight is $7;4^h$ for $\lambda_\odot=$ ♌ 0°. Hence the given moment is $\Delta t=4;4^h=61°$ after sunrise. Alm. II, 8 gives $\rho=106;30°$ as the oblique ascension of ♌ 0°. Thus

$$\rho+\Delta t=106;30+61=167;30°=\rho(H)$$

is the oblique ascension of H[16] and hence, again from Alm. II, 8,

$$H = \text{♍}\,19;40°.$$

This gives for the longitudinal difference from M = ♌ 0° to H

$$\Delta\lambda=49;40°.$$

For the angle ν between ecliptic and horizon one finds from Alm. II, 13 for clima IV[17]

$$\begin{array}{ll} \lambda(H)=\text{♍}\,0° & \nu=74;\ 7° \\ \quad\ \ \text{♎}\,0° & \quad\ \ 77;51 \end{array}$$

and thus by linear interpolation

$$\nu(H)=76;34°.$$

Using these values $\Delta\lambda$ and ν in (1) one obtains $\bar{\theta}=78;50$ or

$$\theta=101;10$$

as compared with $\theta=100$ in the Handy Tables and Theon's 99;45° (which is probably doctored for educational purposes).

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12 Called διάστημα περιφερείας "distance of the arc" (MM' from Δ?) or similar.

13 In connection with the determination of the "inclination" of an eclipse; cf. below p. 997 ff. I know of this procedure only through Delambre HAA II, p. 600 ff.

14 Delambre HAA II, p. 599.

15 Cf. for an application of (1) above p. 37 (4).

16 Cf. p. 41.

17 Cf. p. 47 and Table 3 there.

Table 34

Cl. IV ♏ 0°	ρ(H)	H	$\Delta\lambda$	v	θ computed		text
$-5;25^h$	216;28°	♏ 0	0	74; 7	180		180
-5	222;43	5; 4	5; 4	72;20	163;43 ≈	164	165
-4	237;45	17;14	17;14	68; 5	140;16	140	141
-3	252;46	29;24	29;24	63;49	128; 4	128	128
-2	267;48	♐ 11;46	41;46	58;16	120;30	121	119
-1	282;49	24;47	54;47	52;23	113;19	113	112
n	297;51	♑ 8;36	68;36	46;37	105; 4	105	104
$+1$	312;52	24;16	84;16	40;47	94;21	94	94
2	327;54	♒ 12;13	102;13	35;58	80; 4	80	79
3	342;55	♓ 3;11	123;11	31;54	60;58	61	60
4	357;57	26;41	146;41	30;22	37;18	37	37
5	12;58	♈ 20;34	170;34	31;30	11; 2	11	11
5;25	19;12	♉ 0	180	32; 7	0		0

In order to test the efficiency of this method to a little larger extent I have computed all values of θ for the table of Scorpio and clima IV. The results are shown in Table 34 and leave little doubt that the values in the Handy Tables were obtained by this type of procedure. The discrepancies are small enough to be explicable as the result of different roundings and by the more onerous computations replacing our formula (1).[18]

3. **Tables of Correction.** As we have seen[19] the main tables of parallax[20] were computed under the assumption of the maximum distance for the moon, i.e. for an epicyclic anomaly $\alpha=0$ and an elongation $\bar{\eta}=0$. Even for eclipses only the second assumption is always satisfied and for general positions of the moon any combination of α and $\bar{\eta}$ is permissible. Hence, in order to make the Handy Tables for parallaxes the equivalent of Alm. V, 18, corrections depending on α and (conveniently) $2\bar{\eta}$ must be introduced. This is done in a small table which (at least in our manuscripts[21]) is combined with two unrelated tables which concern eclipses.[22]

The tables as we have them give the impression that one is dealing with one column of numbers with two digits but fortunately one finds in al-Battānī the same numbers separated in two single digit columns with the headings "epicycle" and "eccenter", respectively.[23] The headings in the extant Greek manuscripts

[18] One might perhaps also mention that Ptolemy used for his tables of planetary visibilities values of v which do not agree accurately with the values obtainable from Alm. II, 13 (cf. above p. 236).
[19] Above p. 990 and p. 991.
[20] [H] II, p. 98 to 111; [V] fol. 50ʳ to 56ʳ.
[21] [H] I, p. 146/7 middle section; [V] fol. 48ᵛ last 4 columns.
[22] One concerns the interpolations as function of α in the table of durations (below p. 1000), the other the angles σ of "inclinations" (below p. 997 and p. 1000).
[23] Nallino, II, p. 89. How much of this correct arrangement is actually due to Nallino or to the Arabic version I do not know. Battānī's table is combined, as in [H], with the table of interpolation mentioned in the preceding note.

are meaningless[24] and show that the contamination of the two columns into one must be of an early date.

On the basis of Battānī's text it became clear that we are dealing with two coefficients of corrections, both in the form of sexagesimal fractions.[25] The first column gives a correction $c_1(\alpha)$ such that the adjusted parallax p' found in the main tables becomes

$$p'_1(\alpha, 0) = p' + c_1\, p' \tag{1}$$

for any α and $\bar{\eta} = 0$. For solar eclipses this is the only correction needed. For mean elongations, however, which are different from zero, a second correction $c_2(2\bar{\eta})$ leads to a parallax

$$p'_2(\alpha, \bar{\eta}) = (p' + c_1\, p') + c_2(p' + c_1\, p'). \tag{2}$$

For non-syzygies it is, of course, senseless to take the difference between lunar and solar parallaxes but according to Ptolemy's rule[26] one finds the lunar parallax p from the adjusted parallax p' by taking

$$p = 21/20\, p' \tag{3}$$

which applies to (1) as well as to (2). All these relations obviously also hold for the longitudinal or latitudinal components separately. Rome has shown that parallaxes computed in this fashion agree within 0;1° with the results obtained from the tables in Alm. V, 18.[27]

In order to reconstruct the method of computation for these tables we first remark that it follows from (1) that for $\alpha = 180$ the maximum adjusted parallax for syzygies, i.e. $0;53,34 - 0;2,51 + 0;10,17 = 0;50,43 + 0;10,17°$, should be the same as $(1 + c_1(180))p'_0$ where p'_0 is the maximum adjusted parallax for $\alpha = 0°$, i.e. 0;50,43°. Thus

$$c_1(180) = 0;10,17/0;50,43 = 0;12,10 \approx 0;12.$$

This is indeed the value found in the table of corrections. For any other value of α one obtains

$$c_1(\alpha) \approx 0;12 \cdot c_7(\alpha)$$

where $c_7(\alpha)$ is the coefficient of interpolation given in column 7 of the tables of parallaxes in Alm. V, 18,[28] all products rounded to the nearest sixtieths (cf. Table 35). A similar result is obtainable for $c_2(2\bar{\eta})$ by means of $c_9(\bar{\eta})$ tabulated in Alm. V, 18 (cf. Table 36).[29] I do not know, however, how the maximum 0;32 of c_2 (for $\bar{\eta} = 90$) had been obtained. It would seem plausible to say that the greatest

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[24] Halma, without scruples, accepted as heading "correction du centre de l'épic. et ☾." [V] gives for the independent variables "from 1 to 90 north, from 91 to 280 (sic) south, from 281 to 360 north," giving no heading at all for the column of the tabulated numbers. For an emendation of the version [H] cf. Rome CA I. p. LVI.

[25] This has been shown first by Nallino, Batt. II, p. 239 and in all details for the Handy Tables by Rome CA I. p. LI to LVI.

[26] Cf. above p. 990.

[27] Rome CA I. p. LIV and p. LV.

[28] Cf. p. 113. In using the tables of the Almagest one must not forget that the arguments in column 1 represent only $\alpha/2$.

[29] Cf. also Fig. 101, p. 1237.

Table 35

α	$c_7(\alpha)$	$0;12 \cdot c_7$	Text c_1
0	0	0	0; 0
12	0; 0,42	0; 0, 8,24	0
24	2,42	0,32,24	1
36	5,21	1, 4,12	1
48	9,15	1,51	2
60	14, 0	2,48	3
72	19,36	3,55,12	4
84	25,36	5, 7,12	5
96	31,48	6,21,36	6
108	38, 0	7,36	8
120	44, 0	8,48	9
132	49,30	9,54	10
144	53,48	10,45,36	11
156	57,15	11,27	11
168	59,21	11,52,12	12
180	1; 0. 0	12	12

Table 36

$\bar{\eta}$	$c_9(\bar{\eta})$	$0;32 \cdot c_9$	Text c_2
0	0	0	0; 0
6	0; 0.45	0; 0,24	0
12	3, 9	1,40,48	2
18	6,48	3,37,36	4
24	11,39	6,12,48	6
30	17,18	9,13,36	9
36	23,33	12,33,36	13
42	29,54	15,56,48	16
48	36,12	19,18,24	19
54	42, 3	22,25,36	22
60	47,21	25,15,12	25
66	51,45	27,36	28
72	55,41	29,41,52	30
78	57,54	30,52,48	31
84	59,30	31,44	32
90	1: 0, 0	32	32

adjusted parallaxes were, according to Alm. V, 18,

$$\left.\begin{array}{ll} 0;53,34-0;2,51=0;50,43 & \text{for } \bar{\eta}=0 \\ 1;19,\ 0-0;2,51=1;16,\ 9 & \text{for } \bar{\eta}=90 \end{array}\right. \quad \alpha=0$$

and therefore

$$(1+c_2(180))0;50,43=1;16,9.$$

From this relation one obtains, however,

$$c_2=0;25,26/0;50,43\approx 0;30,5$$

and not 0;32. Other possible combinations lead to even greater discrepancies (e.g. $c_2 \approx 0;39$ for $\alpha=180$).

Use of the Tables. The fact that the tables of parallaxes give only the adjusted parallaxes indicates that the longitude λ in question is the true longitude of the conjunction of sun and moon, hence p'_λ the apparent longitudinal elongation of the moon, and $\beta_☾ + p'_\beta$ the apparent latitudinal distance between the centers of sun and moon. The only question that remains to be clarified is the question of the signs of the tabulated values p'_λ and p'_β.

Since tables of this kind are designed for the use of solar eclipses we know that the latitude of the moon is not far from zero. Hence we assume in the following that we are dealing with a point P of the ecliptic. Its parallax is always directed downward perpendicular to the horizon. Consequently its longitudinal component adds to the longitude of P between the rising point H and the highest point V of the ecliptic; it is negative on the quadrant between V and the setting point Δ. At V itself $p'_\lambda=0$ but this point is generally not tabulated unless it corresponds to an integer hour before or after noon. Only at the solstices, V always coincides with noon. Our texts do not indicate signs for p'_λ and one has therefore to observe the differences in order to see where $p'_\lambda>0$ (in the morning hours)

and where it changes to $p'_\lambda < 0$ (toward the evening hours); cf. Fig. 102 which represents p'_λ graphically in the range from ♈ to ♎ in clima VII.

The situation is much simpler in the case of the latitudinal parallax: as long as the geographical latitude φ is not too far southerly the latitudinal component is always negative. For $\varphi = \varepsilon$, i.e. in clima II, the ecliptic reaches the zenith when ♎ 0° is rising and ♈ 0° is setting. In this case $p'_\beta = 0$ along the whole ecliptic, otherwise negative (cf. Fig. 105). For $\varphi < \varepsilon$, i.e. in clima I, the ecliptic can decline to the north of the zenith (cf. Fig. 103). A point not far from ♎ 0° (e.g. P in Fig. 103) will show in a morning hour a northerly component of parallax, which, however, later in the day will again become negative (Fig. 104). Hence, near ♎ 0° in the morning and near ♈ 0° in the evening positive latitudinal components will occur (cf. Fig. 106[19]). Consequently the Handy Tables distinguish for clima I between northerly and southerly latitudinal components by adding the letters *βο* (for *βόρειον*) and *νο* (for *νότιον*), respectively.

The last (fourth) column in the table of parallaxes has to do with the concept of "*inclination*" (*πρόσνευσις*) of an eclipse. This column was added to the table of parallaxes only for the convenience of tabulation, saving repetition of the same argument columns for all climates and for all zodiacal signs.

In the Almagest (VI, 11) the "inclination" of an eclipse was defined by means of the point P of intersection with the horizon of the great circle that connects the center of the sun (or shadow) with the center of the moon at the characteristic phases of an eclipse.[20] A mathematically exact solution of this problem would require rather long-winded trigonometric procedures, except for the trivial case of an exactly central eclipse ($\beta_☾ = 0$) when the rising (H) or setting (Δ) point of the ecliptic would define the "inclination." In all other cases Ptolemy adopted a rather drastic simplification. He computed accurately and tabulated (as function of the eclipse magnitude) the angles σ between the ecliptic and the above-mentioned great circle connecting the centers of eclipsed and eclipsing body.[21] He also could assume as accurately known the azimuths of H and Δ. But then he placed P simply at an azimuthal distance σ from H or Δ, a procedure with would only be correct if the center of the sun (or of the shadow) were located in the zenith.

The Handy Tables improved on this very inaccurate procedure. The angle $\theta = \Delta M'$ which is tabulated in the 4th column of the tables of parallaxes is defined by means of the point M' of the horizon at which the orthogonal MM' to the ecliptic at the midpoint M of the eclipse intersects the horizon (cf. Fig. 100, p. 1407).

[19] For the sake of clarity I have plotted in Fig. 105 and 106 only values for integer hours nearest to sunrise and sunset. In order to obtain sufficiently smooth curves I deviated from the extant tables in the following cases:

clima I:	♌ $+5^h$ and ♊ -5^h	$-0;18°$	(instead of $-0;19°$)
	♏ -2^h and ♓ $+2^h$	$-0;12$	$-0;14$
	♏ and ♓ noon	$-0;22$	$-0;24$
clima II:	♋ $\pm 3^h$	$-0;\ 7$	$-0;\ 5$
	♑ $\pm 2^h$	$-0;38$	$-0;35$.

Except for the last one these graphically determined corrections are confirmed by explicit computation (as on p. 991) which yields $-0;18,22$, $-0;12,36$, $-0;22,38$, $-0;6,54$, and $-0;35,18$, respectively.

[20] Cf. above p. 141 ff.: for the phases cf. p. 143 there.

[21] Alm. VI, 12 (cf. Fig. 125, p. 1245). This table was repeated, rounded to integer degrees, in the Handy Tables (cf. below p. 1000).

Let us now suppose that for a given phase the great circle at an angle σ to the ecliptic meets the horizon at P as illustrated in Fig. 107. In the Almagest it would have been postulated that $\varDelta P \approx \sigma$ instead of computing accurately $\varDelta P = \tau$. In the Handy Tables τ is approximately determined by assuming that it represents the same fraction of σ as the arc $\varDelta M' = \theta$ is a fraction of the right angle at M. Hence one defines $\varDelta P = \tau$ by

$$\tau = \frac{\theta}{90}\,\sigma \tag{1}$$

and considers the corresponding point P as defining the "inclination" of the given phase. This procedure is not mathematically exact but it constitutes a great improvement over making $\varDelta P = \sigma$.

In a situation as shown in Fig. 108 where MM' and MP meet the horizon nearer to H than to $\varDelta$ we note that $\varDelta H = 180°$; hence, $M'H = 180 - \varDelta M' = 180 - \theta = \bar{\theta}$. Thus one now defines τ by

$$\tau = \frac{\bar{\theta}}{90}\,\sigma. \tag{2}$$

Similarly for a phase in which the circle MP meets the horizon to the north of $\varDelta$ (cf. the schematic Fig. 109). Since by definition $\theta = \varDelta M'' = HM'$ we have $\bar{\theta} = 180 - \theta = \varDelta M'$ and one can assume that

$$\tau = \frac{\bar{\theta}}{90}\,\sigma.$$

Finally a position of P near H but to the north of it leads again to the definition (1) for τ.

This explains the rules given by Ptolemy for the determination of "inclinations" of lunar and solar eclipses in his introduction to the Handy Tables.[22] The relative position of the points P with respect to $\varDelta$ and H for each phase are the same in the Handy Tables as in the Almagest (cf. Figs. 126 and 127, p. 1246f.). The only difference consists in the replacement of the angle σ in the horizon by the angle τ according to the following rules: if the point P which defines the "inclination" at a particular phase falls to the south of $\varDelta$ or to the north of H one applies the horizon arc $\tau = \varDelta P$ or HP by means of (1), where θ is the angle obtained from column 4 of the table of parallaxes. If, however, P falls to the north of $\varDelta$ or to the south of H one has to use (2) with $\bar{\theta} = 180 - \theta$.

For the middle of an eclipse (phase 3) no rule was given in the Almagest. Since in this case $\sigma = 90°$ the formulae (1) and (2) give

$$\tau = \theta \quad \text{or} \quad \bar{\theta}, \tag{3}$$

respectively. Obviously the "inclination" for the middle of an eclipse is exactly known by means of the accurately computed angles θ. Ptolemy defines that for lunar eclipses with northerly latitudes of the moon ($\beta_{☾} > 0$) the point P_3 should be to the south from H, for $\beta_{☾} < 0$ to the north from $\varDelta$. For solar eclipses, however,

[22] Ptolemy, Opera II, p. 178, 23ff. and p. 183, 16ff. The essential step for the understanding of these rules was made by A. Rome [1948]. Halma's translations (HT I, p. 20ff. and p. 24f.) are misleading since he misinterpreted the terms for the phases (e.g. "last moment of totality" instead of "end of restoration of full brightness," i.e. phase 4 instead of phase 5).

the opposite rule should hold. The directions chosen in this way correspond to the side of visible darkness on the eclipsed body at the middle of a partial eclipse.

This completes the prosneusis theory in antiquity. It is no great loss that it was not transmitted into Islamic tables and consequently disappeared from mediaeval astronomy. But there remains the question of the original purpose of this whole prosneusis procedure. Rome thought[23] that the characteristic point established on the horizon had been utilized to find the otherwise difficult observable point on the circumference of the eclipsed body where the first (or last) contact should be expected. Delambre also considered this possibility[24] but did not think that the ancient astronomers were eager to establish the very first moment of an eclipse, in particular in view of the fact that their predictions had to allow for a considerable margin of error. Rome called Delambre's attitude "hypercritical" but I think one could agree with it in this particular instance. Obviously Rome came to his conclusion because the only alternative, astrological significance, seemed to be ruled out since neither the Tetrabiblos nor the material of the CCAG provided any reference to the prosneusis of eclipses.[25] There exists a third possibility; namely, weather prognostications, a doctrine which antedates the ordinary hellenistic astrology. Indeed, Ptolemy in his introduction[26] after his discussion of the case of the middle of the eclipse, speaks about "the other *ἐπισημασίαι*" which is the technical term for weather prognostications.[27] The same terminology already appeared in the Almagest[28] and I think that equally archaic doctrines are also responsible for the association of ortive amplitudes and wind directions.[29] Finally Chap. XIII of the "Liber Hermetis Trismegisti" is called "De nutationibus lunae" which Gundel related to the end of Alm. VI, describing it as a residue from an archaic stratum of hellenistic astrology.[30] All this seems to point in the same direction; namely the "significance" of the phases of eclipses for the prediction of wind and weather, leaving us with no trace of a practical application to observations.

E. Eclipses

As we have seen, the tables of parallaxes were constructed primarily for solar eclipses since they give the adjusted parallaxes only but we also mentioned the rules for finding the solar parallax alone.[1] Consequently it would seem unnec-

[23] Rome [1948], p. 511 and [1950], p. 218.
[24] HAA II, p. 239f., p. 597.
[25] Rome [1950], p. 218 where he explicitly mentions omina concerning countries as not related to prosneusis.
[26] Heiberg II, p. 179, 13 (incorrectly translated by Halma HT I, p. 20).
[27] Cf., e.g., the article Episemasiai by Rehm in RE Suppl. 7 (1940) col. 176 to 198, but restricting the term to stellar phases and parapegmata too narrowly.
[28] Cf. above p. 142 and note 5 there.
[29] Omitted in Manitius' table at the end of Vol. I of his translation: cf. his note p. 404 (a). For the text see Heiberg I, folding plate, and Delambre HAA II, p. 239. Rehm [1916], p. 62, note 1 considers the names of the winds in the circular diagram a later addition, though he himself has underlined (p. 20) the connection between ortive amplitudes and wind directions. Cf. also Theon's reference to winds related to the angle θ (Halma HT I, p. 69, 19–33); finally above IV B 5 on the *βαθμοί*.
[30] Gundel HT, p. 321.
[1] Cf. above p. 990.

essary to compile a special table of solar parallaxes; nevertheless some manuscripts (not [V]) contain such a table, progressing in steps of single degrees of the zenith distance.[2] Since the geographical table with which this table appears is old[3] it is not certain that the table of solar parallaxes is a late addition.

The eclipse tables from Alm. VI, 8[4] appear with some modifications in the Handy Tables as well:

Alm. VI, 8	Handy Tables	
I. Solar ecl., max. dist.	[H] II, p. 90/91 top left;	[V] fol. 49^r 3 middle cols.
min. dist.	right;	last 3 cols.
II. Lunar ecl., max. dist.	bottom left;	[V] fol. 49^v first 4 cols.
min. dist.	right;	last 4 cols.
III. Table of interpolation	[H] I, p. 146/7 first 3 cols.;	[V] fol. 48^v first 3 cols.
IV. Area eclipse magn.	[H] II, p. 94/5 left;	[V] fol. 49^r first 3 cols.

Beyond the differences in arrangement the Handy Tables show other modifications: the "durations" (actually, travelled distances) are now rounded to minutes of arc while the arguments are lunar latitudes (in minutes and seconds), instead of arguments of latitude as in the Almagest.

From the tables of solar and lunar eclipses one can derive the ranges for the underlying apparent radii $r_☾$ and s for the moon and for the shadow under the plausible assumption that the radius $r_\odot$ of the sun had been kept constant at 0;16°. In this way one finds

$$0;16° \leqq r_☾ \leqq 0;18°, \quad 0;41° \leqq s \leqq 0;46°, \quad r_\odot = 0;16°.$$

All these values are simply the roundings to minutes of the parameters used in the Almagest.[5] The magnitudes can reach 12;12 and 21;40 digits for solar and lunar eclipses, respectively.

The tables for the angles of "inclination" σ from Alm. VI, 12[6] also appear in [H] and in [V],[7] but are rounded to integer degrees.[8]

The use of the tables is explained in Ptolemy's introduction.[9] The first step in an eclipse computation is, of course, the determination of the moment t of a true syzygy.

The moment $\bar{t}$ of a mean syzygy is easily found since for it one must have $2\bar{\eta}=0$. Hence one only has to produce in column 3 of the mean motion tables[10] such a combination of entries that the resulting $2\bar{\eta}$ is zero.

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[2] Halma H.T. III. p. 34/5 (from Par. gr. 2493 fol. 51^v), misreading παράλλαξις as παράλληλος.

[3] It is also found in a papyrus of the second century A.D. (P. Lond. 1278; cf. above p. 974 and p. 939). The same geographical table but without the parallaxes appears also in Marc. gr. 314 fol. 218^r, now combined with the list from a geographical astrology (from Tetrabiblos II, 3, Loeb, p. 156–159).

[4] Cf. p. 134 ff.

[5] Cf. p. 125 (1).

[6] Cf. p. 142 and Figs. 124 and 125 (p. 1244 f.); also above p. 997.

[7] [H] I, p. 146/7 last 3 columns, repeated in II, p. 94/5 right half; [V] fol. 48^v middle section.

[8] Cf. above p. 997 and Figs. 107 to 109 there.

[9] Opera II, p. 176 to 185, Nos. 17 to 21. The correct explanation of this procedure was first given by van der Waerden [1958], p. 73 to 78.

[10] Cf. above p. 987.

Let us denote by $\Delta\lambda$ the (positive or negative) distance

$$\Delta\lambda=\begin{cases}\lambda_{☾}-\lambda_{\odot} & \text{for conjunctions}\\ \lambda_{☾}-(\lambda_{\odot}+180°) & \text{for oppositions}\end{cases} \tag{1}$$

where $\lambda_{☾}$ and $\lambda_{\odot}$ are the true longitudes of moon and sun, respectively, computed for the above determined moment $\bar{t}$ of the mean syzygy. Thus, if $\Delta\lambda>0$, the moon is actually ahead of the sun at the moment $\bar{t}$ and a correction Δt must be subtracted from $\bar{t}$ in order to find the moment t of the true syzygy. The opposite holds for $\Delta\lambda<0$. $\Delta t=0$ means that accidentally true and mean syzygy coincide.

In order to find Δt one needs the velocity v of the moon which is a function of the epicyclic anomaly. Let α be this anomaly at the moment $\bar{t}$, $\alpha+\Delta\alpha$ at the moment $t=\bar{t}+\Delta t$. Fortunately one can estimate the amount $\Delta\alpha$ without knowing Δt because

$$\Delta\alpha\approx 1;4\,\Delta\lambda \tag{2}$$

since, e.g., the hourly increment of the elongation is about 0;30,30°, of the anomaly about 0;32,40°,[11] and $32,40/30,30\approx 1;4$. Hence we know that the anomaly changes between $\bar{t}$ and t from α to $\alpha+1;4\,\Delta\lambda$. Consequently one may assume that the velocity on this arc has as average the value which corresponds to the anomaly $\alpha'=\alpha+0;32\,\Delta\lambda$.[12]

In order to find the velocity which belongs to α' one proceeds as follows. In [H] I, p. 146/7 (or in the equivalent table III of Alm. VI, 8[13]) we find coefficients of interpolation $c(\alpha)$ which concern the transitions between apogee ($\alpha=0$) and perigee ($\alpha=180$) of the epicycle, i.e. between lowest and highest velocity. The minimum hourly velocity is about 0;30°, the maximum about $0;36°=0;30+0;6$. Hence one assumes that for the anomaly α' the lunar velocity is about

$$v(\alpha')=0;30+c(\alpha')\cdot 0;6^{°/h}. \tag{3}$$

In [V] we find not only (fol. 48ᵛ) the table for $c(\alpha)$ but also (fol. 46ʳ) a special column[14] for $v(\alpha)$, computed according to (3) and using exactly the values of $c(\alpha)$.[15]

The total distance which the moon must cover to come from the mean to the true syzygy is about $13/12\,\Delta\lambda$,[16] hence the time interval

$$\Delta t=13/12\,\Delta\lambda/v(\alpha'). \tag{4}$$

This gives $t=\bar{t}\mp\Delta t$ as moment of the true syzygy.

Formula (3) is of general usefulness since it provides a reasonably accurate estimate for the hourly lunar velocity at a given anomaly α'.

[11] Alm. IV, 4.

[12] Ptolemy's introduction has $\alpha'=\alpha+(1/4+1/30)\,\Delta\lambda=\alpha+0;17\,\Delta\lambda$ instead of $\alpha+(1/2+1/30)\,\Delta\lambda$. The error must be old since already Theon used 1/4 instead of 1/2 in his commentaries (cf. Halma H.T. I, p. 75, 6 and Rome CA I, p. LX note (1)). It was van der Waerden ([1958], p. 74f.) who brought sense to this passage.

[13] Above p. 1000.

[14] Column 3 is headed "Table of the hourly motion of the moon (as function) of the anomaly." Columns 1 and 2 concern the lunar latitude.

[15] Surprisingly the results are expressed in minutes and unit fractions of minutes instead of in minutes and seconds (as also the column heading assumes). We find, e.g., 0;30 1/4 1/20 instead of $0;30,18^{°/h}$.

[16] Cf., e.g., p. 122 (3).

5. The Planets

The computations of planetary longitudes closely follow in the Handy Tables the pattern established in the Almagest and consequently produce identical results, except perhaps some small differences caused by different roundings. For the latitudes, however, and for the planetary phases the theoretical basis of the tables was greatly modified (cf. below the sections V C 4, 5 B and 5 C). The planetary phases lost all importance for the development of astronomy, whereas the theory of the latitudes was destined to play a central role in the transformation of the Ptolemaic astronomy to Kepler's theory. The astronomy of the Renaissance, however, again took its departure from the theory of the Almagest while the Handy Tables, by their character as computing aides, had long been superseded by similar and supposedly improved tables, from Battānī to the Toledan and Alphonsine tables and many others. This may explain why the theoretical progress incorporated in the Handy Tables remained without effect on the development of planetary theory.

A. Longitudes

1. The Use of the Tables. The planetary tables naturally fall into two classes, the tables for mean positions,[1] and the tables for equations.[2] As is the case for sun and moon[3] the tables of mean motions are arranged for direct entry with a given date, progressing in 25-year steps of the Era Philip, extending over one Sothic period,[4] followed by the tables for single (egyptian) years, months, etc. The mean anomalies $\bar{\kappa}$ and $\bar{\alpha}$ (cf. Fig. 110) are tabulated for each planet, preceded by a column for the longitude λ_R of Regulus[5] because the apogees of the planets are assumed to remain at fixed distances from this star.[6]

With $\bar{\kappa}$ and $\bar{\alpha}$ known for the given moment t one enters the second group of tables which are essentially the equivalent of the tables Alm. XI, 11.[7] The arguments, however, are now spaced uniformly 1° apart and the two parameters $c_3(\bar{\kappa})$ and $c_4(\bar{\kappa})$ are conveniently combined into one number $k_3(\bar{\kappa})$ such that (cf. Fig. 110)[8]

$$k_3(\bar{\kappa}) = \eta = c_3(\bar{\kappa}) + c_4(\bar{\kappa}) \tag{1}$$

and therefore the true anomalies

$$\alpha = \bar{\alpha} + k_3 \tag{2}$$

and

$$\kappa_0 = \bar{\kappa} - k_3 \tag{3}$$

with

$$k_3 \gtrless 0 \quad \text{for} \begin{cases} 0 \leqq \bar{\kappa} \leqq 180 \\ 180 \leqq \bar{\kappa} \leqq 360. \end{cases}$$

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[1] [H] II, p. 112–133; [V] fol. 57r–62v.
[2] [H] II, p. 134–193; [V] fol. 63r–77v.
[3] Above p. 963 and Table 32, p. 989.
[4] Cf. above p. 971.
[5] Cf. above p. 986.
[6] Cf. below p. 1003 (7); above p. 150.
[7] Cf. above p. 183.
[8] Cf. also Fig. 186, p. 1264.

With α as argument one finds the equation $k_6(\alpha)$ under the assumption that the epicycle is located at mean distance. If $k_6(\alpha)-k_5(\alpha)$ is the equation at anomaly α for the maximum distance of the epicycle, $k_6(\alpha)+k_7(\alpha)$ for the minimum distance, one finds the equation θ for a general position from

$$\theta(\alpha, \kappa_0)=k_6(\alpha)+k_4(\kappa_0)\cdot\begin{cases}k_5(\alpha) & \text{if } k_4\leqq 0\\ k_7(\alpha) & \text{if } k_4\geqq 0\end{cases} \tag{4}$$

with

$$k_5\geqq 0, \quad k_6\gtrless 0 \quad \text{for } \begin{cases}0\leqq\alpha\leqq 180\\ 180\leqq\alpha\leqq 360,\end{cases} \quad k_7\geqq 0.$$

The values of k_5, k_6, k_7 are the same as c_5, c_6, c_7, respectively, in the Almagest[9] (ignoring occasional deviations of 0;1° or 0;2°). The interpolation coefficients $k_4(\kappa_0)$, however, are not numerically identical with the corresponding coefficients $c_8(\bar{\kappa})$ in the Almagest in so far as the argument is now κ_0 and not $\bar{\kappa}$. The same values $k_4(\kappa_0)$ are used once more for the similar interpolations for latitudes; in the latter context we shall explain the method used in the computation of these coefficients.[10]

With all these coefficients known one now forms

$$\kappa=\kappa_0+\theta. \tag{5}$$

In order to obtain the longitude λ of the planet one still needs the longitude λ_A of the apogee which has a fixed distance c from Regulus[11]:

$$\lambda_A=\lambda_R+c. \tag{6}$$

The values of c are[12]

$$\begin{array}{llll} ♄ & c=110;30^\circ & ♀ & c=292;30^\circ \\ ♃ & 38;30 & ☿ & 67;30 \\ ♂ & 353;0 & & \end{array} \tag{7}$$

Finally

$$\lambda=\lambda_A+\kappa_0+\theta \tag{8}$$

for the true longitude of the planet.

Example. Find the longitude of Saturn for Era Philip 684 IV 28 5^h after noon.[13] For this date one finds the mean positions

$$\lambda_R=124;44, \quad \bar{\kappa}=276;38, \quad \bar{\alpha}=25;18$$

hence $k_3(\bar{\kappa})=-6;26$ and therefore $\kappa_0=283;4$ and $\alpha=18;52$. From these values it follows that

$$k_4(\kappa_0)=-0;12, \quad k_5(\alpha)=0;6, \quad k_6(\alpha)=1;50$$

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[9] Cf. Fig. 188 and 189, p. 1265 f.

[10] Cf. below p. 1013 f.; cf. also Fig. 120, p. 1416 in relation to Fig. 190, p. 1267.

[11] This constant is called ἐπίλειψις (e.g. in a marginal note in Vat. gr. 208 fol. 59r) or ἐπίληψις (e.g. Halma, H.T. II, p. 134–192; Heiberg, Anon., p. 115, 4). Ptolemy uses instead διάστασις (Canobic Inscr., Opera II, p. 152, 21).

[12] From Ptolemy's Introduction No. 10 (Heiberg, p. 169, 11 to 13; Halma H.T. I, p. 50) or [H] II, p. 134, 146, 158, 170, 182 respectively. The same numbers also in the Canobic Inscription (above Table 23). Cf. also Fig. 111 which shows the relative positions of the apsidal lines.

[13] Cf. for this date above p. 984; also Halma H.T. I, p. 50.

thus

$$\theta = k_6 + k_4 k_5 = 1;50 - 0;12 \cdot 0;6 = 1;49.$$

Consequently

$$\lambda = \lambda_R + c + \kappa_0 + \theta = 124;44 + 110;30 + 283;4 + 1;49 = 160;7 = ♍ 10;7.$$[14]

Graphical Methods. In Sect. 9 of his Introduction[15] Ptolemy gives the rules, first for Mercury, then for the remaining planets, for how to graphically find their longitudes for a given moment. These rules contain nothing new. On a board a circle is drawn with properly divided circumference. On it one marks the position of Regulus for the given time and with respect to it the position of the apogee of the planet, i.e. $\lambda_A = \lambda_R + c$ as in (6), p. 1003. Now one draws the eccenter and the epicycle with their proper parameters and places the planet onto the epicycle; then one reads on the circumference the longitude of the point which is met by the straight line from the center to the planet. For Mercury the center of the eccenter has to be placed on its little eccenter but all these rules are nothing more than the description of the basic model.

This "graphical" method avoids the use of the tables for the equation, but this cannot mean much gain of time since all mean positions have to be computed in any case, and there is certainly a loss in accuracy to be expected.[16]

2. Epoch Values. The new tables were based on the parameters of the Almagest, as can be shown, e.g., by comparing the epoch values for Nabonassar 1 and for Philip 1. We first compute the longitudes λ_A of the apsidal lines from

$$\lambda_A = \lambda_R + c$$

(cf. above p. 1003) with $\lambda_R = 117;54 = ♋ 27;54$ for Regulus.[1] The results are shown in Table 37A; they agree exactly with the positions given for Philip 1 in the "Planetary Hypotheses."[2] Table 37B gives also the positions for Nabonassar 1 as listed in the tables in Alm. IX, 4. From this it is seen that all longitudes, with one exception, have increased by 4;14° during the 424 Egyptian years between the two epochs, in agreement with Ptolemy's constant of precession.[3] Replacing 2;9 by 2;10 would also eliminate the exceptional value for Jupiter. In Table 37C the mean motion $\Delta\bar{\lambda}$ for 424 years is added to the epoch value for Nab. 1 as given in Alm. IX, 4. The same result should be obtained from the Handy Tables by adding $\bar{\kappa}$, as given for Philip 1, to λ_A. Except for two deviations of 0;1°[4] this is indeed the case. The epoch values for the epicyclic mean anomaly $\bar{\alpha}$ of the Handy Tables for Philip 1 are obtained exactly by adding the increments $\Delta\bar{\alpha}$

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[14] Tuckerman, Tables, give $\lambda = 163;10$ for A.D. 360 Sept. 19. Precession accounts for an error of 1° but at epoch Regulus had the longitude $\approx 117;40°$ instead of 117;54°.

[15] Opera II, Heiberg, p. 167, 22–169, 5. Halma H.T. I, p. 10f.

[16] The beginning of the text seems to be corrupt since it speaks about the "latitude" of Regulus and about a "return" of the apsidal line.

[1] Cf. p. 986.

[2] Cf. above Table 23.

[3] Indeed 424/100 = 4;14,24.

[4] In the case of Saturn the deviation amounts to 0;2° when we use ♑ 26;43 for the epoch value, as found in all MSS of the Almagest. Ptolemy's own computation in Alm. XI, 8 would, however, result in ♑ 26;44 and, if accurately carried out, even in ♑ 26;45. This would eliminate the above mentioned deviation. Cf. also p. 182, note 15 and p. 912, n. 5.

Table 37

	c	$\lambda_A = \lambda_R + c$
♄	110;30	228;24 = ♏ 18;24
♃	38;30	156;24 = ♍ 6;24
♂	353; 0	110;54 = ♋ 20;54
♀	292;30	50;24 = ♉ 20;24
☿	67;30	185;24 = ♎ 5;24

A

	Alm. IX, 4 Nab. 1	H.T. Phil. 1	$\Delta\lambda_A$
♄	♏ 14;10	♏ 18;24	4;14
♃	♍ 2; 9!	♍ 6;24	4;15!
♂	♋ 16;40	♋ 20;54	4;14
♀	♉ 16;10	♉ 20;24	4;14
☿	♎ 1;10	♎ 5;24	4;14

B

	Alm. IX, 4		Phil. 1	H.T.		Phil. 1
	Nab. 1	$\Delta\bar{\lambda}$	$\bar{\lambda}$	λ_A	$\bar{\kappa}$	$\bar{\lambda}$
♄	♑ 26;43	142;41	79;24	228;24	211; 2	79;26
♃	♎ 4;41	264; 2	88;43	156;24	292;20	88;44
♂	♈ 3;32	103;29	107; 1	110;54	356; 7	107; 1
♀	♓ 0;45	256;55	227;40	50;24	117;17!	227;41!
☿				185;24	42;16	227;40

C

	Alm. IX, 4		Phil. 1	H.T.
	Nab. 1	$\Delta\bar{\alpha}$	$\bar{\alpha}$	$\bar{\alpha}$
♄	34; 2	114;14	148;16	148;16
♃	146; 4	352;53	138;57	138;57
♂	327;13	153;26	120;39	120;39
♀	71; 7	10;54	82; 1	82; 1
☿	21;55	192;45	214;40	214;40

D

from Alm. IX, 4 to the epoch values for Nab. 1 (cf. Table 37 D). This agreement would be disturbed if the equation of time (about $-1/2$ hour[5]) were taken into consideration,[6] as Ptolemy instructs us to do[7] and as actually was done for the moon.[8]

3. Stationary Points. The tables in Alm. XII, 8 give for each of the five planets the epicyclic anomaly α at which the planet appears stationary as function of the mean eccentric anomaly $\bar{\kappa}$ of the center of the epicycle.[1] These tables proceed in steps of 6° from $\bar{\kappa}=0°$ to $\bar{\kappa}=180°$ and back again.

[5] Cf. above p. 984.
[6] The deviations would reach 0;4° in the case of Mercury. Less outspoken, but still noticeable, would be the disagreement for $\bar{\lambda}$.
[7] Introduction No. 2 (Heiberg, p. 163, 7 to 23; Halma H.T. I, p. 5f.).
[8] Above p. 987f.
[1] Cf. p. 202f.

Similar tables are given in the Handy Tables,[2] proceeding, however, in steps of 3° from 3° to 180° (and back to 357°). The extremal values are practically identical with the parameters in the Almagest but for intermediate values one finds systematic deviations which can exceed 1° in the case of Mars.[3] Obviously the independent variable is no longer $\bar{\kappa}$ but the true epicyclic anomaly κ.[4] The transformation of the old tables to the new ones is theoretically not difficult but must have involved a great deal of labor.

B. Latitudes

1. Introduction and Outline. The theoretical and empirical foundations upon which rest the tables for the planetary latitudes in the Almagest are fully explained in the accompanying text. For the Handy Tables, however, little more is at our disposal than the tables themselves. Thus one is faced with the problem of determining the underlying parameters as well as the theoretical model from the numbers in the tables and from the rules for their use. We owe the solution of this problem to van der Waerden[1] whose method we essentially follow in the subsequent discussion.

Using the Handy Tables the latitude of a planet has to be computed from two components, β_0 and β_1. The first, β_0, depends on the fixed angle i_0 between ecliptic and deferent, the second, β_1, on i_1 which is the angle between deferent and epicycle. Both components are functions of the given epicyclic anomaly α of the planet.

The function $\beta_0(\alpha)$ is computed under the assumption that the plane of the epicycle coincides with the plane of the deferent and that the center C of the epicycle is at mean distance from O. Let ω be the argument of latitude counted from the northernmost point N of the deferent (cf. Fig. 112). Then the planet has the argument of latitude

$$\omega_P = \omega_A + \kappa. \tag{1}$$

Consequently (cf. Fig. 113) the latitude of P under the specified conditions is given by

$$\beta_0 = \pm i_0 c_7(\omega_P) \quad \begin{cases} \text{if } 270° < \omega_P < 90° \\ \text{if } \;\; 90° < \omega_P < 270°. \end{cases} \tag{2}$$

The modern equivalent of the tabulated function $c_7(\omega_P)$, including the sign rules, is simply $\cos\omega_P$.[2] For the values of i_0 cf. below p. .

For the computation of β_1 the assumption is made that P is at maximum distance

$$H = r \sin i_1 \tag{3}$$

to the north of the plane of the deferent (cf. Fig. 117, p. 1415), whatever the given value of the epicyclic anomaly α. Under this assumption three functions are computed. First $c_5(\alpha)$ which represents the distance from the plane of the deferent of the planet P as seen from O, assuming that OC is the mean distance. This

[2] [H] III, p. 11–15; [V] fol. 83ᵛ–85ʳ.
[3] E.g. 162;47 against 161;44 in the Almagest at 84°.
[4] Cf. also Halma I, p. 59 f.
[1] Van der Waerden [1953] and [1958, 1].
[2] The values given for c_7 in the Handy Tables agree with the values of c_5 in Almagest XIII, 5 when rounded to nearest minutes; cf. also p. 219 (6) and note 3.

relative latitude β' becomes smaller when C is at maximum distance (thus $\kappa_0=0$), and greater for minimum distance OC. The absolute values for the corresponding increments at the two extremal distances are tabulated as $c_4(\alpha)$ and $c_6(\alpha)$, respectively (cf. Figs. 119 and 120, p. 1415f.). All three functions c_4, c_5, c_6 are always reckoned as positive.[3]

Finally a coefficient of interpolation $c_3(\kappa_0)$ is tabulated which has the value -1 at the maximum distance of C from O ($\kappa_0=0°$), $+1$ at the minimum distance ($\kappa_0=180°$, or 120° for Mercury). For an arbitrary value of κ_0 the relative latitude β' is given by

$$\beta'(\alpha,\kappa_0)=c_5(\alpha)+c_3(\kappa_0)\cdot\begin{cases}c_4(\alpha) & \text{if } c_3(\kappa_0)\leqq 0\\ c_6(\alpha) & \text{if } c_3(\kappa_0)\geqq 0.\end{cases} \tag{4}$$

The function β' is also always positive.

Since this computation is based on the assumption that P was at maximum positive distance from the plane of the deferent one has now to reduce β' depending on the angular distance

$$\Omega_P=\Omega_A+\kappa_0+\alpha \tag{5}$$

of P from the highest point B of the epicycle (cf. Fig. 114).[4] With this Ω_P as argument one finds in the last column of the tables, as in (2), the coefficient c_7:

$$c_7(\Omega_P)\begin{cases}\geqq 0 & \text{if } 270°\leqq\Omega_P\leqq 90°\\ \leqq 0 & \text{if } 90°\leqq\Omega_P\leqq 270°.\end{cases} \tag{6}$$

This then gives the (positive or negative) latitudinal component $\beta_1=c_7\beta'$ or with (4):

$$\beta_1(\alpha,\kappa_0)=c_7(\Omega_P)\cdot\left(c_5(\alpha)+c_3(\kappa_0)\cdot\begin{cases}c_4(\alpha)) & \text{if } c_3\leqq 0\\ c_6(\alpha)) & \text{if } c_3\geqq 0.\end{cases}\right. \tag{7}$$

Finally, with (2), one has

$$\beta=\beta_0+\beta_1 \tag{8}$$

for the latitude of the planet.

2. Examples. In the following we compute the latitudes of Mars (Table 38) and of Venus (Table 39) for the same sequence of dates for which we determined the latitudes by means of the tables of the Almagest. The values of α, κ_0, and κ are taken from the previously computed examples[5] since the results obtained from the Almagest and from the Handy Tables can only differ by some insignificant rounding errors.

The computation of latitudes with the Handy Tables is a little longer than with the Almagest but much easier with respect to interpolation because of a uniform interval for all entries (steps of 3°) and by restricting all numbers to minutes only. This latter fact, however, sometimes produces rather irregular first differences.

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[3] In Halma's text (H.T. III, p. 1 to 10, with one exception) c_4 is denoted as μεγίστου πλάτους "plus grande latitude." c_6 as ἐλαχίστου πλάτους "moindre latitude." This makes no sense; indeed Vat. gr. 1291 (fol. 78r to 82v) gives ὑπεροχή "excess" instead of latitude.

[4] Equivalent to (5) is $\Omega_P=\Omega_A+\bar{\kappa}+\bar{\alpha}$ because the planet moves with respect to B with constant angular velocity.

[5] Above Table 21 and Table 22, respectively (p. 220 and 225).

Table 38

No.	Date: Nab.	Date: julian	$\omega_p = \kappa$	$\beta_0 = c_7(\omega_p)$	α	$c_5(\alpha)$	κ_0	$c_3(\kappa_0)$	$c_6(\alpha)$	$c_3 \cdot c_6$	$c_5 + c_3 \cdot c_6$	$\Omega_p = \alpha + \kappa_0 + \Omega_A$	$c_7(\Omega_p)$	$\beta_1 = c_7(c_5 + c_3 c_6)$	β: $\beta_0 + \beta_1$	β: Alm.	β: modern	No.
	450	−297																
1	X 8	Aug. 10	296;23	+0;26	129;46	1;54	253;40	+0;17	0;32	+0; 9	2; 3	203;26	−0;55	−1;53	−1;27	−0;34	−1; 8	1
2	18	20	301;31	0;31	134; 3	2; 1	259;15	+0;12	0;36	+0; 7	2; 8	213;18	0;50	1;47	1;16	−0;26	0;59	2
3	28	30	306;12	0;35	138;25	2;10	264;44	+0; 7	0;40	+0; 5	2;15	223; 9	0;43	1;37	1; 2	−0;14	0;49	3
4	XI 8	Sept. 9	310;16	0;39	142;53	2;20	270; 8	+0; 3	0;46	+0; 2	2;22	233; 1	0;36	1;25	0;46	0	0;37	4
5	18	19	313;38	0;41	147;26	2;31	275;26	−0; 3	0;28	−0; 1	2;30	242;52	0;27	1; 8	0;27	+0;16	0;22	5
6	28	29	316; 9	+0;43	152; 4	2;43	280;40	−0; 9	0;32	−0; 5	2;38	252;44	−0;17	−0;45	−0; 2	+0;34	−0; 4	6
7	XII 8	Oct. 9	317;36	0;44	156;49	3; 0	285;46	0;15	0;37	0; 9	2;51	262;35	−0; 7	−0;20	+0;24	0;54	+0;16	7
8	18	19	317;50	0;44	161;41	3;24	290;45	0;21	0;42	0;15	3; 9	272;26	+0; 2	+0; 6	+0;50	1;17	+0;40	8
9	28	29	316;35	0;43	166;37	3;41	295;41	0;26	0;47	0;20	3;21	282;18	+0;12	+0;40	+1;23	1;42	+1; 6	9
10	I 3	Nov. 8	314;14	0;41	171;37	3;59	300;32	0;31	0;51	0;26	3;33	292; 9	+0;22	+1;18	+1;59	2; 7	+1;34	10
	451																	
11	13	18	310;52	+0;39	176;41	4;13	305;20	−0;35	0;56	−0;33	3;40	302; 1	+0;32	+1;57	+2;36	+2;29	+1;59	11
12	23	28	307; 3	0;36	181;49	4;16	310; 2	0;39	0;56	0;36	3;40	311;51	0;40	2;27	3; 3	2;46	2;20	12
13	II 3	Dec. 8	303;32	0;33	187; 4	4; 3	314;39	0;43	0;53	0;38	3;25	321;43	0;47	2;41	3;14	2;57	2;34	13
14	13	18	301; 7	0;31	192;20	3;45	319;14	0;46	0;48	0;37	3; 8	331;34	0;53	2;46	3;17	3; 0	2;41	14
15	23	28	299;49	0;30	197;40	3;27	323;46	0;49	0;42	0;34	2;54	341;26	0;57	2;45	3;15	2;56	2;42	15
		−296																
16	III 3	Jan. 7	299;56	+0;30	203; 3	3; 1	328;14	−0;51	0;37	−0;31	2;30	351;17	+0;59	+2;28	+2;58	+2;50	+2;40	16
17	13	17	301; 1	0;31	208;29	2;42	332;39	0;53	0;32	0;28	2;14	1; 8	1; 0	2;14	2;45	2;39	2;36	17
18	23	27	303; 4	0;33	213;57	2;28	337; 3	0;55	0;28	0;26	2; 2	10;59	0;59	2; 0	2;33	2;28	2;31	18
19	IV 3	Febr. 6	305;53	0;35	219;26	2;15	341;25	0;57	0;24	0;23	1;52	20;51	0;56	1;45	2;20	2;15	2;25	19
20	13	16	309;18	0;38	224;58	2; 3	345;45	0;59	0;22	0;21	1;42	30;43	0;52	1;28	2; 6	2; 5	2;19	20
21	23	26	313;17	+0;41	230;31	1;54	350; 3	−0;59	0;19	−0;19	1;35	40;34	+0;45	+1;11	+1;52	+1;54	+2;13	21
22	V 3	March 7	317;40	0;44	236; 4	1;45	354;22	1; 0	0;17	0;17	1;25	50;26	0;38	0;56	1;40	1;42	2; 7	22
									$c_4(\alpha)$	$c_3 \cdot c_4$	$c_5 + c_3 \cdot c_4$			$c_7 \cdot (c_5 + c_3 c_4)$				

Table 39. Venus, Handy Tables

No.	Date Nab.	Date julian		$\omega_p = \kappa$	$c_7(\omega_p)$	$\beta_0 = i_0 \cdot c_7$	α	$c_5(\alpha)$	κ_0	$c_3(\kappa_0)$	$c_6(\alpha)$	$c_3 \cdot c_6$	$c_5 + c_3 c_6$	$\Omega_p = \alpha + \kappa_0 + \Omega_A$	$c_7(\Omega_p)$	$\beta_1 = c_7 \cdot (c_5 + c_3 c_6)$	β: $\beta_0 + \beta_1$	β: Alm.	β: modern	No.
	442	−305																		
1	XII 13	Oct.	16	188; 3	−0;59	−0;10	159;29	6;14	148;50	+0;51	0;25	+0;21	6;35	218;19	−0;46	−5; 3	−5;13	−3;46	−5; 0	1
2	18		21	190;16	0;59	0;10	162;23	6;43	153;57	0;54	0;29	0;26	7; 9	226;20	0;41	4;53	5; 3	3;31	4;55	2
3	23		26	191;43	0;58	0;10	165;17	7;12	159; 4	0;56	0;32	0;30	7;42	234;21	0;35	4;30	4;40	3; 8	4;41	3
4	28		31	192;17	0;58	0;10	168;10	7;40	164;11	0;58	0;34	0;33	8;13	242;21	0;28	3;50	4; 0	2;39	4;16	4
5	ep. 3	Nov.	5	191;53	0;58	0;10	171; 3	8; 2	169;19	0;58	0;36	0;35	8;37	250;22	0;20	2;52	3; 2	2; 0	3;38	5
	443																			
6	I 3		10	190;28	−0;59	−0;10	173;56	8;24	174;27	+0;59	0;38	+0;37	9; 1	258;23	−0;12	−1;48	−1;58	−1;13	−2;47	6
7	8		15	188;19	0;59	0;10	176;48	8;39	179;35	1; 0	0;39	0;39	9;18	266;23	−0; 4	−0;37	−0;47	−0;20	−1;43	7
8	13		20	185;39	0;59	0;10	179;40	8;50	184;44	1; 0	0;40	0;40	9;30	274;24	+0; 4	+0;38	+0;28	+0;39	−0;28	8
9	18		25	182;53	1; 0	0;10	182;33	8;42	189;52	0;59	0;39	0;38	9;20	282;25	+0;12	+1;52	+1;42	+1;34	+0;50	9
10	23		30	180;36	1; 0	0;10	185;26	8;27	194;59	0;58	0;38	0;36	9; 3	290;25	+0;20	+3; 1	+2;51	+2;23	+2; 2	10
11	28	Dec.	5	179; 4	−1; 0	−0;10	188;19	8; 7	200; 7	+0;56	0;36	+0;34	8;41	298;26	+0;28	+4; 3	+3;53	+3; 5	+3; 4	11
12	II 3		10	178;28	1; 0	0;10	191;12	7;45	205;15	0;55	0;35	0;32	8;17	306;27	0;35	4;50	4;40	3;38	3;52	12
13	8		15	178;46	1; 0	0;10*	194; 7	7;18	210;21	0;52	0;33	0;29	7;47	314;27	0;42	5;27	5;17	4; 1	4;26	13
14	13		20	180; 0	1; 0	0;10	197; 2	6;49	215;26	0;49	0;29	0;24	7;13	322;28	0;47	5;39	5;29	4;17	4;47	14
15	18		25	182; 3	1; 0	0;10	199;57	6;19	220;32	0;47	0;26	0;20	6;39	330;29	0;52	5;46	5;36	4;22	4;58	15
16	23	−304	30	184;44	−1; 0	−0;10	202;53	5;51	225;37	+0;42	0;24	+0;17	6; 8	338;30	+0;56	+5;43	+5;33	+4;20	+4;59	16
17	28	Jan.	4	188; 0	0;59	0;10	205;49	5;25	230;41	0;38	0;22	0;14	5;39	346;30	0;59	5;33	5;23	4;12	4;54	17

$\omega_A = 0°$, $i_0 = 0;10°$, $\Omega_A = -90°$

Comparison of the present results (Figs. 115 and 116) with the latitudes from the Almagest[6] and with modern values[7] shows some large deviations from the Almagest and a better agreement with modern values. But only a systematic analytic representation could show how far this is generally the case. It seems clear, however, that the better results for Venus are related to a definite improvement for the inclination i_1 (cf. below p. 1015). In comparing the ancient computations with modern results one should also keep in mind that we have chosen a date 4 1/2 centuries distant from Ptolemy's time; yet the general agreement is still remarkably good. The error in the constant of precession should cause longitudes of about 1 1/2° too great but actually in most cases the differences are much smaller.

3. Numerical Data. From the introduction to the Handy Tables or from Theon's Commentary we know that the following parameters were used for the computation of the tables:

	i_0	ω_A	Ω_A	
♄	2;30°	40°	220°	
♃	1;30	−20	160	$=\omega_A+180$
♂	1;0	0	180	
♀	0;10	0	270	$=\omega_A-90$
☿	0;10	180	90	

For the outer planets the values of i_0 are exactly the same as in the Almagest. For the inner planets a fixed inclination of 0;10° has replaced the variable inclinations with amplitudes in opposite directions.

The nodal lines agree with the Almagest, excepting Saturn for which the nodal line has been moved 10° forward in longitude. For the outer planets the nodal lines of the epicycles are parallel to the nodal lines of the deferent (cf. Fig. 114, p. 1412). For the inner planets the epicyclic nodal lines are parallel to the apsidal lines and therefore perpendicular to the nodal lines of the deferent.

Determination of i_1. The values assumed for the inclinations i_1 between the planes of the epicycles and of the deferents are not explicitly mentioned in the available texts. Consequently one has to determine i_1 for each planet from the numerical tables. Best suited for this is the function $c_5(\alpha)$ because it gives for the mean distance OC (cf. Fig. 117) the latitude of the planet relative to the plane of the deferent under the assumption that the planet is at its maximum distance $H = r \sin i_1$ from the plane of the deferent.

To derive from the tabulated values of $c_5(\alpha)$ the value of the underlying angle i_1 one can apply the same method which Ptolemy used for the determination of the inclination of the epicycle from observations.[1] We can interpret (cf. Fig. 117) the value $c_5(0)$ as the epicyclic equation which belongs to an anomaly of $\pm i_1$ and

[6] Above Fig. 228a, p. 1283 and Fig. 233a, p. 1285.
[7] Taken from Tuckerman, Tables I.
[1] Above p. 209 and p. 215.

similarly $c_5(180)$ as the equation for the anomaly $180 \pm i_1$. Thus we can use the tables for the planetary equations[2] to find the angle i_1 which produces the given equations c_5. In this way[3] one obtains the following values for i_1:

	♄	♃	♂	♀	☿	
from $c_5(0)$	4;30°	2;33°	2;15°	3;29°	6;28°	
from $c_5(180)$	4;34	2;30	2;15	3;30	6;29	
rounded	4;30	2;30	2;15	3;30	6;30	
Almagest	4;30	2;30	2;15	2;30	6;15	i_1
				3;30	7;0	i_2

Computation of $c_5(\alpha)$. With i_1 known one can now compute the latitudinal components $c_5(\alpha)$ for arbitrary values of α, i.e. the latitudes of a planet which is at a distance $H \approx r i_1$ above the plane of the deferent. Thus we should have

$$r i_1 \approx c_5(\alpha) \cdot \rho$$

where $\rho = \mathrm{OP}$ in Fig. 118. This distance can be computed from

$$\rho = \frac{R + r\cos\alpha}{\cos\theta}$$

where θ is the equation which belongs to the anomaly α tabulated as $c_6(\alpha)$ in the Almagest (XI, 11) or in the Handy Tables. The subsequent table gives a comparison between values computed by this method and the corresponding numbers found in the Handy Tables.

	Saturn		Jupiter		Mars		Venus		Mercury	
α	comp.	H.T.	comp.	H.T.	comp.	H.T.	comp.	H.T.	comp.	H.T.
0	0;27°	0;27°	0;25°	0;25°	0;54°	0;54°	1;28°	1;28°	1;46°	1;46°
60	0;29	0;29	0;27	0;28	1; 2	1; 1	1;41	1;44	1;59	1;58
90	0;30	0;30	0;29	0;30	1;14	1;11	2; 3	2; 5	2;17	2;18
120	0;32	0;32	0;32	0;32	1;41	1;39	2;49	2;57	2;47	2;49
180	0;33	0;34	0;37	0;36	4;20	4;20	8;58	8;51	3;54	3;52

The deviations generally remain $\leq 0;3°$ except for Venus where one finds irregular differences up to 0;8°.

For a graphical representation of the tabulated values of $c_5(\alpha)$ cf. Fig. 119. The trend of these curves clearly reflects the dimensions of the geometrical model schematically represented in Fig. 117, but very different in size for the different planets.

[2] Almagest XI, 11 or the equivalent Handy Tables.

[3] If one repeats the process for maximum or minimum distances OC one finds only insignificantly different values for i_1.

Computation of $c_4(\alpha)$ and $c_6(\alpha)$. We operate here with the same method which we used for the determination of i_1 (above p. 1010). For $\alpha=0°$ and $\alpha=180°$ we can make use of Fig. 117 (p. 1415), only changing the distance OC to its maximum value when we wish to find $c_4(0)$ and $c_4(180)$; i.e. the decrease of $c_5(0)$ and $c_5(180)$, respectively; and similarly for c_6 by giving OC its minimum value.

We again interpret the situation at $\alpha=0$ as the equivalent of a position of the planet at the epicyclic anomaly i_1 whereas $\alpha=180$ corresponds to the anomaly $180 \pm i_1$. Hence we can use the tables for the epicyclic equations to find the absolute values of the increments at extremal distances OC.

In order to distinguish the numbers obtained from the epicyclic equations from the coefficients c_4 and c_6 in the tables of latitudes we denote the values which come from the tables of epicyclic equations by primes, i.e. c'_5 for the case of far distances, c'_7 for the near distances. Then we should expect

$$c'_5(i_1)=c_4(0), \qquad c'_7(i_1)=c_6(0),$$
$$c'_5(180+i_1)=c_4(180), \quad c'_7(180+i_1)=c_6(180).$$

The following table shows that these relations are indeed satisfied except for $c_4(0)$ and $c_6(0)$ for Saturn and, with a very small deviation, for $c_4(180)$ for Venus.

		Saturn	Jupiter	Mars	Venus	Mercury
	i_1	4;30	2;30	2;15	3;30	6;30
epicyclic equations	$c'_5(i_1)$	0; 1,30	0; 1	0; 3,15	0; 1	0;11
	$c'_7(i_1)$	0; 2	0; 1	0; 3,15	0; 1	0; 5,30
	$c'_5(180+i_1)$	0; 3	0; 2,30	0;57,15	0;35,30	0;45,30
	$c'_7(180+i_1)$	0; 3	0; 2,30	1;46,15	0;40	0;30,30
latitudes	$c_4(0)$	0; 1	0; 1	0; 3	0; 1	0;11
	$c_6(0)$	0; 1	0; 1	0; 3	0; 1	0; 5
	$c_4(180)$	0; 3	0; 3	0;57	0;37	0;45
	$c_6(180)$	0; 3	0; 3	1;46	0;40	0;30

In order to compute c_4 and c_6 for general values of α we denote by $\rho_M(\alpha)$ the distance OP when C is at its maximum distance from O, and similarly $\rho_m(\alpha)$ at the minimum distance OC. Then we have

$$\begin{aligned}\rho_M(\alpha)\cdot(c_5(\alpha)-c_4(\alpha))&=r i_1.\\ \rho_m(\alpha)\cdot(c_5(\alpha)+c_6(\alpha))&=r i_1\end{aligned} \tag{1}$$

for the same reason that we had for the mean distance

$$\rho(\alpha)\cdot c_5(\alpha)=r i_1. \tag{2}$$

From (1) and (2) it follows that

$$c_4(\alpha)=r i_1\left(\frac{1}{\rho(\alpha)}-\frac{1}{\rho_M(\alpha)}\right)$$
$$c_6(\alpha)=r i_1\left(\frac{1}{\rho_m(\alpha)}-\frac{1}{\rho(\alpha)}\right).$$

Except for Mercury we may assume that approximately

$$\rho_{\mathrm{M}} - \rho = \rho - \rho_{\mathrm{m}} = e \tag{3}$$

e being the eccentricity. For Mercury, however, we have

$$\rho_{\mathrm{M}} - \rho = 3e, \qquad \rho - \rho_{\mathrm{m}} = 60 - 55;34 = 4;26.^{4}$$

Thus we have in general

$$c_4(\alpha) \approx \frac{e r i_1}{\rho(\alpha)\rho_{\mathrm{M}}(\alpha)} \qquad c_6(\alpha) \approx \frac{e r i_1}{\rho(\alpha)\rho_{\mathrm{m}}(\alpha)} \tag{4}$$

and similarly for Mercury with $3e$ instead of e in the first formula and with 4;26 instead of e in the second.

The resulting values agree well with the values found in the tables, except for occasional deviations up to 3 minutes. These deviations are probably caused in part by differences between our trigonometric functions and values which had to be derived in each case from a table of chords only; in fact Fig. 120 shows clearly that certain sections were obtained by linear interpolation causing irregular differences. But there can be little doubt that the functions c_4, c_5, c_6 had been computed with methods corresponding to the procedures outlined here.

Computation of $c_3(\kappa_0)$. The latitude of a planet at maximum height above the deferent, at anomaly α, and at mean distance of the epicycle from the observer at O is given by $c_5(\alpha)$. At maximum distance, but otherwise identical conditions the latitude is $c_5(\alpha) - c_4(\alpha)$, at minimum distance $c_5(\alpha) + c_6(\alpha)$. For all planets the maximum distance of the epicycle corresponds to $\kappa_0 = 0$; the minimum distance occurs for Mercury at $\kappa_0 = 120$, for all other planets at $\kappa_0 = 180$. We now must construct a function $c_3(\kappa_0)$ which has the value -1 for $\kappa_0 = 0$ and the value $+1$ for $\kappa_0 = 120$ or $\kappa_0 = 180$, respectively. Intermediate values of κ_0 should produce (in the form (4), p. 1007) a continuous transition between the situation at apogee and at perigee.

Exactly the same interpolation problem arose in the computation of the epicyclic equation for the longitudes of the planets. The solution given in the Almagest operated with the following idea.[1] One considers an interpolation function valid for each constant anomaly if it is correct for the maximum equations.[2] Then one constructs a function ($c_8(\bar{\kappa})$ in the tables of Almagest XI, 11) which is given by the ratio $\dfrac{\theta(\bar{\kappa}) - \mu}{\mu - M}$ for the interval from maximum to mean distance, and by $\dfrac{\theta(\bar{\kappa}) - \mu}{m - \mu}$ from mean to minimum distance, where m, μ, M, represent the values of the maximum equation at minimum, mean, and maximum distance, respectively.

In the present case we are dealing with latitudes instead of with equations. But one could form similar functions of interpolation by computing, e.g., the maximum latitudes (i.e. for $\alpha = 180$) at minimum, mean, and maximum distance of the center

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[4] Cf. p. 164.

[1] Cf. above p. 185. Similarly for the moon: p. 94 ff.

[2] Which, incidentally, do not occur for all distances at the same anomaly.

C of the epicycle from O. This, however, leads to rather poor agreement with the values tabulated as $c_3(\kappa)$ in the Handy Tables. Apparently there is no compelling reason for the choice of the latitudes at $\alpha=180$ as the key values for the interpolation at all other anomalies.

The solution of the problem is found in a different direction: one simply used the same interpolation function both for equations and for latitudes. This is an interesting case, and probably the earliest one, of an attitude toward problems of interpolation which became quite common in mediaeval (particularly Indian) astronomy; namely, the use as interpolation function of a known smooth function (e.g. trigonometric function) which satisfies the proper boundary conditions but no longer has a direct relation to the specific geometric situation.

The identity of the functions used for the interpolation as a function of the distance CO for the epicyclic equations and for the latitudes is not directly apparent from the tabulated numbers. The reason for this lies in the change of the independent variable from $\bar{\kappa}$ (i.e. the mean eccentric anomaly of C with respect to the equant) to κ_0 (i.e. the true eccentric anomaly with respect to O). Furthermore one obtains accurate numerical agreement only if one computes with the numerical values given in the Handy Tables, tabulated for integer values of the argument, because interpolations and roundings occasionally produce values slightly different from those one would obtain from the tables in the Almagest (XI, 11).

To illustrate the procedure we take, e.g., $\bar{\kappa}=60^\circ$ for Mars. Then the Handy Tables give for the equation of the eccenter $c_3(\bar{\kappa})=-9;24$.[3] This tells us that C has the true eccentric anomaly $\kappa_0=60-9;24=50;36$. We now use the same tables once more with the argument $\kappa_0=50;36$ and find as coefficient of interpolation $c_4(\kappa_0)=-0;38$. It is this value which one also finds in the tables for latitudes for the argument $\kappa_0=51^\circ\approx 50;36$.

The following Table 40 shows for a few typical entries the excellent agreement obtained in this way.[4] The few deviations of at most 2 minutes are easily explicable as caused by rounding errors.[5]

In the graph for $c_3(\kappa_0)$ in Fig. 120 the differences between the four planets, except Mercury,[6] are so small that one single curve suffices. As stated before the coefficients $c_3(\kappa_0)$ are identical with the coefficients $k_4(\kappa_0)$ for the planetary equations[7].

4. **Extremal Latitudes.** The Handy Tables list the following extremal planetary latitudes:

$$♄:\begin{cases}+3;2^\circ\\-3;6\end{cases}\quad ♃:\begin{cases}+2;3^\circ\\-2;9\end{cases}\quad ♂:\begin{cases}+4;23^\circ\\-7;\ 6\end{cases}\quad ♀:\pm 8;56^\circ\quad ☿:\pm 4;18^\circ. \qquad (1)$$

For the outer planets these values deviate only in some cases by 1 or 2 minutes from the extrema obtained in the Almagest.[1] The extrema for Mercury are $\pm 0;13^\circ$

[3] This is the same as $c_3(\bar{\kappa})+c_4(\bar{\kappa})$ in Alm. XI, 11.

[4] (λ) means the tables concerning the epicyclic equations, (β) the tables for latitudes.

[5] Of course ignoring the misprints and errors of reading and transcription in Halma's edition.

[6] When C is at mean distance from O then $c_3(\kappa_0)=0$. According to Almagest XI, 11 this occurs at $\kappa_0=64;35$ ($\bar{\kappa}=67;13$) whereas I find $\kappa_0=65;5$ ($\bar{\kappa}=67;45$).

[7] Above p. 1003.

[1] Cf. p. 226.

Table 40

$\bar{\kappa}$	Saturn (λ) $c_3(\bar{\kappa})$	Saturn (λ) κ_0	Saturn (λ) $c_4(\kappa_0)$	Saturn (β) $c_3(\kappa_0)$	Jupiter (λ) $c_3(\bar{\kappa})$	Jupiter (λ) κ_0	Jupiter (λ) $c_4(\kappa_0)$	Jupiter (β) $c_3(\kappa_0)$	Mars (λ) $c_3(\bar{\kappa})$	Mars (λ) κ_0	Mars (λ) $c_4(\kappa_0)$	Mars (β) $c_3(\kappa_0)$	$\bar{\kappa}$
30	−3; 6	26;54	−0;54	−0;52	−2;31	27;29	−0;54	−0;54	− 5;16	24;44	−0;54	−0;55	30
60	−5;29	54;31	−0;35	−0;35	−4;27	55;33	−0;34	−0;34	− 9;24	50;36	−0;38	−0;38	60
90	−6;31	83;29	−0; 4	−0; 4	−5;15	84;45	−0; 3	−0; 3	−11;23	78;37	−0;10	−0;10	90
120	−5;49	114;11	+0;25	+0;26	−4;40	115;20	+0;25	+0;25	−10;22	109;38	+0;21	+0;21	120
150	−3;24	146;36	+0;50	+0;50	−2;43	147;17	+0;59	+0;50	− 6;10	143;50	+0;48	+0;48	150

$\bar{\kappa}$	Venus (λ) $c_3(\bar{\kappa})$	Venus (λ) κ_0	Venus (λ) $c_4(\kappa_0)$	Venus (β) $c_3(\kappa_0)$	Mercury (λ) $c_3(\bar{\kappa})$	Mercury (λ) κ_0	Mercury (λ) $c_4(\kappa_0)$	Mercury (β) $c_3(\kappa_0)$	$\bar{\kappa}$
30	−1;12	28;48	−0;53	−0;53	−1;17	28;43	−0;45	−0;45	30
60	−2; 4	57;56	−0;32	−0;32	−2;25	57;35	−0;10	−0;10	60
90	−2;23	87;37	−0; 1	−0; 1	−3; 1	86;59	+040	+0;40	90
120	−2; 5	117;55	+0;29	+0;29	−2;41	117;19	+1; 0	+1; 0	120
150	−1;12	148;48	+0;51	+0;51	−1;32	148;28	+0;49	+0;49	150
					0; 0	180; 0	+0;40	+0;40	180

greater than in the Almagest; but for Venus they are changed from $\pm 6;22°$ to $\pm 8;56°$, now practically correct.[2] This increase is also reflected in a change of the inclination i_1 (cf. Fig. 219, p. 1279) from 2;30° to 3;30° which makes $i_1 = i_2$. This represents a definite improvement over the original parameters for the latitude of Venus; our computed examples (Fig. 116a and b, p. 1414) illustrate the results.

The values given above in (1) can be checked by direct computation from the tables, taking as epicyclic anomaly $\alpha = 180°$ and for the center of the epicycle of the outer planets the points of extremal latitude of the deferent, for the inner planets the nodes of the deferent.

Outer Planets. If the center of the epicycle is either at the northernmost or at the southernmost point of the deferent we have, respectively

$$\omega_P = \Omega_P = 0 \qquad \text{or } 180$$

thus

$$c_7(\omega_P) = c_7(\Omega_P) = +1 \qquad \text{or } -1.$$

Consequently one finds $\beta_0 = \pm i_0$ as is also evident from the geometric situation.

The computation of the corresponding latitudes, as shown in the subsequent table, fully confirms the values listed in (1), p. 1014.

[2] According to modern tables (Tuckerman) the extremal values are about

$$♄: \pm 2;53° \quad ♃: \pm 1;49° \quad ♂: \begin{cases} +4;38° \\ -6;53 \end{cases} \quad ♀: \begin{cases} +8;35° \\ -8;47 \end{cases} \quad ☿: \begin{cases} +3;52° \\ -4;44 \end{cases}.$$

	Northern Latitudes			Southern Latitudes			
	Saturn	Jupiter	Mars	Saturn	Jupiter	Mars	
β_0	2;30	1;30	1; 0	−2;30	−1;30	−1; 0	β_0
$c_5(180)$	0;34	0;36	4;20	0;34	0;36	4;20	$c_5(180)$
κ_0	320	20	0	140	200	180	κ_0
$c_3(\kappa_0)$	−0;45,20	−0;57,20	−1	0;45,20	0;56,20	1	$c_3(\kappa_0)$
$c_6(180)$	0; 3	0; 3	0;57	0; 3	0; 3	1;46	$c_6(180)$
$c_3 c_6$	−0; 2,16	−0; 2,52	−0;57	0; 2,16	0; 2,49	1;46	$c_3 c_6$
β_1	0;31,44	0;33,8	3;23	−0;36,16	−0;38,49	−6; 6	β_1
β	3; 1,44	2; 3,8	4;23	−3; 6,16	−2; 8,49	−7; 6	β
Text	3; 2	2; 3	4;23	−3; 6	−2; 9	−7; 6	Text

Inner Planets. In order to obtain extremal latitudes for the inner planets we must place the center of the epicycle in the nodal line of the deferent. Then the component β_0 is zero and we are only concerned with the influence of the inclination i_1, taking as epicyclic anomaly $\alpha = 180°$.

It is evident that the tables produce exactly symmetric values for the northern and southern extrema since the nodal line is assumed to be perpendicular to the apsidal line. Consequently the values of $c_5 + c_3 c_6$ are the same for northern and for southern latitudes and only $c_7(\Omega_p)$ is either $+1$ or -1 as Ω_p shifts from 0° to 180°. Hence we have the following numerical data:

	Venus	Mercury
$c_5(180)$	8;51	3;52
κ_0	270 or 90	90 or 270
$c_3(\kappa_0)$	0; 2	0;44
$c_6(180)$	0;40	0;30
$c_3 \cdot c_6$	0; 1,20	0;22
β	±8;52,20	±4;14
Text	±8;56	±4;18

I cannot explain the consistent deviation of $\pm 0;4°$ between the extrema which result from computation with the tables and the values given for these extrema at the headings of the tables. The latter values cannot be simple errors since they appear again as extremal latitudes for the "steps," ($\beta\alpha\vartheta\mu o i$) which are exactly 1/6 of the respective greatest latitudes[3] as is seen from the following tabulation:

	Saturn	Jupiter	Mars	Venus	Mercury
extremal latitudes	+3; 2° −3; 6	+2; 3° −2; 9	+4;23° −7; 6	±8;56°	±4;18°
steps	+0;30,20 −0;31	+0;20,30 −0;21,30	+0;43,50 −1;11	±1;29,20	±0;43

[3] Cf. for the "steps" above IV B 5. The numerical data are found in Theon's commentary (Halma HT I, p. 59) and on the margin of the tables for the planetary latitudes (Halma HT III, p. 1–10 with misprints, correct in Vat. gr. 1291 fol. 78r–82v).

C. *Visibility ("Phases")*

1. Summary. In discussing the concluding chapters of the Almagest we have shown[1] that the elongation $\Delta\lambda$ of a planet from the true sun at the moment of the planet's first or last visibility was obtained from

$$\Delta\lambda = \frac{h}{\sin\nu} - \beta\cot\nu \tag{1}$$

where ν represents the eastern angle between ecliptic and horizon,[2] h the "normal arcus visionis,"[3] β the latitude of the planet. Paradoxically the tables in the Almagest which concern only one geographical latitude ("Phoenicia", between climata III and IV) are understood to a lesser degree than the corresponding tables for all seven climata in the Handy Tables.[4] For the tables in the Almagest we do not know how the angles ν were determined[5] and the computation of the latitudes for the inner planets leads to the intricate question of the choice of the epicyclic anomaly.[6]

For the Handy Tables, however, Aaboe has shown[7] that computational simplicity is the governing principle. The angles ν are taken from the Almagest (II, 13) and all latitudes are computed for $\alpha = 0°$ or $\alpha = 180°$, respectively.

New values were adopted for the arcus visionis h, perhaps based on more recent observations:

		H.T.[8]		Alm.[9]	
Saturn		13°		11°	
Jupiter		9		10	
Mars		14;30		11;30	(2)
Venus	Σ, Ξ:	7	Ξ:	5	
	Ω, Γ:	5		10	
Mercury		12		10	
			Can. Inscr.	10;30[10]	

It is an oversimplification without empirical support that h should be independent of the geographical latitude within such a wide a range as required by the seven climata.

The values given in (2) for the Handy Tables are also found in the "Planetary Hypotheses," with the only difference that an additional arcus visionis is assigned to fixed stars of the first magnitude ($h = 15°$); Ptolemy furthermore says that for

[1] Above p. 242ff.

[2] Normed according to p. 240.

[3] Cf. p. 234.

[4] [H] III, p. 16–29; [V] fol. 85^{r}–88^{v}.

[5] Excepting, of course, the trivial case of the solstices.

[6] Cf. p. 252ff.

[7] Aaboe [1960], p. 11–17.

[8] As van der Waerden has remarked ([1953], p. 271 f.) h can be obtained directly from the tables for clima II for which $\varphi = \varepsilon$ and thus $\nu = 90°$ when ♎0° is rising (Γ, Σ) and $\bar{\nu} = 270°$ when ♈0° is setting (Ω, Ξ). Hence, $\Delta\lambda = h$ in these cases.

[9] Cf. p. 235.

[10] Ptolem. Opera II. p. 153, 15.

the outer planets at acronychal rising (i.e. essentially at opposition) only about half of h is required to make the planet visible.[11]

The entries of the tables refer, as in the Almagest, to a position of the planet at ♈0°, ♉0°, etc. and the same holds for the angles ν.[12] It should also be noted that the tables were computed for conditions valid at the time of the Almagest but subject to precession, according to Ptolemy's own theory. For example, the angles between ecliptic and horizon depend on tropical longitudes, while the planetary apsides and nodal lines are sidereally fixed.[13] No provision is made to take these secular changes into account; even during the Middle Ages Ptolemy's tables were retained unchanged.

2. The Tables, Numerical Details

Symmetries. The Figs. 121 to 131 (p. 1417 ff.) give a graphic representation of the elongations $\Delta\lambda$ at which the planetary phases are supposed to occur, as found in the Handy Tables[1] for the beginning of the 12 signs and for the 7 climata. The majority of the curves reveal at a glance a more or less accurate symmetry with respect to ♈ and ♎, a fact which must provide us with information about the underlying computational assumptions. Consequently we ask first under what condition "symmetry" with respect to ♈/♎ can be expected to hold, i.e. the relation

$$\Delta\lambda(\lambda) = \Delta\lambda(-\lambda). \tag{3}$$

Exact symmetry is found in the tables of Jupiter (Fig. 121) and Mars (Fig. 122). For Jupiter we have for Γ and Ω separately

$$\Delta\lambda_\Gamma(\lambda) = \Delta\lambda_\Gamma(-\lambda) \quad \text{and} \quad \Delta\lambda_\Omega(\lambda) = \Delta\lambda_\Omega(-\lambda) \tag{4}$$

whereas for Mars even

$$\Delta\lambda_\Gamma(\lambda) = \Delta\lambda_\Gamma(-\lambda) = -\Delta\lambda_\Omega(\lambda+180) = -\Delta\lambda_\Omega(-\lambda+180) \tag{5}$$

is true.

Since the three functions

$$\nu(\lambda), \quad A = h/\sin\nu, \quad \cot\nu$$

are symmetric with respect to ♈/♎ it is necessary and sufficient for the symmetry of

$$\Delta\lambda = A - \beta\cot\nu$$

that the latitudes β are symmetric.

Since we know that the apogee of Mars is located in about ♌0°, together with the northernmost point of the deferent,[2] the latitudes cannot be symmetric to ♈/♎ unless $\beta = 0°$. If, however, $\beta = 0$ we have

$$\Delta\lambda = h/\sin\nu \tag{6}$$

[11] Cf. Goldstein [1967], p. 9, Sect. 6.
[12] Cf. p. 245.
[13] Cf., e.g., below p. 1019.
[1] In constructing the above-mentioned figures I have made use not only of [H] but also of [V]. For the problems raised by the numerous scribal errors cf. below p. 1020 ff.
[2] Cf. p. 208 and p. 1010.

and the fact that[3]

$$\nu(\lambda) = -\bar{\nu}(\lambda + 180)$$

also explains the identity of $\Delta\lambda_r$ and $-\Delta\lambda_\Omega$ in (5). Since $\Delta\lambda$ is found in the tables and $h = 14;30°$ is known[4], one can compute $\sin\nu$, and hence ν, from (6). The resulting values are close enough to the values obtainable from the tables in Alm. II, 13[5] that there can be no doubt that these tables were used. This is fully confirmed by using these same ν in the subsequent computations for the other planets.

In order to explain the symmetry (4) for Jupiter we compute its latitude under the assumption $\alpha = 0$ which is necessary, as we have seen,[6] for the symmetry of β. We furthermore ignore the eccentricity, calling the resulting latitude $\bar{\beta}$ instead of β; hence we assume

$$c_4(0) = c_6(0) = 0.$$

Then,[7] since $\kappa = \kappa_0$ for $\alpha = 0$,

$$\bar{\beta} = i_0\, c_7(\omega_P) + c_5(0)\, c_7(\Omega_P)$$

with

$$\omega_P = \omega_A + \kappa = \omega_A + \lambda - \lambda_A$$
$$\Omega_P = \Omega_A + \kappa = \omega_A + 180 + \kappa = \omega_P + 180.$$

Since the coefficients $c_7(\theta)$ are the equivalent of $\cos\theta$ we have for the latitude

$$\bar{\beta} = (i_0 - c_5)\cos(\lambda + \omega_A - \lambda_A). \tag{7}$$

The same relation holds also for Saturn, only the numerical values of the constants are different. We know[8] that for the time of the Almagest approximately

$$\omega_A - \lambda_A = \begin{cases} -20 - 160 = -180 = 180 & \text{for Jupiter} \\ 40 - 230 = -190 = 170 & \text{for Saturn.} \end{cases}$$

Combined with (7) this shows that for Jupiter

$$\bar{\beta} = -(i_0 - c_5)\cos\lambda$$

is indeed exactly symmetric, for Saturn slightly asymmetric (cf. Figs. 121 and 123, 124).

For Jupiter as well the latitude becomes asymmetric if one takes the eccentricity into consideration. Then one has[9]

$$c_4(0) = c_6(0) = 0;1 = c, \qquad c_3(\kappa_0) = c_3(\kappa) \approx -\cos\kappa = -\cos(\lambda - \lambda_A)$$

and hence for the latitude

$$\beta = -\big(i_0 - c_5 + c\cos(\lambda - \lambda_A)\big)\cos\lambda$$

which is no longer symmetric, though the effect of the term with $c = 0;1$ is very small. Nevertheless the strict symmetry of the tables for Jupiter implies that they

[3] Cf. p. 245 (3).
[4] Cf. above p. 1017 (2); Aaboe [1960], p. 12.
[5] Cf. Table 3, p. 47.
[6] Cf. p. 246.
[7] Cf. p. 1006 f.
[8] Cf. p. 1010 and p. 208.
[9] Cf. Fig. 120, p. 1416.

were based on $\alpha=0$, ignoring eccentricity. If one computes the $\Delta\lambda$ for Saturn under the same assumptions one obtains fair agreement with the tabulated values.

If we again assume $\alpha=0$ for the inner planets at Σ and Ξ and if we disregard eccentricities we have

$$\omega_P=\omega_A+\kappa=\omega_A+\lambda-\lambda_A$$
$$\Omega_P=\Omega_A+\kappa=\omega_A-90+\lambda-\lambda_A$$

thus

$$\bar{\beta}=i_0 \cos(\lambda+\omega_A-\lambda_A)+c_5 \sin(\lambda+\omega_A-\lambda_A)$$

with[10]

	i_0	$c_5(0)$	$\omega_A-\lambda_A$
Venus:	0;10	1;28	$0-\ 55=-55$
Mercury:	0;10	1;46	$180-190=-10.$

Hence $\Delta\lambda$ for Σ and Ξ cannot be symmetric, though the asymmetry is not very outspoken (cf. Figs. 125 and 128, 129).

For Γ and Ω, however, one has with $\alpha=180$

$$\Omega_A=\omega_A+90+\lambda-\lambda_A$$

and therefore

$$\bar{\beta}=i_0 \cos(\lambda+\omega_A-\lambda_A)-c_5 \sin(\lambda+\omega_A-\lambda_A)$$

with

$$c_5(180)=\begin{cases}8;51 & \text{for Venus}\\ 3;52 & \text{for Mercury.}\end{cases}$$

Since $c_5(180)$ is so much greater than $c_5(0)$ we now observe far greater asymmetries than at superior conjunction (cf. Figs. 126, 127 and 130, 131).

Remark. The peculiar shape of the curves for Ω and Γ of Venus (cf. Figs. 126 and 127) may be explained by the example of Ω for climata IV and V (cf. Fig. 132). The curves for $A=h/\sin\nu$ are exactly symmetric to ♈/♎. As function of φ, within the seven climata, these curves vary not very much and have similar shapes. The amplitude of about $\pm 9°$ for β (independent of φ), is increased by the factor $\cot\nu$, mainly between ♋ and ♑. Since $\beta=0$ at about ♉25 and ♏25 also $B=\beta\cot\nu$ changes its sign at these points, independent of φ. Consequently all the curves $\Delta\lambda=A+(-B)$ go nearly to the same point, roughly where $\beta=0$.

Special Data. The elimination of the numerous scribal errors and the choosing between variants constitutes a very laborious task, the details of which cannot be presented here. It will suffice to say that, as always, the graphical representation (Figs. 121 to 131) is of great help to detect errors and to emend such common substitutions as 1 for 4 or 3 for 6, and conversely. Difficult to explain is the frequent interchange of 10 and 50 — an Arabic intermediary is, of course, out of the question.

Another great help in establishing the correct text is the existence of symmetries, e.g. the fact that the same numerical values appear at the solstices for related morning and evening phases since, e.g., ν(♋0°)$=-\bar{\nu}$(♋0°):

$$\left.\begin{array}{l}\Delta\lambda_\Gamma(\lambda)=-\Delta\lambda_\Omega(\lambda)\\ \Delta\lambda_\Sigma(\lambda)=-\Delta\lambda_\Xi(\lambda)\end{array}\right. \quad \text{for } \lambda=\text{♋}0° \text{ and ♑}0°. \tag{8}$$

[10] Cf. p. 1010 and Fig. 119, p. 1415; for λ_A at the time of Ptolemy cf. p. 213.

Table 41

From $\Delta\lambda_r$

λ	I	II	III	IV	V	VI	VII	β comp.
♈	−1;27,46	−1;47,18	−1;45,17	−2; 4,49	−2; 2,39	−2; 6,19	−1;42,49	−2; 1
♉	−1;34,23	−1;45,22	−1;51,49	−1;52,19	−2; 2,43	−1;44,10	−2;13,15	−1;55
♊	−1;15,43	−1;11,59	−1;13,43	−1; 3,42	−1;29, 8	−1;38,54	−1;28,53	−1;19
♋	−0;26,36	−0;17, 1	−0;54,48	−0;23,41	−0;35,41	−0;27, 6	−0;21,36	−0;21
♌	+0; 2,58	+0;38,16	+0;43,34	+0;45,44	+0;44,45	+0;31,22	+0;35,57	+0;41
♍	+1;49,34	+1;49,34	+1;26, 8	+1;20,18	+1;29,43	+1;38,48	+1;27, 2	+1;33
♎	+1;59,44	−	+2; 2,42	+1;51,14	+1;31,13	+1;54,50	+2; 0,39	+2; 1
♏	+2;17, 8	+2;23,11	+1;43, 0	+1;44,54	+1;53, 8	+2; 1,23	+1;51, 0	+1;55
♐	+1;31,43	+1;21, 8	+1;27, 6	+1;28, 2	+1;21,42	+1;15,50	+1;19.23	+1;19
♑	+0;22,30	+0;31,41	+0;11,36	+0;26,24	+0;16,57	+0;23,44	+0;22.28	+0;21
♒	−0;32,41	−0;27,22	−0;29,59	−0;20,35	−0;44, 9	−0;53,59	−0;44, 7	−0;41
♓	−1;11,11	−1;20,39	−1;26,54	−1;27,13	−1;37,29	−1;16,33	−1;41,38	−1;33

From $\Delta\lambda_n$

λ	I	II	III	IV	V	VI	VII	β comp.
♈	−1;51,14	−	−2;19,55	−2;10,17	−1;50,29	−1;57,35	−2; 3, 3	−2; 1
♉	−1;50,57	−1;21, 6	−1;56,19	−2; 7, 3	−1;55,55	−1;25,24	−1;59.47	−1;55
♊	−4;12,14	−1;13,19	−1;10,41	−1; 9. 6	−1;18,58	−1;25,50	−1;21,14	−1;19
♋	−0;26,36	−0;17, 1	−0;54,48	−0;23,41	−0;35,41	−0;27, 6	−0;21,36	−0;21
♌	+0;46,31	+0;44, 1	+0;41,33	+0;49,40	+0;28,52	+0;19,36	+0;29, 9	+0;41
♍	+1;43,50	+1;37,21	+1;33,48	+1;32,20	+1;23,46	+1;40, 7	+1;14,25	+1;33
♎	+2; 4,29	+2; 3,40	+2;10,29	+1;56,48	+1;58,25	+2;18,15	+2;12. 8	+2; 1
♏	+2; 6,59	+2; 2, 5	+1;58,44	+1;57,27	+1;49, 9	+2; 4,23	+1;40.25	+1;55
♐	+1;22,40	+1;28,38	+1;25,16	+1;32.47	+1;13,51	+1;11,33	+1;13,54	+1;19
♑	+0;22,30	+0;31,41	+0;11,36	+0;26,24	+0;16,57	+0;23,44	+0;22,28	+0;21
♒	−0;14,50	−0;30,24	−0;27, 9	−0;18,45	−0;35,13	−0;41,23	−0;37, 1	−0;41
♓	−1;37,30	−1;21, 4	−1;33,49	−1;42, 9	−1;32,29	−1;23,17	−1;35,59	−1;33

Saturn, Latitudes

For example the curves of Γ in Fig. 123 would intersect the curves in Fig. 124 exactly in the same points at ♋ and ♑, respectively. The same holds for the two halves of Fig. 121, or of Figs. 126 and 127.

After removal of the mere scribal errors one can ask the question of how the tables were actually computed. Since we know the values of the angles v and of the arcus visionis h accurately one can compute the latitudes β from the tables for $\Delta\lambda$. This should result not only in reasonably smooth sequences for β as function of λ but also in independence of φ, i.e. of the specific clima. Table 41 shows in the example of Saturn (computing with $\alpha=0$ and ignoring eccentricity) that this is by no means the case. There exist bad fluctuations from climate to climate and the conclusion seems inescapable that the tables were computed rather carelessly.[11]

[11] It should be noted, however, that $\beta=(h-\Delta\lambda \sin v)/\cos v$ is rather sensitive to variations of v since $\cos v$ can become quite small.

Table 42

♂		I		II		III		IV	
Ω	Γ	comp.	H.T.	comp.	H.T.	comp.	H.T.	comp.	H.T.
♎	♈	19; 0,34	18;55	21;32,36	21;29	24;47,53	24;30	28;52,18	29; 0
♍ ♏	♉ ♓	18;21,39	18; 8	20;37,27	20;28	23;35,55	23;31	27;16,22	27;11
♌ ♐	♊ ♒	16;46, 6	16;44	18;25,56	18;19	20;34,31	20;28	23;14,29	22;54
♋ ♑	♋ ♑	15;14,58	15;16	16; 9,57	16; 7	17;23,39	17;34	18;55,36	18;55
♊ ♒	♌ ♐	14;33,38	14;37	14;53,19	14;52	15;26,54	15;24	16;11, 9	16; 7
♉ ♓	♍ ♏	14;32,11	14;30	14;31,27	14;30	14;43,44	14;45	15; 4,25	15; 8
♈	♎	14;37,19	14;37	14;30	14;30	14;35,35	14;37	14;50, 1	14;52

V		VI		VII		♂	
comp.	H.T.	comp.	H.T.	comp.	H.T.	Ω	Γ
34; 1,44	34; 7	40;12, 5	40;22	47;53,23	47; 2	♎	♈
31;57,56	32;13	37;33,14	37;30	43;43, 7	44;37	♍ ♏	♉ ♓
26;33, 5	26;45	30;28,22	31; 4	35;11,56	35;31	♌ ♐	♊ ♒
20;46,19	20;48	22;51,52	22;54	25;18,10	25; 4	♋ ♑	♋ ♑
17; 4,52	17; 3	18; 3,21	18; 7	19; 6,50	19: 7	♊ ♒	♌ ♐
15;31,52	15;32	16; 2,12	15;58	16;31, 8	16:34	♉ ♓	♍ ♏
15;10,12	15; 8	15;32,58	15;32	15;57,30	15;58	♈	♎

The situation for Mars is even worse. According to (6), p. 1018 the $\Delta\lambda$'s can be computed directly by dividing $h=14;30$ by $\sin\nu$. Since the ancient tables of chords agree very well with modern tables of sines one should expect nearly the same results. Table 42 shows that this is not the case. One cannot blame the manuscript condition for the discrepancies because each value of $\Delta\lambda$ appears four times in the table and scribal errors are no longer in our way (cf. also Fig. 122). Only carelessness within simple divisions seems left as an explanation.

An interesting algebraic error can be noticed in the tables for clima I. For this latitude ($\varphi=16;27°$) there exist cases for which the absolute value of ν exceeds 90°. In the east this happens for ♍, ♎, ♏, in the west for ♓, ♈, ♉.[12] The fundamental relation (1) p. 1017 should then be used in the form

$$[\Delta\lambda]=A+\beta|\cot\nu|, \qquad A=h/\sin\nu. \tag{9}$$

Ptolemy, however, (or whoever computed the tables) followed mechanically the rule which is valid in all other cases

$$\Delta\lambda=A-\beta|\cot\nu|. \tag{10}$$

Fig. 133[13] illustrates the consequences in the case of Venus at Γ. The points in [ ] represent the correctly computed values; all other points are based on (10), rightly or wrongly. It is, of course, the absence of an algebraic notation for the trigonometric functions that causes an error of this type.

[12] Cf. Table 3, p. 47.

[13] Fig. 133 is an enlargement of a section in Fig. 126.

An interesting copying error, common to [H] and [V],[14] is found in the tables for Γ and Ω of Mercury in Sagittarius. The two following lists compare the values given for $\Delta\lambda$ in the text with modernly computed values (hence the differences of a few minutes). For Γ one finds

Clima:	I	II	III	IV	V	VI	VII
Text:	11;52[15]	11;40	11;42	11;56[16]	12;51	13;14	12;14
computed:	11;48	11;40	11;45	12; 0	12;23	12;51	13;24

Obviously the scribe skipped one number in clima V and went one step ahead. The numbers Ω show furthermore that he simply continued because his last value for Γ (12;14) is exactly the first one for Ω:

Clima:	I	II	III	IV	V	VI	VII
Text:	12;54	14;5	15;26	17;41	20; 7	25;59[17]	22;20
computed:	12;14	13;3	14;11	15;42	17;39	20; 1	22;54

The last value in Ω (22;20) seems to be an arbitrary number added by the scribe in order to fill the last space. An error of this type indicates the copying of line by line over two adjacent tables. This seems to suggest for the archetype a roll, not a codex.

Both the Almagest (XII, 10) and the Handy Tables contain a table for the maximum elongations of Mercury[18] according to which the planet can never be more than about 28 1/2° distant from the sun, occasionally only about 16° (cf. Fig. 238, p. 1288). Nevertheless the Handy Tables list for the phases of Mercury elongations which often by far exceed these limits, reaching, e.g., for Ω in clima VII values above 41° (cf. Figs. 128 to 131, p. 1424ff.). We see here a drastic recurrence of the situation already encountered in the Almagest. Although Ptolemy was fully aware that Mercury under certain conditions remained invisible he nevertheless entered a required elongation in the tables for the phases.[19] Now we see him follow the same formal procedure: the theory of latitudes produces for given longitudes and for $\alpha=0$ or $\alpha=180$ a certain value of β. Substituting it in the formulae (1), p. 1017 one obtains a $\Delta\lambda$ and it is left to the user to realize that such an elongation may not exist.

In fact this procedure is not at all useful for limiting cases. The tables in the Almagest took at least for the critical situation the latitude β which corresponded to maximum elongation. The Handy Tables, however, uniformly use $\alpha=0$ or 180 and it is not at all certain that a phase which seems excluded for β at $\alpha=0$ or 180 could not become visible at maximum elongation. If one probes more deeply into the procedures concerning the planetary phases one cannot avoid getting the

[14] [H] III, p. 27 and 29; [V] fol. 88r.
[15] [H]: 11;12.
[16] [H]: 11;16.
[17] Sic, instead of 22;59. This error is common to [H] and [V].
[18] Cf. p. 232f. and (identical but for scribal errors) [H] III, p. 32.
[19] Cf. p. 255.

impression of a certain mediaevalism in which there is no reasonable relationship between the juggling with numbers and observable empirical data.

3. **Byzantium.** Obviously a later addition to the Handy Tables is a planetary visibility table for the geographical latitude of Byzantium[1] for which a longest daylight of $15;15^h$ is assumed, i.e. the mean value between the climata V and VI.[2] The numerical data, however, reveal that the tables themselves were not simply obtained by interpolation between the tables for clima V and VI although it is evident that the basic method of computation belongs to the Handy Tables type.[3] This is clear evidence for the presence of competent computers in Byzantium, who were instructed enough to determine for any given geographical location the angles ν between ecliptic and horizon — again not by simple interpolation[4] — and who were familiar with the methods of the Handy Tables for computing planetary latitudes. Since these Byzantine additions[5] are found in the Vat. gr. 1291 of the 9th century already affected by serious copyist errors[6] one cannot doubt that an astronomical tradition existed in Byzantium long before the 9th century. This is fully confirmed by direct evidence from the early seventh century[7].

That the deviations of the Byzantine tables from interpolations cannot be explained as computational inaccuracies is evident from the systematic character of these deviations. As we have seen Γ and $-\Omega$ as well as Σ and $-\Xi$ should have identical values for ♋ and ♑. Fig. 135 shows that in the case of Venus, as an example, the respective curves do not intersect at ♋ and ♑. Another difficulty is shown in Fig. 136 for Mars by the exact symmetry with respect to ♈/♎ of Γ as well as of Ω. We know[8] that this implies $\beta=0$ and hence not only

$$\Delta\lambda = h/\sin\nu \tag{1}$$

but also

$$\Delta\lambda_\Gamma(\lambda) = -\Delta\lambda_\Omega(\lambda+180). \tag{2}$$

It follows, however, from Fig. 136 that (2) is not accurately satisfied.[9]

The same difficulty also manifests itself in another fashion. One can again[10] make use of (1) and determine the angles ν from the tabulated values $\Delta\lambda$ and $h=14;30$.[11] This leads to two different sets of angles ν (cf. Table 43), neither one being the same as the mean value obtainable from Alm. II, 13, climata V and VI. The astronomically implausible hypothesis of different values used for the arcus visionis at Ω and at Γ finds no support on numerical grounds. Computing separately one finds $h_\Omega \approx 14;35$ and $h_\Gamma \approx 14;25$, values which are too near to the attested

[1] [H] III, p. 32f.; [V] fol. 8r.
[2] The corresponding latitude is, according to the texts, $\varphi=43;5$ thus accepting, e.g., the norm of Ptolemy, Geogr. III, 11, 5 and VIII, 11, 7 (ed. Nobbe, p. 188, 17 and p. 210, 29); actually $\varphi=41;1$.
[3] Cf. Fig. 134, p. 1429.
[4] Cf. below p. 1025, Table 43.
[5] Similar additions are tables for rising times ([H] II, p. 58 to 65), for the lunar parallax ([H] II, p. 194f.), and the ortive amplitudes (cf. above p. 983).
[6] Interchange of Σ and Ξ for Venus and Mercury; cf. also above p. 260f.
[7] Cf. below V C 5,2 B 5.
[8] Above p. 1018f.
[9] For the sake of greater clarity of Fig. 136 the curves for Γ and $-\Omega$ are not superimposed. The registered points should lie on equal levels and they do not.
[10] Cf. p. 1018.
[11] Cf. above p. 1017 (2).

Table 43

λ		$\Delta\lambda$		ν			
Γ	Ω	Γ	$-\Omega$	Γ	Ω	$\approx$	1/2 (V+VI)
♈	♎	37;52	37;48	22;31	22;33	22;30	23;10
♉ ♓	♏ ♍	37;17	35;19	24;16	24;14	24;15	24;51
♊ ♒	♐ ♌	28;44	28;53	30:18	30; 8	30;15	30;45
♋ ♑	♑ ♋	21;49	21;56	41;39	41;23	41;30	41;48
♌ ♐	♒ ♊	17;28	17;36	56; 7	55;28	55;45	55;45
♍ ♏	♓ ♉	15;41	15;45	67;36	67; 1	67;15	66;51
♎	♈	15;16	15;20	71;46	71; 2	71;30	70;52

value $h = 14;30$ to be significant. Consequently one has to assume an intermediary common set of values ν which cannot be much different from the list of round values given in Table 43 (third column for ν). As to be expected these values are not the mean values between the climata V and VI.

What seems to defy explanation is the origin of the differences between the tabulated values for $\Delta\lambda_{\Gamma}$ and $\Delta\lambda_{\Omega}$ if based on a relation as simple as (1). Since we are on insecure grounds even for the tables of Mars the chances for a successful recomputation of the Byzantine tables are very slim indeed.

6. Appendix. Supplementary Material

The sections of the Handy Tables classified here as "supplementary" deserve this name only in so far as they are non-mathematical in character. For the practice of ancient astronomy, however, these tables were just as useful as the proper astronomical tables. Lists of "Important Cities" with geographical coordinates were needed for the reduction of observations or of predictions. The lack of a generally accepted chronological era required a schematic king list. And a table of coordinates of bright stars near the path of the planets is an obvious necessity for observational astronomy.

We mentioned before the geographical tables in connection with mathematical geography.[1] The existence of a monograph on this subject containing an edition of relevant texts (Honigmann, SK) relieves us from further discussion. The chronological tables and their ancestors in ancient oriental king lists contain many difficult historical problems but are fortunately of no concern to us here. Therefore a few bibliographical references must suffice as a guide to the literature. A last section (p. 1026ff.) will deal with the tables of reference stars, more or less concluding the Handy Tables.

A. Royal Canon

The king list in the Handy Tables is often called "Ptolemaic Canon." In modern literature one can find the statement that this canon is part of the Almagest, a work obviously never opened by the author.

[1] Cf. above p. 939.

In fact it was the list of ancient rulers which by its intrinsic historical interest first attracted attention (in the 18th century) to the Handy Tables.[1] The first edition is due to Halma (1822), H.T. I, p. 139–143. A modern edition, by H. Usener, is found in the Monumenta Germaniae Historica, Auctores Antiquissimi 13, Chronica Minora 3 (1898), p. 438–455; cf. also p. 359ff. and R.E. 23, 2 col. 1824, 30–66.

In spite of being associated with the Handy Tables this Royal Canon begins with Nabonassar. To all rulers is given an integer number of (Egyptian) years and the totals correspond therefore to the years of the Era Nabonassar as used in the Almagest. In all extant manuscripts the list of rulers is continued beyond the time of Ptolemy (or Theon), often supplemented by entries from the hands of the last users. The usefulness of such lists is, of course, not restricted to astronomical tables. Fragments have been found on papyri, e.g. P. Oxy. 35 with a list from Augustus to Decius (A.D. 250). Another interesting but badly damaged fragment, from Oxyrhynchus, for about the same period but in astronomical context was published by Sattler.[2] It testifies again to the existence of variants of the Handy Tables.

B. Reference Stars

The natural reference system for the motion of the planets, moon and sun, are the fixed stars, in particular the stars near the ecliptic. The latitude of Venus can reach almost $\pm 9°$. This obviously explains the selection of bright stars, i.e. stars of the first four magnitudes, with latitudes $|\beta| \leqq 10°$ and their arrangement in a list with increasing longitudes. A tabulation of some 180 such stars is found at the end of the Handy Tables,[1] beginning with Regulus ($=\alpha$ Leo) which serves as zero point for all other longitudes. Hence we have the same convenient norm, independent of precession, also found in the "Planetary Hypotheses" and in the "Canobic Inscription."[2] The transformation from these sidereal coordinates λ^* to tropical coordinates λ is made easy by the tabulation of the tropical longitudes λ_R of Regulus in a special column of the tables for planetary mean positions.[3] For the time of the Almagest we have

$$\lambda_R = 122;30°. \qquad (1)$$

The list in the Handy Tables is based on the Catalogue of Stars in the Almagest (VII. 5/VIII, 1). One obtains the stars of the new list by excerpting from the Almagest all stars with $|\beta| \leqq 10°$[4] and at least 4th magnitude, using (1) for obtaining the sidereal longitudes λ^*. Not only the zodiacal constellations were taken into account but the few constellations as well with stars near enough to the ecliptic,

[1] The early literature is mentioned by Ideler, Astron. Beob., p. 36ff. (The author of the anonymous work mentioned p. 37, note 1 is Van der Hagen, not Masson; cf. Usener, Monumenta l.c. p. 441, note 1.)

[2] Sattler, Stud., p. 39–50.

[1] [H], p. 44–58; [V] fol. 90ʳ–94ᵛ; also in Par. gr. 2400 fol. 123ʳ–126ʳ. Cf. also below p. 1050.

[2] Above p. 915.

[3] Above p. 1002.

[4] The value $|\beta| = 10°$ occurs three times: at α Ari (adopting the factually correct value found in MS D and in both [H] and [V]; cf. also Peters-Knobel, Ptol. Cat. p. 35), at c^1 Tau ([H] incorrectly giving 5;0 here but 10;10 for α Tau, instead of 5;10; [V] correct), and at ξ Ori. [H] giving 10;15 for τ Tau is an error for 0;15.

i.e. Ophiuchus, Orion, and Cetus. The only omissions I found are ν and μ Ceti[5] and f Lupi ($\beta = -10°$).

Nevertheless the new list is not simply a mechanical selection and rearrangement from the catalogue in the Almagest. For example four stars in Taurus are now denoted as "nebulous"[6] which in the Almagest were classified as of 5th magnitude.[7] Conversely Sgr ν^1 ν^2 is nebulous according to the Almagest, of 4th magnitude in the H.T.[8] Repeatedly intermediate magnitudes of the Almagest were replaced by a definite magnitude, usually favoring greater brightness. Sgr d is of 5th magnitude according to the Almagest, listed as 4th magnitude in [H] but omitted in [V]. Oph d has become of 4th magnitude in [H] and [V], against 5 in the Almagest.

A great variety of changes can be observed in the description of the constellations. Obviously this fact should raise many questions which have been completely ignored in the modern discussion of stellar iconography. It has also escaped proper notice that some Hipparchian descriptions are mentioned which drastically differ from Ptolemy's version.[9] For example ζ Virg according to Hipparchus lies "in the end of the right hand" whereas the H.T. associate it with the "northern buttock".[10] This confirms the drastic regrouping of the stars in Virgo, mentioned by Ptolemy in Alm. VIII, 4 at the end.

It is not surprising that a list of this type is prone to be marred by scribal errors. The majority of these errors presents no serious problems and with the exception of a few cases the identification with the stars in the Almagest can be made without doubt. In general [V] is about as bad as [H] except for the complete omission of 18 lines between fol. 90^v and 91^r and of 5 or 8 lines at the end (after fol. 94^v).[11] In a few cases both [H] and [V] follow a variant known from the Almagest.[12]

From a historical point of view it is of interest that the list of reference stars in the Handy Tables has its boundaries $\beta = \pm 10°$ in common with the Babylonian set of "Normal Stars."[13] Since the latter is restricted to the first four magnitudes (although this concept does not exist explicitly in Babylonian astronomy) it is not surprising that all 31 Normal Stars are among the stars of the Handy Tables. There is no reason, however, to assume a closer relationship of the Ptolemaic list with its predecessor. In particular the choice of Regulus as the origin of sidereal coordinates seems to be Ptolemy's innovation. In Alm. VIII, 3. at the description of the construction of a celestial globe, Ptolemy considers Sirius as the star from which longitudinal differences should be counted.[14] For the reference stars near the ecliptic this would not be a suitable choice since the latitude of

[5] Even this omission is not certain since the text is in bad shape for this region (near $\lambda^* = 250$).
[6] [H]. p. 54/55.
[7] These are 16. 17. η Tau and a duplication (?) for η Tau.
[8] [H] and [V] agree; ε Cnc is given as nebulous in Alm. and [V]. as $m = 4$ in [H].
[9] These cases are: τ Leo. γ and ζ Virg. 42 and γ Aqr. δ Gem. To this list may be added α Ari in the Almagest.
[10] [H]. p. 44/45. In the Almagest it is the "right buttock."
[11] Cf. above p. 970. note 8.
[12] Three times following MS D, twice from A and B.
[13] Cf. above p. 545.
[14] Cf. above p. 891.

Sirius is $-39;10°$. The shift to Regulus is again a step toward greater practical convenience.

The star list of the Handy Tables seems to have not influenced later star catalogues. These belong either to the same class as the Catalogue of Stars in the Almagest (with more than 1000 stars), or they are related to the astrolabe and therefore restricted to a small number of about 30 bright stars but of all latitudes.[15]

§ 5. The Time from Theon to Heraclius

1. Chronological Summary

Continued from p. 943.

Macrobius	A.D. 400		
Martianus Capella	420		
Syrianus	430		
Proclus	412 to 485	P. Mich. Inv. 1454 (ephemeris)	467
Marinus	485 ff.	Notes to Handy Tables	477
"Palchus"	480		
Ammonius			
Eutocius			
Heliodorus	500		
Rhetorius			
Stobaeus			
Boethius	480 to 524		
Philoponus	530		
Simplicius	530		
Lydus	490 to 554		
Olympiodorus	564		
Cassiodorus	d. 583		
Isidore of Seville	560 to 636		
Stephanus	610 to 650		
Heraclius, emperor	610 to 641		

Notes

Proclus: cf. Neugebauer-Van Hoesen, Greek Hor., p. 135 f. (No. L 412).

"Palchus": this is not a Greek personal name but a late Byzantine rendering of al-Balkhī (full Arabic version: "the dragoman from Balkh"; cf. Pingree [1968, 3], p. 279 and [1971], p. 204). Much older material is incorporated in this collection, e.g. horoscopes from around 480 (cf. Neugebauer-Van Hoesen, Gr. Hor., p. 187).

Heliodorus: cf. Westerink [1971].

Rhetorius: cf. above p. 960, n. 4.

Olympiodorus: cf. below, V C 5, 2 B 4.

Notes to Handy Tables: cf. Tihon [1973], p. 106.

[15] Cf. for details Kunitzsch, Sternverz. The same holds for the 30 stars in the "Phaseis" (above p. 928).

2. Fifth to Seventh Century

A. Popularization

A modest measure of astronomical knowledge was considered a basic educational requirement in late antiquity. This is evident, e.g., from a remark by Augustinus[1] who complains that his erstwhile friends, and now bitter enemies, the Manichaeans, could not properly explain solstices and equinoxes, eclipses, nor other phenomena that are correctly described in secular books[2]. It is interesting to see that Augustinus raises objections to the Manichaean request to accept their cosmic doctrines merely "on faith", a request not unheard of in other circles. Augustinus' statement[3] that solar and lunar eclipses had been computed for many years ahead comes from secular sources, such as Cicero[4] or Pliny[5].

"No fame can be expected from the antipodes" (obviously because of their awkward position) says Cicero in his "Scipio's Dream"[6]. Such is the aspect of the world, seen from heaven by the two Scipios in the dream imagined by Cicero. Some four centuries later this essay inspired a huge commentary by Macrobius, a work destined to exercise great influence on medieval thought.

Scipio (i.e. Cicero) describes, in his customary superficial fashion, the arrangement of the nine spheres, from the fixed stars down to the earth. He mentions the central position of the solar sphere and he admires the harmonies produced by the planetary motions. Macrobius greatly expands the discussion on these topics without adding much new information. Cicero's clear reference to the central position of the solar sphere is distorted by Macrobius into a support of the sequence postulated by Plato; namely earth-moon-sun-planets.[7] For the motion of all planets he assumes identical orbital speed without drawing the numerical consequences for the relative distances, except for some purposely vague remarks about the inner planets in relation to the sun.[8]

We mentioned in a previous context the absurd parameters which Macrobius associated with the size and the distance of the sun.[9] He pretends that they are the result of careful observations at a hemispherical sundial at equinox. Needless to say the "observation" as described by Macrobius is purely fictitious and would never furnish the desired results, not to mention that the apparent solar diameter $d_{\odot}$ would not be given simply by the "observed" time Δt it takes the solar disk to cross the horizon without considering geographical latitude and solar declina-

[1] Born in Thagaste in 354, died 430 at Hippo, North Africa.
[2] Confessions V 3,6 (Budé I, p. 96).
[3] Confessions V 3,4 (Budé I, p. 94; Corpus S.E.L., Vol. 33, p. 91, 14–22).
[4] Cicero. De divinatione, 17: "predicted for many years" (Loeb. p. 389); De natura deorum II. 17: "predicted for all future" (Loeb, p. 271); cf. also above p. 319.
[5] Pliny. NH II. 53 (above I E 5, 2 A). Augustinus speaks of Pliny as "doctissimus homo" (De civitate dei, XV, 9: Corpus Script. Eccl. Lat., Vol. 40.2, p. 76,4) and he quotes him repeatedly, often without mentioning his name (cf. the index in Corpus S.E.L., Vol. 40, 2, p. 688).
[6] Chap. 6 (Stahl. Macrob., p. 74; Boyancé. Études, p. 28/29).
[7] Somnium I 19 (trsl. Stahl. p. 162–168).
[8] Somnium I 14. 26 and 21, 6 (trsl. Stahl. p. 148, p. 176).
[9] Above p. 661.

tion. Macrobius pretends[10] that one found $\Delta t = 1/9^h = 1;40^\circ$ – a rather senseless result.

If the "Somnium" appears to a modern reader as a dreadful piece of literature the "Marriage of Philology and Mercury" contains exactly what one might expect from an author (Martianus Capella) who chose such a title.[11] Among modern historians Martianus, Macrobius, and Chalcidius, nevertheless, occupy a prominent place because they provide references – real or imaginary – to heliocentric theories; it is not necessary to return to this topic here.[12]

The usefulness of Book VIII of Martianus' work lies for us in the remnants of older material, scattered over its different sections.[13] Martianus obviously made an attempt to present a summary of elementary astronomical concepts and data in a relatively orderly way. After a long mythological introduction[14] a first part[15] deals with the conventional definitions in spherical astronomy, related to fixed stars and constellations, including some garbled pronouncements on rising- and setting-times of zodiacal signs.[16] The second part[17] concerns the planets, sun and moon. Here one finds again the postulate that all seven planets move with equal speed in their respective orbits, such that the sun's orbit is 12-times as extended as the moon's; consequently Mars receives the factor 24, Jupiter 144, and Saturn 360. The conversion to absolute distances is based on the assumption that the lunar orbit amounts to 100 times the circumference of the earth[18] in consequence of another of these fictitious "observations," this time with a water clock, ignoring well established data for the moon's distance.

Martianus considers the luni-solar cycle of 235 months or 19 years as a cycle for the lunar latitude while he mentions a sidereal period of 55(?) years.[19] Of interest is the fact that he uses a terminology which is related to the archaic concept of "steps"[20] for the branches of the lunar latitude.

Finally the discussion of the solar motion leads to a short excursus on the geographical "climata." A reference to Cyrene-Carthage is probably motivated by Martianus' place of origin. The counting of Rome as fifth clima is another concession to the contemporary situation. However, the statement "*climata 8 sunt*" because of a clima "*ultimum ultra Maeotis paludes*"[21] is quite uncanonical.

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[10] Somnium I 16, 10 and 20, 30 (trsl. Stahl, p. 154, p. 174).
[11] For bibliographical data cf. Stahl [1965].
[12] Cf. above IV C 2, 2.
[13] Cf. the subject index (below p. 1151) s.v. Martianus.
[14] Sect. 803 to 816 in the Teubner edition (Dick).
[15] Sect. 817 to 849.
[16] Sect. 844/845; cf. above p. 723 f.
[17] Sect. 850 to 887.
[18] Cf. above p. 664 (16); for other estimates see below p. 1044.
[19] Sect. 868.
[20] Cf. above p. 671, Sect. 869: *ὕψος* or *ταπείνωμα ὑψουμένη* and *ταπεινουμένη*.
[21] Sect. 876, ed. Dick, p. 462, 3 f. and 14 f. The values for the longest daylights are garbled by trivial scribal errors, taken much too seriously by Honigmann, SK, p. 52.

B. The Latest Schools

Two major pagan schools still existed in the fifth and sixth century: the "Academy" in Athens and the school of Alexandria.[1] In both places the discussion centered mainly on the explanation of the works of Plato and Aristotle, perhaps with a stronger emphasis on Plato in Athens, and on Aristotle in Alexandria.[2] The scholars of this period are known as "Neoplatonists."

Both schools maintained an interest in astronomy and in astrological doctrine. Proclus in particular had an excellent command of Ptolemaic theory and it is probably due to his influence that astronomical knowledge remained alive among his pupils and successors. Unfortunately, with the exception of Proclus' "Hypotyposis", only fragments of their teaching has reached us, frequently only in the form of notes (ἀπὸ φωνῆς) on lectures (πράξεις), made by some student.

In spite of the rivalry between them the two schools kept in close contact. The short summary of data given in the following paragraphs will make this clear, even if we arrange the material in two different sequences.

Academy. Proclus (born 412 Febr. 8[3] in Byzantium, died 485 Apr. 17 in Athens) was the pupil of Syrianus and became his intimate friend and successor in the directorship of the Academy; we owe his biography to his successor Marinus.[4] The years following Proclus' death seem to be years of rapid decline of the Academy until a recovery took place under Damascius,[5] its last head. To him we are also indebted for a biography (of Isidorus, a pupil of Proclus) which, in spite of its fragmentary state of preservation,[6] gives us important insight into the life and work of these late scholars.[7] Damascius was at the head of the school when, in 529, Justinian issued his famous decree against teaching by pagans, a step which caused the emigration of Damascius and his pupils to Persia. Disappointed, they soon returned to Athens where they were still able to maintain reasonable working conditions.[8] Simplicius[9], at least, worked on his commentaries to Aristotle until about 560.

Alexandria. Hermias and his brother Gregorius[10] studied in Athens under Syrianus at the same time as Proclus. Hermias married Aedesia, a relative of Syrianus, and returned with her to Alexandria. Again, two of their sons, Ammonius[11] and Heliodorus[12], studied under Proclus in Athens but eventually returned to Alexandria to succeed their father. Among the pupils of Ammonius[13]

[1] For the intricate chronological and historical problems of this period see in general Saffrey [1954], Westerink, Anon. (1962), Cameron [1969], Lemerle, Hum. Byz. (1971); also Frantz [1975].
[2] For differences in the financial status of the two schools cf. Westerink, Anon., p. XIV.
[3] Cf. Neugebauer-Van Hoesen, Greek Hor., p. 135f.
[4] Published, with Latin translation, by J. F. Boissonade in Proclus, Opera inedita (Paris 1864), p. 1–66.
[5] Born around 455, died after 538; cf. Cameron [1969], p. 286/7.
[6] R. Asmus, Das Leben des Philosophen Isidoros von Damaskos (Leipzig 1911); Damascii vitae reliquae, ed. Clemens Zintzen (Hildesheim 1967).
[7] For Isidorus cf. RE 9, 2 col. 2062/4 No. 17).
[8] Cf. Westerink, Anon., p. XV; Cameron [1969].
[9] Born about 490; cf. Cameron [1969], p. 287.
[10] Damascius, Vita Isidori, trsl. Asmus, p. 45/46; Suidas, ed. Adler I, 1, p. 543, 8–15 = ed. Zintzen, p. 104(75)/105(123).
[11] Born about 440, died between 517 and 526; cf. Westerink, Anon., p. X; Warnon [1967], p. 204, etc.
[12] Younger than Ammonius.
[13] Cf., e.g., RE 1, 2 col. 1864, 4–6.

are known Philoponus,[14] Asclepius, Olympiodorus, Damascius and Simplicius. The mathematician Eutocius[15] seems to have been the successor of Ammonius as the head of the Alexandrian school. He was followed by Olympiodorus[16] who directed the school from about 550 to 565, in spite of remaining a pagan.[17] Elias, David, and Stephanus, however, who were probably pupils of Olympiodorus, were Christians.[18] By that time Constantinople had become a cultural centre strong enough to attract scholars from Alexandria to the new capital. Thus Stephanus worked in Byzantium from about 615,[19] probably at the court of Heraclius.

Victory over the Persians, then disastrous defeat at the hands of the Muslims are the great events in the reign of Heraclius that inaugurate a new epoch in the Mediterranean world. Not until the ninth century do we again see more clearly what became of Greek science at the courts of the Caliphs and of the Byzantine emperors.[20]

1. **Proclus.** We know from Marinus' biography of Proclus that he was born in Byzantium and his horoscope fixes the date to 412 Febr. 8. He died in Athens in 485, the year after a total solar eclipse;[1] another eclipse of the sun, partial at Byzantium, followed in 486.[2]

The data of the horoscope allow us to check computations carried out in the fifth century. The following elements can be compared directly with modern results:

	Text	modern		Text	modern	
♄:	♉24;23	♉27;33	♀:	♓23[ ](?)	♒23;34	
♃:	♉24;41	♉25;47	☿:	♒ 4;42	♒ 7;23	(1)
♂:	♐29;50	♑ 3;14	node:	♏24;33	♏26;58	
			conj.:	♒ 8;51	♒10;38 Jan. 29	

To do the same for sun and moon one should know the hour. For it, however, we only have the information that H = ♈8;19 was the ascendant, M = ♑4;42 midheaven, while the sun was at ♈16;26 (or ♈16). In order to obtain agreement with the data shown in (1) the sign ♈ for the sun must be emended to ♒. The position given for Venus is probably corrupt in the extant texts.[3]

According to the Handy Tables the normed right ascension for M = ♑4;42 is $\alpha' = 5;10$. This should be equal to the oblique ascension $\rho(H)$ for H = ♈8;19.[4]

[14] Philoponus never held a chair of "philosophy" since he is always called "grammaticus"; cf. Westerink, Anon., p. XIII, Cameron [1969], p. 282.
[15] Cf. Westerink, Anon., p. XIII.
[16] Born between 495 and 505; cf. Westerink, Anon., p. XIII.
[17] For his numerous commentaries on the works of Plato and Aristotle cf. RE 18, 1 col. 207–227.
[18] Westerink, Anon., p. XX–XXIV.
[19] Lemerle, Hum. Byz., p. 80f.
[20] See for the time of Leo the Mathematician: Lemerle, Hum. Byz., Chap. VI et passim.
[1] Visible in Athens shortly after sunrise, Jan. 14 484.
[2] May 19. For both eclipses cf. Ginzel, Kanon, Map XIV.
[3] For the textual variants cf. Neugebauer-Van Hoesen, Greek Hor., p. 135f. The modern data are taken from Tuckerman's and Goldstine's tables. The arithmetical mean of the deviations is $\approx -2;25°$. This agrees very well with the error in precession: the longitude of Regulus found from the Handy Tables is 125;15 against actually 127;50, thus $\Delta = -2;35°$.
[4] Cf. above V C 4, 2 A.

Since ρ(♈ 8;19) is found to be

5;29° for Alexandria
5;10 for Rhodes
4;43 for Hellespont

it is certain that H and M were computed for the latitude of Rhodes. This is perhaps permissible for an astrologer in Athens but excludes Alexandria as well as Byzantium.

For the solar longitude ♒ 16;26 the normed right ascension is $\alpha' = 48;58$. Since $\alpha' = 5;10$ for M we see that the sun was located $48;58 - 5;10 = 43;48° \approx 2;55^h$ before noon. Actually, however, the longitude of the sun must have been found first for the given hour (which is unfortunately not mentioned in our sources) and from it H or M determined. The determination of H seems to have been based on the following approximate consideration: assume for the given time 4 seasonal hours after sunrise. For $\lambda_{\odot} =$ ♒ 16;26 and Rhodes one finds from the Handy Tables[5] that $1^{s.h.} = 12;58,18°$, hence $4^{s.h.} = 51;53°$. Adding this to the solar position one finds exactly ♈ 8;19, the longitude given for the ascendant. This procedure is, of course, incorrect since one applies a difference in right ascension directly to the ecliptic. Yet, the numerical agreement is accurate and it seems more plausible to assume as given time 4 seasonal hours of daytime than 2;55 equinoctial hours before noon (or $2;16^h$ after sunrise). It seems that we are dealing with a crude practitioner who did not mind inaccurate shortcuts. And we also know that Paulus Alexandrinus (around 375) tought exactly this procedure[5a].

We can now approximately determine the longitudes of sun and moon, assuming as time about 2 hours before Alexandria noon. To introduce any correction for geographical longitude would mean making arbitrary assumptions instead of simply accepting the norm Alexandria of the Handy Tables.

Thus one finds

	Text	modern	
sun:	(♒) 16;26	♒ 19;52	(2)
moon:	♊ 17;29	♊ 16;38.	

We now turn to the planetary positions. As we have seen, the elements from spherical astronomy (H and M) were undoubtedly obtained with the Handy Tables. It is only reasonable to assume the same for the longitudes of the planets.

The date of the horoscope as determined from (1) corresponds to the year Philip 735 X 2 about 10 a.m. Computing accurately with the Handy Tables for this moment does not lead to the longitudes given in the text. The cause of the discrepancy is easily detected in the case of Mars: one obtains (with a slight deviation of 0;2°, explicable by differences in roundings) the numbers of the text if one ignores the hours after X 1 noon, although they amount to almost one day. Assuming the same procedure for Jupiter and Saturn only brings us close to the numbers of the text. Exact agreement is reached if one assumes a

[5] Halma H.T. II, p. 33.
[5a] Cf. above p. 956.

computing error that can occur very easily: the correction $k_4 \cdot k_7$ is in both cases taken negative instead of positive.[6]

For the ascending node of the moon one obtains for X 1 noon, again ignoring hours, ♏24;32 instead ♏24;33 in the text.[7]

The deviations for the inner planets and for sun and moon remain inexplicable:

	Text	H.T.		Text	H.T.
Venus:	(♒)23[ ](?)	♒0;19	sun:	(♒)16;26	♒17;36
Mercury:	♒ 4;42	♒5;35	moon:	♊ 17;29	♊ 16;18.

The longitude ♊16;18 of the moon is found for noon of X 2 but even this does not quite reach the position given in the text. For the sun the opposite holds: ♒17;36 is found for noon of X 1 but the text is still one day short. Mercury, computed for X 1 noon is about one day ahead of the text. Venus is retrograde at the time and this may be the cause for the apparent great discrepancy. As a result it seems that the positions of sun and moon and of the inner planets were very carelessly computed.

It is, impossible to say when, and by whom, the horoscope was cast. Proclus himself is not excluded but his authorship seems unlikely in view of the geographical inaccuracy established above. On one occasion[8] he remarks that one disregards precession in the computation of nativities, a curious way of supporting his dislike for a secular correction which was unknown to Plato (but is, of course, embedded in the Handy Tables). Proclus also declares that Plato did not explain the irregularities of the planetary motions by means of such "artificial" devices as epicycles,[9] although he admits their usefulness and he displays his learning by giving the list of the numerical values for their radii, taken from the Almagest[10] – a completely useless digression in the given context.[11]

It is also interesting to see how little insight a prominent scholar in the fifth century A.D. had into the character and purpose of early Greek science. Proclus seriously believes that much of Euclid's "Elements" was developed because of its utility for astronomy; the investigation (in Book IV) of the regular 15-gon, e.g., should have been caused by its significance for the obliquity of the ecliptic ($\varepsilon = 24°$).[12] One such postulated connection between an Euclidean theorem and astronomy remains obscure. Proclus says that Euclid I, 7, a congruence theorem for triangles,[13] had been used to prove that no two consecutive intervals between

[6] Cf. for the computation of planetary equations above p. 1003 (4). Errors of signs also occur in our Handy Tables, in part introduced by Halma (for Saturn, Jupiter, and Mercury in k_4 and k_7).

[7] Additional hours would only increase the difference because of the retrograde motion of the nodes. Hence we are dealing here with a trivial adding error.

[8] Proclus, Comm. Tim. IV, 125; trsl. Festugière IV, p. 161 (where in line 17 "retrograde" should be replaced by "direct"). Cf. also Hypotyposis, p. 234/5 (ed. Manitius).

[9] Proclus, Comm. Tim. III, 96; trsl. Festugière IV, p. 125.

[10] Proclus, Comm. Rep. II, 222; trsl. Festugière III, p. 174. For the moon Proclus accepts the norm used in the "Planetary Hypotheses" and in the "Canobic Inscription"; cf. above p. 903 (9).

[11] This is nicely illustrated by the modern commentators, Kroll as well as Mugler, who on this occasion present real caricatures of the Ptolemaic models (Kroll, Rep. II p. 415; Mugler in Festugière, Rep. III, p. 175). Kroll, Rep. II, p. 414, n. 2 in criticizing Proclus obviously did not understand the connection between eccenter and epicycle models.

[12] Proclus, Comm. Euclid. trsl. Morrow, p. 210; cf. above p. 733.

[13] Heath, Euclid I, p. 258–261.

eclipses can be equal.[14] In this general formulation the statement is obviously wrong but the numerical example that mentions two consecutive intervals of 6 months and 20 days[15] is correct insofar as the first and the third eclipse would be separated by more than 13 months and this is impossible.[16]

Proclus also refers to the interest of astronomers in the possibility of obtaining complicated motions through the combination of simple ones. He mentions, e.g., the spiral motion produced on a cylinder by the combination of rotation and axial motion; or the diagonal motion which is the resultant of two motions in different directions; or the elliptic or circular path of a point on a line segment that moves with its end points on two perpendicular axes.[17]

The special case of a circular motion, generated by the device described in the last-mentioned theorem, is of great historical interest because it became a crucial element in the planetary theory of Naṣir al-Dīn al-Ṭūsī and his school (13th cent.) as well as for Copernicus. In its astronomical applications Proclus' theorem appears in a slightly modified form. In the above given version it is obvious that the midpoint F of the line segment HK (cf. Fig. 137a, p. 1431) describes a circle with center D as the points H and K move on orthogonal axes since $DF = FH = FK = r$. If we now make F the center of a circle of radius r (Fig. 137b) the point G will move on a circle of center D and radius $2r$. At the same time the point H will remain on the diameter AB. Hence two uniform motions about D and F, respectively, result in a simple harmonic motion of a point on a line segment AB.

Copernicus quotes Proclus for his theorem in the original version[18] but he uses it (in the theory of Mercury) in the expanded form (Fig. 137b) which is also found in Ṭūsī, including the same lettering.[18a] Ṭūsī's application of this mechanism to planetary theory was known in Byzantium in the early 14th century as is evident from figures in a codex (Vat. gr. 211) written around 1300. Our Plate IX reproduces two of these diagrams, one which concerns the eccentric solar orbit, the other, in the lower part, showing four characteristic positions of the roling device, depicted also in identical fashion by Ṭūsī.[18b] It would be surprising if Italian astronomers around 1500 were not familiar with these techniques.

A small treatise (in two Books) "Elements of Physics,"[19] based on Aristotle, deals with the basic concepts of time, motion, continuity and indefinite divis-

[14] Proclus, Euclid, p. 268 f. ed. Friedlein; trsl. Morrow, p. 209 f.

[15] Probably $6^m 20^d$ ($= 200^d$) is some mistake (177 + 29?).

[16] Cf. above I B 6,4.

[17] Proclus. Euclid, p. 105 f. ed. Friedlein; trsl. Morrow, p. 85 f. The same examples are also in Comm. Rep. II, 234, trsl. Festugière III, p. 189, where he also mentions the computation (*λογισμοῖς χρώμενοι*) of parapegmata and the making of gold.

[18] Explicitly in De revol. V, 25 and by reference to the general case (ellipse) in III, 4 in a passage deleted in the manuscript (Gesamtausgabe. p. 339, 22–24 and p. 151, note ad 16). Cf. also Swerdlow [1973] Figs. 39 and 40.

[18a] Noticed by Hartner [1971]. p. 616.

[18b] Cf., e.g., Hartner [1971], p. 617.

[19] "Elementatio physica" (or "De motu") in the medieval Latin version. Cf. for it Helmut Boese, Die mittelalterliche Übersetzung der *στοιχείωσις φυσική* des Proklus (Akademie-Verlag, Berlin 1958) who showed that the translator is the same who produced in Sicily, around 1160, a translation of the Almagest into Latin.

ibility, etc.[20] It imitates Euclidean style, providing a "proof" for each theorem. In particular, in Book II, eternal existence is promised to all objects that move, according to their nature, in circular orbits.

Proclus, in spite of all his philosophical prejudices had sound astronomical knowledge as is shown by his "Hypotyposis." This work provided the reader with a good introduction to Ptolemaic astronomy – one might say, the first and last summary of the contents of the Almagest from antiquity. Proclus shows special interest in the design of instruments described in Alm. I, 12 (determination of the obliquity of the ecliptic) and in V, 1 (the "ringed" or spherical "astrolabe"[21]). Here, Proclus leads us beyond the information one finds in the Almagest[22] by giving dimensions of such instruments. A "plate" (πίναξ) for the nomographic determination of the true solar longitude to given mean position, obviously equivalent to the method described in Ptolemy's Introduction to the Handy Tables, is of much less practical importance.[23] In the chapter on the moon Proclus gives 5;30° as the maximum lunar latitude but later he returns to the customary value of 5°.[24] Two other topics, not known from the Almagest, have been mentioned before: the theory of trepidation[25] and the hypothesis of nested spheres which we now know from Ptolemy's "Planetary Hypotheses."[26]

A small treatise called "Sphaera" is alleged to be by Proclus but is only an excerpt, often verbatim, from Geminus' "Isagoge."[27] There seems to be no good reason for considering Proclus as the author of this compilation which was extremely popular in the 16th and 17th century.[28] The "Paraphrase to Ptolemy's Tetrabiblos" edited by Melanchthon (Basel 1554) and other astrological writings[29] that go under Proclus' name are also of doubtful origin. Proclus accepted unquestionably the validity of astrological doctrine. One only needs to read, e.g., what he says about the influence of the planets on human pregnancy[30]: for its duration he accepts the same pattern of 10 sidereal months $(=273;20^d) \pm 15^d$ or 7 sidereal months $(=191;20^d) \pm 15^d$ (the sidereal month reckoned as $27;20^d$) which we also know from Hephaistio.[31]

Proclus' successor, Marinus, in some measure, seems to have maintained the traditional interest in scientific matters. We know[32] that he lectured on

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[20] Edited, with German translation, by A. Ritzenfeld: Proclus. Institutio physica, Leipzig, Teubner, 1912.

[21] Proclus devotes a whole chapter on this instrument but he did not write a treatise on the "astrolabe" as the title of a work by G. Valla (Paris 1546 etc.) leads one to assume. Cf. Tannery, Mém. Sci. IV, p. 244 or above p.

[22] Cf. Rome [1927].

[23] Hypotyposis, p. 72–77; cf. above p. 984.

[24] Hypotyposis, p. 86–89 and p. 116–119.

[25] Above p. 633, n. 14.

[26] Cf. above p. 919.

[27] Manitius, Geminus, Introd., p. XXIII–XXV.

[28] Laurence Jay Rosán, The Philosophy of Proclus (New York 1949), p. 252–254 enumerates some 75 editions and translations.

[29] Cf. Tetrabiblos, Loeb edition, Introd., p. XIVf. The article in RE 23, 1 col. 204 lacks competence.

[30] Comm. Rep., trsl. Festugière II, p. 165ff.

[31] Hephaistio II, 1, ed. Pingree I, p. 82.

[32] Thanks to A. Tihon [1971], p. 329, [1973], p. 77.

Pappus' commentary to Book V of the Almagest (in particular his discussion of parallax); and there are still extant lecture notes on the Data of Euclid[33].

2. Ammonius, Heliodorus, Philoponus

Ammonius. Damascius tells us that Ammonius excelled among the pupils of Proclus in geometry and astronomy.[1] Ammonius in turn instructed Olympiodorus in these subjects.[2] We even know that he made some observations: in A.D. 503 he observed with his brother Heliodorus an occultation of Saturn by the moon[3] and in the presence of Simplicius[4] he observed Arcturus with the armillary sphere[5] to determine the star's longitude and he thus confirmed Ptolemy's constant of precession. If this indeed was his result we cannot consider Ammonius a good observer. In the Almagest Arcturus is given the longitude ♍27. Some 3 1/2 centuries of Ptolemaic precession would move the star by the time of Ammonius to about 180;30° of longitude whereas the actual longitude in A.D. 500 was 183;22°. Thus Ammonius' observation shows an error of about 2;50°.

In his treatise on the plane astrolabe Philoponus refers to a similar work by his teacher Ammonius,[6] but nothing of this earlier treatise has reached us. A group of later manuscripts, however, which concern the theory of the astrolabe, name Ammonius as the author though it is quite obvious that the real author is Nicephorus Gregoras (14th century).[7]

The name of Ammonius, in more or less dubious spellings, also appears in Islamic astronomy in connection with planetary tables which are preserved, however, only in the late version by al-Zarqālī[8] (epoch: 1088 Sept. 1). The method of computing planetary positions with these tables radically differs from Ptolemy's procedures as we know them from the Almagest or the Handy Tables. The basic idea consists in preparing tables for the positions of a planet during the whole length of a cycle of synodic periods, known from Babylonian astronomy, the so-called "goal-year periods," e.g. 59 for Saturn, 83 for Jupiter, etc.[9] Then, for a given date, one only has to determine its place within the cycle to find the desired longitude directly. Everything we know about Proclus and the late Neoplatonic schools shows a strict adherence to Ptolemy's methods and it seems unlikely therefore that Ammonius should have followed a totally different approach.[10]

[33] Cf. M. Michaux, Le commentaire de Marinus aux *Data* d'Euclide (Univ. de Louvain, Rec. de Travaux d'Hist. et de Philol. 3. sér., 25, 1947); also above p. 840/841.
[1] Damascius, Vita Isidori, trsl. Asmus, p. 49; ed. Zintzen, p. 110.
[2] Cf. Warnon [1967], p. 210f.
[3] Cf. below p. 1040.
[4] Simplicius, Comm. Arist. de caelo, CAG VII, p. 462, 20–30.
[5] *διὰ τοῦ στερεοῦ ἀστρολάβου.*
[6] Cf. Tannery, Mém. Sci. IX, p. 342.
[7] Cf. Neugebauer [1949, 2], p. 254 and above V B 3, 7 F.
[8] Published by Millás Vallicrosa, Est. Azar., p. 153ff. and p. 379ff. Cf. also Boutelle [1967] and Steinschneider, Europ. Übers. aus d. Arab. I, p. 51, No. 72.
[9] Cf. above p. 151 (5), excepting Jupiter which is assigned a cycle of 83 (=71+12) years; cf. for it p. 391 (12).
[10] I do not deny the possibility that the above described method could have existed in hellenistic-Roman astronomy and provided the ultimate source for Zarqālī's "Almanac." I only doubt that the Neoplatonists adopted such methods. The only thing we know is that the tables of Heraclius and Ammonius agree with Theon's tables in so far as epoch and calendar are concerned (cf. CCAG 2, p. 182, 17f.).

Heliodorus. The only factual information we have about the astronomical activities of Heliodorus are records of observations made between A.D. 475 and 510 that accidentally found a place in manuscripts which also contain the Almagest [11] and related material [12] (e.g. an anonymous introduction to the Almagest, most likely the work of Eutocius [13]).

All these observations concern occultations or near conjunctions. Counting them from I to VII in the order they appear in the text we have the following combinations:

I. Mars and Jupiter	IV. Jupiter and α Leo	VI. Mars and Jupiter
II. Moon and Saturn	V. Moon and α Tau	VII. Venus and Jupiter.
III. Moon and Venus		

The purpose of these observations is nowhere stated; only the last two mention discrepancies between observed and computed positions.[14] It does not seem very likely, however, that this reveals a program to check and correct existing tables; a few data scattered over several decades would not suffice for such a task.

The chronological sequence of the observations slightly differs from the textual order:

III. 475 Nov. 18	IV. 508 Sept. 27	VI. 509 June 13
I. 498 May 1/2	V. 509 March 11/12	VII. 510 Aug. 21/22.
II. 503 Febr. 21/22		

In the text all dates are given in the era Diocletian and in months and days of the Alexandrian calendar.

Perhaps it is only accidental that four of the seven observations involve Jupiter: I (in 498), IV (in 508), VI and VII (in 509/10); the three remaining ones involve the moon: III (in 475), II (in 503), V (in 509). We shall discuss these observations presently in this order, after having described their historical setting.

The list of observations begins with the sentence "I wrote the following from the copy (ἀντιγράφου) of the Philosopher." It appears from the text of the observations that the "philosopher" is Heliodorus: "I, Heliodorus, observed in the year ..."; similar in II: "I and my dear brother (i.e. Ammonius) found ..." or in V: "I saw ..." Only the observations V to VII do not mention the observer but it seems clear from the chronological context that these were also made by Heliodorus, presumably in Alexandria. Only the earliest observation (i.e. III) was made in Athens, as stated in the text.

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[11] The text was discovered by Boulliau (perhaps in Par. gr. 2390) and in detail analyzed in his Astronomia Philolaica (Proleg., p. 14 and, in the order of the observations, I: p. 326, II: p. 246, III: p. 172, IV: p. 278, V: p. 172, VI: p. 327, VII: p. 346). Modern edition by Heiberg in Ptol. Opera II, Introd., p. XXXV–XXXVII from three MSS: C = Marc. gr. 313 (10th cent.), F = Par. gr. 2390, and G = Vat. gr. 184 (both 13th cent.). French paraphrase in Delambre. HAA I, p. 318f., translation in Halma, Chron. II, p. 10–12, both to be used with caution.

[12] For the arrangement of this material in F. cf. Mogenet [1956]. p. 10f., correcting a misleading description given by Tannery, Mém. Sci. II, p. 452. The text in C is preceded by Ptolemy's "Canobic Inscription."

[13] Cf. below p. 1042.

[14] Cf. below p. 1039.

The observations III and IV are headed "observation of θεῖος".[15] It is a much repeated story that Boulliau took θεῖος as a proper name "Thius" and that Delambre and Halma ascribed all seven observations, or at least III to VII, to this otherwise unknown astronomer, until Tannery assumed that θεῖος, meaning "godlike", referred to the venerable Proclus. This interpretation ignores the fact that only III (in 475) could be an observation made by Proclus while IV (in 508) was made after his death. Hence one either drops (without further discussion) the repetition of the "θεῖος" or assumes that the reference to the θεῖος was made at the beginning as well as on the end of III, which leaves IV without a heading. Westerink remarked[16] that θεῖος could simply mean "uncle" and thus refer to Georgius, Hermias' brother.[17] This is itself is the most plausible interpretation. However the chronological difficulty remains since it is hardly possible that Hermias' brother made observations as late as 508. Hence one is compelled to follow Tannery in ignoring the second θεῖος or in admitting that we do not know what is meant by θεῖος. In any case there is no good reason to associate the observation III with Proclus.

Turning to the observational material we illustrate the situation at I, VI, and VII in Fig. 138. The positions of the planets are marked in 10-day intervals,[18] counting in I from 498 Apr. 8, in VI from 509 May 30, in VII from 510 Aug. 13. It is easy to see to what extent these data agree with the statements made in the text. At I it is said that in the night of Pachon 6/7 (= May 1/2) at 2^h of night (presumably Alexandrian time) Mars came so near to Jupiter that hardly any space was left between the planets. In fact at minimum distance, at about ♍ 0;44, the latitude of Mars was about 0;5° greater than Jupiter's.

Observation VI was made on Payni 19[19] (= June 13, 509) after sunset. Mars is said to have been at a distance of 1 finger to the west, 2 finger to the north of Jupiter. Fig. 138 shows that indeed Jupiter was below Mars by about $0;10° = 2^f$. The text notes that the conjunction should take place "according to the tables (ἀπὸ τοῦ κανόνος) and the Syntaxis" on the 23rd, contrary to observation.

A year later, Diocletian 226, it was observed (VII) that Venus had moved on the 28th (month omitted) from a position 8^f (= 0;40°) west of Jupiter to a position 10^f (= 0;50°) to the east, both planets having the same latitude (cf. Fig. 138). Boulliau determined the date as 510 August 20 = Mesore 28.[20] Again a disagreement is noted with "the ephemerides" which give Mesore 30 as date of the conjunction.

Modern computation is in agreement with the observations VI and VII. The position of Jupiter in the evening of June 13, 509 was ♌ 12;10 ($\beta = +1;1$), of Mars ♌ 12;0 ($\beta = +1;13$), i.e. about 2^f difference in longitude as well as in latitude. For

[15] Τοῦ θείου τήρησις, written, according to Heiberg's edition, at the beginning of III and IV, respectively. According to Boulliau, however, these words are written at the end of III while IV begins with τοῦ θείου τήρησιν. ἀπὸ Διοκλητιανοῦ σκε θὼθ λ (the words ἀπὸ Δ. ... are not reported by Heiberg). Cf. also below p. 1109, n. 7.

[16] Westerink [1971], p. 20, n. 27.

[17] Cf. above p. 1031, n. 10.

[18] Using Tuckerman's tables.

[19] The day number 19 is given in words in C while F and G have $\overline{\theta\iota}$, explained by Pingree [1973], p. 33, n. 6 as a scribal error for (night of) 9/10. Halma's translation has incorrectly 29.

[20] Bullialdus, Astron. Phil., p. 346–348; Delambre HAA I, p. 319.

VII one finds

A.D. 510	Diocl. 226	Jupiter		Venus		$\Delta\lambda$
Aug. 20	XII 27	♍ 19;31	+1;11	♍ 19; 1	+1;6	$-0;30=-6^f$
21	28	19;44	+1;11	20;14	+1;5	$+0;30=+6^f$

in fair agreement with the observations.

The complaint against the tables cannot be based on very accurate computations. For VI the Handy Tables give at Diocl. 225 Payni 23[21] for Jupiter the longitude ♌10;30, for Mars ♌6;6, i.e. a situation still before conjunction. For VII one finds at Diocl. 226 Mesore 30[22] Jupiter in ♍17;38. Venus in ♍16;13, i.e. still a little before conjunction.

Finally the observation IV found Jupiter on Thoth 30 (=508 Sept. 27) at a distance of 3^f ($=0;15°$) to the north of Regulus, in direct motion. This agrees with modern data: the planet was on Sept. 27 at ♌9;8 $\beta=+0;43°$; very near to Regulus at ♌9;3 $\beta=+0;25$, i.e. et a distance of about 3 1/2 fingers.

The lunar observations concern:

II: occultation of Saturn, 503 Febr. 21/22 (=Mekhir 27/28)
III: occultation of Venus, 475 Nov. 18 (=Athyr 21)
V: near approach to Aldebaran (α Tau), 509 March 11/12 (=Pham. 15/16).

No. III is the "θεῖος"-observation made at Athens;[23] the two other ones were made at Alexandria.

In II Heliodorus and his brother Ammonius watched the moon at the 4th hour of night[24] as the moon came near Saturn. Then the planet vanished behind the moon and was seen reappearing at the middle of the illuminated arc[25] at about 5 3/4 seasonal hours. From this the observers concluded that the planet was at about 5 $1/8^{s.h.}$ behind the center of the moon, having traversed a full lunar diameter (i.e. remaining invisible for about 1 $1/4^h$).

At the conclusion of this observational report there is the following remark "the third circle was about 2 1/2 degrees." This statement must be related to the time of reappearance of the planet at about 5;45 seasonal hours after sunset, determined on the astrolabe, as is explicitly stated in the text. This "astrolabe" is the plane astrolabe because only its design provides us with seasonal hours.[26] Since 2 1/2° correspond to 10 minutes of time this number may have to do with the interval between the observation and midnight. I do not know, however, which circle on the astrolabe counts as the "third."[27]

[21] Philip 833 III 1=509 June 17.
[22] Philip 834 V 6=510 Aug. 23.
[23] Cf. above p. 1039.
[24] Heiberg took the reading "first hour" into the text because it is the reading found in C, whereas F and G have the obviously correct δ. Heiberg's date 502 is a mistake.
[25] New moon was at Febr. 12, full moon at Febr. 27, hence the moon was about 3/5 illuminated.
[26] This connection between astrolabe and seasonal hours is also emphasized by Paulus of Alexandria (ed. Boer, p. 80, 12f.); cf. also above p. 956.
[27] For the determination of fractions of hours on the astrolabe cf., e.g., Philoponus' treatise, Sect. VI (Tannery, Mém. Sci. 9, p. 351–353).

Modern computations agree with the observations very well.[28] The emersion of the planet took place at about 23;31^h (Alexandrian mean time), i.e. only about 10 minutes earlier than determined by the astrolabe.[29] The moon's center did not pass exactly over the planet but the planet's latitude was about 0;7° greater. Thus the occultation lasted only 1;5^h instead of the estimate of 1;15 seasonal hours, i.e. about 1;20 equinoctial hours.[30]

Equally good is the agreement for the occultation of Venus in A.D. 475 Nov. 18, observed by "ϑεῖος" at Athens. It began (at 16;25^h) before sunset at Athens but the emersion was visible (at 17;46^h). The center of the moon passed 0;7° to the south of Venus (which was at ♑14;58 $\beta = -2;44$) at an elongation of 47° from the sun. The text[31] gives for the moon ♑13° and 48° as elongation, thus assuming ♏25 as solar longitude. Computation with the Handy Tables for Alexandria gives for the sun at noon ♏25;16 and six hours later ♏25;32. For the moon one finds ♑11;15 at noon, ♑14;48 at 18^h. For Venus, however, one obtains almost exactly ♑13. Hence we see that the position of the moon was taken from the position of the planet.

The last lunar observation. A.D. 509 March 11, took place after darkness when the moon was seen at a distance of at most 6 fingers (=0;30°)[32] from αTau. "Because the star was near the midpoint of the convex limb of the illuminated part of the moon" Heliodorus concluded that the moon had passed over the star. In fact, however, the moon had not occulted the star; at the closest approach, which had taken place before sunset, the star was 0;41° distant from the moon's limb. The moon's longitude at the observation is given as ♉16 1/2, undoubtedly computed with the Handy Tables which give ♉16;31 for 6^h after noon in Alexandria.

Philoponus. Ioannes Grammaticus, nicknamed Philoponus,[33] was a prolific writer. He was a pupil of Ammonius,[34] later turned christian and polemicized against the doctrines of the pagan philosophers, e.g. Proclus. His theological position, however, inclined toward monophysite teachings, and it was eventually condemned as being heretical.[35] His connection with monophysite circles explains the influence his writings had among Syriac scholars. Sergius of Reš'aina was his pupil, Severus Sebokht in the 6th century studied his treatises[36] and thus transmitted his influence to Jacob of Edessa (d. 708) and to Ananias of Shirak

[28] The modern computation concerning the three lunar observations described here were carried out by Dr. F. R. Stephenson. For the secular acceleration he used Schoch's value (Langdon-Fotheringham-Schoch. The Venus Tablets of Ammizaduga, Oxford 1928, Chap. XV).

[29] This determination involves the observation of a fixed star and the computation of the solar longitude; cf., e.g., Philoponus' treatise, Sect. VIII (Tannery, Mém. Sci. 9, p. 354f.).

[30] Duration from 22;31 to 23;36 Alexandrian mean time. The Handy Tables give for the solar position ♓2;3, hence for one seasonal hour of the night 64 1/3 equinoctial minutes.

[31] Translating line 5 as "(Venus), visible after conjunction, at Athens, (the moon being) located in ♑13."

[32] Delambre and Halma read incorrectly 1/2 instead of 6.

[33] For the meaning of this name cf. above p. 878, n. 8.

[34] So he tells us, e.g., in his treatise on the astrolabe (Tannery, Mém. Sci. 9, p. 342).

[35] At the Council of Constantinople in 680 (cf. Mansi, Council XI col. 501/502C).

[36] On the astrolabe (cf. above V B 3, 7 F); on the general conjunction of 529, mentioned by Philoponus in his polemics against Proclus (cf. Neugebauer [1959, 2]).

in Armenia in the 7th century.[37] Bīrūnī mentions Philoponus on several occasions, e.g. in his "Transits" for the problem of the position of Venus in relation to the sun.[38] All this has, no doubt, also contributed to the spread of the treatise on the astrolabe and thus to the use of the instrument during the Islamic and Byzantine period. This is certainly a major contribution to the preservation of sound astronomical knowledge during the Middle Ages.[39]

Philoponus' interests also ranged into mathematics, at least of the type represented in the number theory of Nicomachus of Gerasa (about A.D. 100) on which he wrote a commentary.[40] Arabic tradition even associates Philoponus with the transmission of medical works but Meyerhof has shown that this tradition is spurious.[41] It illustrates, however, the high opinion which one had of Philoponus' scholarship through many centuries.

3. Eutocius. Two famous architects, Anthemius of Tralles, and after his death (around 534), Isidorus of Miletus; were entrusted by Justinian with the supervision of the building of the Hagia Sophia. To Anthemius, who had also written on optics,[1] Eutocius dedicated his commentary on the Conics of Apollonius. Earlier commentaries, one of which is dedicated to Ammonius, deal with works of Archimedes.[2] Finally Mogenet [1956] suggested with good arguments that an anonymous Introduction to Book I of the Almagest should be added to the list of Eutocius' writings. If a horoscope, cast for 497, is correctly ascribed to Eutocius[3] we would have evidence for some astrological writings of his.

The above-mentioned dedications establish the years around 500 for the work of Eutocius. Westerink suggested[4] considering him as the head of the Alexandrian school between Ammonius and Olympiodorus. Tannery concluded from the tenor of the dedication to Ammonius that Eutocius had been his pupil,[5] which would perhaps explain how a scholar mainly interested in mathematical topics could become the head of a school which was traditionally under the control of philosophers.

The above-mentioned Introduction to Book I of the Almagest is the only extant work of Eutocius which is related to astronomy. However this is true only to the extent that certain passages in the Almagest suggested a discussion of some

[37] Cf. N. Pigulewskaja, Byzanz auf den Wegen nach Indien (Berlin 1969), p. 116–118. Also Lemerle, Human. Byz. (Paris 1971), p. 82–84.
[38] Bīrūnī, Transits, p. 16 (13;18) trsl. Kennedy.
[39] Kroll (RE 9, 2 col. 1793, 16–24) considers it possible that Philoponus is the author of the Pseudo-Geminus parapegma because one manuscript mentions "Ioannes of Alexandria" as its author (cod. Berol. 161 = Phill. 1565 fol. 190; cf. CCAG 7, p. 42), a manuscript overlooked both by Wachsmuth (in his edition of Lydus) and by Manitius (in his edition of Geminus). Needless to say, such an association is chronologically excluded (cf. above p. 581).
[40] Cf. L. Tarán, Asclepius of Tralles, Comm. to Nicomachus, Trans. Amer. Philos. Soc. NS 59, 4 (1969).
[41] Max Meyerhof, Joannes Grammatikos (Philoponos) von Alexandrien und die arabische Medizin. Mitteilungen des deutschen Instituts für ägyptische Altertumskunde in Kairo 2 (1932), p. 1–21.
[1] Cf. Heath, GM II, p. 541–543.
[2] Cf. Heath, GM II, p. 540.
[3] For the rather inconclusive textual evidence cf. Neugebauer-Van Hoesen, Greek Hor., p. 188f.
[4] Westerink, Anon., p. XIII.
[5] Tannery, Mém. Sci. 2, p. 121.

mathematical problem or of computational methods. In other words Eutocius did not write a "commentary" of the ordinary type that follows a given text chapter by chapter. Mogenet gave a table of contents[6] from which it is clear that the main part concerns methods of sexagesimal computation: multiplication,[7] division,[8] square roots, etc. Another chapter concerns isoperimetric problems,[9] followed by a short section about shape and size of the earth, based on Ptolemy's norm of 500 stades for the equatorial degree.[10] Obviously nothing of real astronomical interest has come down from Eutocius.

4. Olympiodorus. In 1958 E. Boer published the first modern edition of the "Isagoge" of Paulus of Alexandria[1] and continued in 1962 with the editio princeps of a commentary to Paulus by "Heliodorus, *ut dicitur*". The caution in this formulation was very appropriate, since it soon became evident that Heliodorus was not the author of this commentary but that it reproduced lectures by Olympiodorus, given in Alexandria. This insight was first reached by Warnon [1967] and again, independently, by Westerink [1971]. Both scholars realized that the original version reproduced "lectures" (πράξεις)[2] by Olympiodorus which later were cast into a more bookish form by some Byzantine user. The original lectures were held in the months from May to August 564, as is shown by several dates mentioned in the text, probably 26 in number of which the first five are lost.[3]

Evidence of the superposition of different versions can be found in numerical parameters. For example[4] one redaction assumes 21° as the oblique ascension of Aries and Pisces, which is the traditional value from System B for Babylon.[5] A second version, however, replaces this value by 16;43°, i.e. by the trigonometrically determined value, correct for Byzantium.[6] Similarly, $\rho(♌)=35°$ is said to be valid for "the golden Alexandria; i.e. clima III" as is indeed the case for System A in the version of Hypsicles.[7] In Lecture 28, however, all oblique ascensions are trigonometrically computed, based on the "tables of Ptolemy"[8] for the latitude of Alexandria.

Continued changes in numerical data can also be observed in other instances. Paulus, e.g., uses in an example the following positions:[9]

♃: ♓2 ☉: ♓17 ♄: ♓26

[6] Mogenet [1956], p. 4.
[7] Published by Tannery, Diophantus II, p. 3, 18–15, 17.
[8] Published Mogenet [1951], describing a method found in Pappus' (lost) Commentary to Almagest III.
[9] Published by Hultsch, Pappus III, p. 1138–1165.
[10] Published by Hultsch, Pappus III, p. XX f. For a description of the rather sad state of publication cf. Mogenet [1956], p. 5-8. Mogenet's own publication, announced in 1956 (p. 8, note 2) has not yet appeared (1975).
[1] Cf. above p. 955.
[2] Cf. Warnon [1967], p. 202, etc.; Westerink [1971], p. 7f.
[3] Westerink [1971], p. 16 and p. 9f.
[4] Boer, Heliod., p. 8, 1.
[5] Cf., e.g., above p. 368 (1).
[6] Halma, H.T. II, p. 58/59.
[7] Above p. 717 (10): Boer, Heliod., p. 130, 15–30.
[8] Boer, Heliod., p. 94, 19; p. 95, 2; p. 96
[9] Paulus Alex., ed. Boer, p. 29, 24–30, 2.

while Olympiodorus' commentary has[10]

♃: ♈2 ☉: ♈15 ♄: ♈27.

It is clear we have here a quite fluid tradition in which one can take specific value only with great caution.[11]

The fact that the "Heliodorus" commentary actually belongs to Olympiodorus does not exclude the possibility that a commentary to Paulus by Heliodorus did nevertheless exist. Evidence for such a work was collected by Warnon[12] who suggested that *διδασκαλία ἀστρονομική* was its title.[13]

It is obvious that Olympiodorus had some technical competence. He also assumed that his readers (or students) were able to determine the position of a planet from "the tables of Ptolemy"[14] and he had intended to discuss these tables in his lectures.[15] Yet, some data which he presented to his audience are rather crude roundings of parameters found in the Almagest or in Proclus' Hypotyposis. For example, using the distance of the moon as unit, he makes[16] the distance of the sun 20 while the length of the shadow cone of the earth is assumed to be 5 lunar distances.[17]

A definite lack of understanding is revealed in his discussion of Ptolemy's circular diagram of the sun's rising amplitudes in the seven climata (Alm. VI, 11).[18] Neither he nor his "ancestor, the philosopher" (i.e. Ammonius)[19] realized the schematic character of the diagram and therefore assumed that the wind directions, written between the radial lines, were assumed to differ 30° from one to the next.

Olympiodorus seems to have known Ptolemy's "Analemma" as well. At

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[10] Boer, Heliod., p. 13, 1–9. In line 7 it is stated incorrectly that Jupiter is in morning setting; line 9 and Paulus (p. 30, 3) speak correctly about morning rising.

[11] I do not think that schematic examples, like Boer, Heliod., p. 31, 10f. and 23f. should be used for chronological purposes (Pingree, in Boer, Heliod., p. 149). Three of these positions represent Jupiter, Sun, and Moon in trine at longitudes 0°, 120°, 240° (*μοῖρα α* means, of course, "beginning" of a sign; cf., e.g., above p. 596, n. 19; the other positions are exactly 1/2 sign distant: Jupiter and Moon at 15°, Mars at 0° (the moon is said to be at 12° only in the late version while version A gives 15°). The positions in Boer, Heliod., p. 48, 6–8 for sun and moon are much too frequent to be chronologically useful.

[12] Warnon [1967], p. 212–214.

[13] Many fragments of such a treatise are preserved but no systematic edition exists; cf. Warnon [1967], p. 212, note 78. Tihon [1973], p. 100ff. considers the scholion on planetary phases, discussed above p. 259, as one of these fragments.

[14] Boer, Heliod., p. 39, 1f; cf. also the index s.v. Ptolemy (Boer, p. 176). One has to be careful, however, to distinguish between the different versions of the text.

[15] Westerink [1971], p. 19 on the basis of Boer, Heliod., p. 24, 11f. Apparently *κανὼν Πτολεμαίου* means the "Handy Tables." Paulus (p. 33, 11f. and p. 79, 9f.) says *πρόχειρος κανὼν Κλαυδίου Πτολεμαίου*. Theon is never mentioned by Olympiodorus or by Simplicius (according to the indices in CAG).

[16] CAG XII, 2, p. 68, 20–27; p. 72, 14–16. Corresponding sizes: p. 19, 20–20, 3.

[17] These roundings should be the equivalent of 64;10, 1210, and 268 earth radii, respectively (e.g. from Proclus, Hypotyposis, p. 135, Manitius).

[18] For the modern equivalent of this diagram cf. above p. 38. The ancient version is shown on the folding plate at the end of Heiberg's edition of the Almagest (Vol. I). The corresponding plate in Manitius' translation omitted the names of the winds. Stüve, the editor of CAG XII, 2, was as bewildered as Olympiodorus and Ammonius (cf. his note to p. 188, 33).

[19] CAG XII, 2, p. 188, 34–189, 10.

least he refers to the term "hektemoros" but he says,[20] incorrectly, that this was Ptolemy's name for the horizon.[21]

The lectures on Paulus, delivered in 564, were followed by a course on Aristotle's "Meteorology,"[22] dated by a reference to a comet, visible in March and April of 565.[23] The path of the comet is described as leading from a position in the northern sky near Draco, then moving across the Milky Way, to end in Capricorn.[24] Such an orbit is rightly considered as incompatible with the theory of a zodiacal origin of comets. It is in this context that Olympiodorus assigns the zodiac a "width" of 20°,[25] in contrast to the conventional 12°; we shall return to this norm presently.[26]

Olympiodorus is credited sometimes with a commentary on the alchemical works of Zosimus of Panopolis[27] (about A.D. 300?). The fact alone that the author of this commentary is obviously a christian and addresses himself to Justinian makes the identification with our Olympiodorus very doubtful.[28]

5. Stephanus of Alexandria. It seems to be a well established fact that Stephanus, a pupil of Olympiodorus, was called to Constantinople[1] at the time of the emperor Heraclius (610 to 641), but most details in the life and work of Stephanus (or his namesakes) remain in the dark.[2]

There is evidence that Stephanus was the author of an Introduction to the Handy Tables, extant in several manuscripts. Five of these were used by Usener, who first drew our attention to this work by publishing six out of its 30 chapters[3]. Four of these, however, are not genuine. As far as I know only one of these manuscripts names Stephanus, "the great philosopher from Alexandria"[4] as the author and uses the title "Explanation through individual examples of Theon's method for the Handy Tables."[5] Perhaps the best manuscript,[6] unknown to Usener, adds to the title "but it is not by Theon, but by someone later." Obvi-

[20] CAG XII, 2, p. 261, 34f.

[21] Cf. above p. 849 (Fig. 25).

[22] Published in CAG XII, 2; cf. also RE 18, 1 col. 220, 67–221, 26.

[23] Incorrectly dated in the edition (p. 52, note 31) to 565 August/September, ignoring that the date refers to the Egyptian calendar (κατὰ Αἰγυπτίους), as is not surprising for an astronomical event that requires, e.g., comparison with a solar longitude. The wrong date is repeated, ever since, in the literature. Even the year became eventually wrong (564), e.g. in Vancourt, Comm., p. 1 and Warnon [1967], p. 203.

[24] CAG XII, 2, p. 52, 30–53, 2. In RE 11, 1 col. 1193, 16–24 Olympiodorus' report is mixed up with other records which have nothing to do with the comet of 565 March/April.

[25] CAG XII, 2, p. 52, 24ff.; p. 53, 6.

[26] Cf. below p. 1050.

[27] Festugière, RHT I, p. 239; cf. also RE 1, 1 col. 1349, 11–50; Hammer-Jensen, Die älteste Alchymie, p. 125f. (Danske Vid. Selsk., Hist.-fil. Medd. 4, 2 [1921]).

[28] Westerink, Anon., p. XV; Warnon [1967], p. 206 who also observes that the division of the commentary into "lectures" (πράξεις) is missing, otherwise the rule with the Neoplatonists.

[1] Cf., e.g., Westerink, Anon., p. XXV.

[2] Cf. Lemerle, Hum. Byz., p. 80f. and note 29 there; also Vancourt [1941], p. 26–33.

[3] Usener, Kl. Schr. III, p. 294–317. Additional manuscripts are listed by R. Browning, Class. Review 15 (1965), p. 263. The numbering of the chapters is (as usual) not the same in the few manuscripts which count the chapters.

[4] He is also called "Oecumenical Teacher." Cf. for this title Lemerle, Hum. Byz. p. 80f., p. 85ff.

[5] Usener, Kl. Schr. III, p. 295, 1–3.

[6] Marc. gr. 325, fol. 10r–81v, 2 (unpublished).

ously Stephanus' authorship was not generally known. On the other hand his name is still mentioned in the 14th century as an authority with Ptolemy and Theon.[7]

The date and locality of the treatise are firmly established since all numerical examples range over the years A.D. 617 to 619 and are computed for Constantinople. This supports the authorship of Stephanus but close cooperation with the emperor Heraclius seems to me extremely unlikely. Usener assumed his authorship for the last three sections[8] (28 to 30 by his count) which no longer have any relation to the Handy Tables, since they deal with trivial calendaric matters and with the Easter computus.[9] The first section of the treatise is also not part of the original composition but is one of the long scholia (compiled, e.g., by a teacher) intended to explain the commonly used technical terms for a beginner student.[10] The same text, written in a later hand, is also found on a page prefixed to the famous copy Vat. gr. 1291 of the Handy Tables.[11] There the title is "Scholion, with the help of God, to Ptolemy's Handy Tables, from the lectures (ἀπὸ φωνῆς[12]) of Heraclius, our king by the Grace of God." A similar phrase also occurs in section 2, i.e. in the first section of the treatise proper: "in the 9th year of our kingship which it pleased God to grant." In spite of these passages it is hard to believe that Heraclius, among the military disasters of the first ten years of his reign had the time to lecture on the use of the Handy Tables.

The table of contents, published by Usener, shows that the "Explanation" is a comprehensive introduction to the use of the Handy Tables, similar to Ptolemy's Introduction.[13] However, it is much longer (as is in part justified by the inclusion of numerical examples, e.g. for eclipses, requiring extensive computations). The comparison with Ptolemy and with Theon's "Small Commentary"[14] shows that the order of topics was somewhat changed by discussing the planets after sun and moon, whereas Ptolemy and Theon place the eclipse computations after the planets. For a large scale introduction the new arrangement is clearly preferable.

The sections published by Usener only give a very incomplete and atypical picture of the treatise since the published excerpts are much more calendaric in character than the remaining sections which deal with the practical use of the tables. The initial step consists in the determination of the mean position of a celestial body by means of the "five chapters," i.e. the tables (1) for 25-year steps, (2) for single years, (3 to 5) for months, days, and hours.[15] Since the years in these tables are Egyptian years the transformation from other calendaric norms requires

[7] I. Ševčenko, Études sur la polémique entre Théodore Métochite et Nicéphore Choumnos, p. 114, note 2 (Bruxelles 1962).

[8] Usener, Kl. Schr. III. p. 292; also Monumenta 13,3. p. 362.

[9] Marc. gr. 325 has only one of these sections (Usener's No. 29). All three sections are also published by Halma, H.T. III. p. 101–111 with French translation (cf. Usener, p. 291, note 24). For the Easter computus cf. Schissel [1934], p. 287–292.

[10] For similar additions turning into an introduction cf. Theodosius, Dieb. (above p. 756).

[11] Vat. gr. 1291 fol. 1ʳ, 13–36, published by Heiberg, Ptol. Opera II, p. CXCIf., note 1, corresponding to Usener, p. 296, 25–298, 9. Cf. also above p. 970.

[12] For this terminology cf. Lemerle, Hum. Byz., p. 80, note 27.

[13] Cf. above p. 969f.

[14] Cf. above p. 977.

[15] These are the "πέντε κεφάλαια" (cf. Neugebauer [1960, 1], p. 10).

a lengthy discussion. This is what is published in section "2" (i.e. the beginning of the treatise proper).[16]

The other section published by Usener (No. 12 in his count) is preparatory to the computation of the lunar eclipse in the year Philip 942 Phaophi (II) 12 (= A.D. 618 Oct. 9). The text in question, however, concerns only the preliminary problem: on what day and hour in the month Phaophi of the current year does the true opposition occur?[17] The answer to this question is found by computing the double elongation for the beginning of the month in question;[18] then one makes up for what is missing from 360° by means of the tables for days and hours. This provides the moment of mean opposition. Computing for it the true positions of the luminaries leads to a correction that makes the true elongation 180°. This is exactly the procedure prescribed by Ptolemy in his Introduction to the Handy Tables.[19]

The actual computation is complicated by purely calendaric problems caused by the fact that the tables are based on the Egyptian calendar while the given date refers to the Alexandrian calendar. Consequently one needs the number of intercalary days since the time in the reign of Augustus when the Alexandrian pattern was introduced. Our text presupposes that Thoth 1 was the same day for both calendars in the year Augustus 5, as is indeed the case.[20] Consequently, if n is the number of years since Augustus 5 the number of intercalary days is $[n/4]$ (i.e. the integral part of the quotient $n/4$). Since Augustus 5 = Philip 299 we have for Philip 942 the difference $n = 643$, thus $[n/4] = 160$ intercalary days. Hence we know that the Egyptian month we are investigating is 160 days = 5 months 10 days ahead of the Alexandrian month II.

The next step turns to the epact of a given year N. If

$$N \equiv r \text{ mod. } 19 \qquad 0 \leqq r < 19 \tag{1a}$$

then the epact e (ἐπακτή) of N is given by

$$e \equiv 11 \cdot r \text{ mod. } 30 \qquad 0 \leqq e < 30. \tag{1b}$$

Hence the date of the full moon in month m of this year will be

$$15 - \left(e + \frac{m}{2}\right) = d. \tag{2}$$

The text states that N may represent either the years in the era Philip or the years of Constantine. Because[21]

$$\text{Constantine } 1 = \text{Philip } 628 \tag{3}$$

[16] What Usener called "section 3" is actually only a short paragraph at the end of section 2. In Marc. gr. 325 the order of the sections is 4–3–5 of Usener's table of contents (4 = fol. 12ᵛ, 26–13ᵛ, 16; 3 = 13ᵛ, 16–17ʳ, 6; 5 = 17ʳ, 6–17ᵛ, 13).

[17] The determination of the circumstances of the eclipse follows in a later section ("16" in Usener ≈ Marc. gr. 325 fol. 36ᵛ, 6–44ᵛ).

[18] Column 3 in the tables of mean motions of sun and moon (cf. above p. 987).

[19] Cf. above p. 1000f.

[20] Usener, p. 300, 10. Cf. above p. 966.

[21] Cf., e.g., the Royal Canon of the Handy Tables (Monum. 13, 3. p. 454, ignoring Constans): Philip 942 = Augustus 648 = Diocletian 335 = Constantine 317 = *indictio* 7 (because *indictio* 1 = Diocletian 29 and therefore Diocletian 335 = *indictio* 307 ≡ 7 mod. 15).

and because $628-1=627\equiv 0$ mod. 19 the two eras produce the same epact and are therefore equivalent for the present purpose.[22]

It is interesting to see that the Handy Tables discarded the 25-year table of mean syzygies in the Almagest[23] and replaced it by estimates based on the 19-year cycle, starting from some given new- or full-moon date. For the determination of the true syzygies the estimates thus obtained are, of course, just as useful as the more accurate Ptolemaic mean syzygies.[24] The main advantage of the epact computations probably lies in the use of the civil calendar as against Egyptian years.

We find in our case for $N=942$ (Philip): $r=11$ thus $e=121\equiv 1$ mod. 30; hence for $m=2$ the full moon date $d=15-(1+1)=13$, i.e. 2 days earlier than at epoch. Since we have found that the Egyptian date would be 5 months and 10 days ahead of the Alexandrian date we may assume that the full moon in question belongs to month VIII of the Egyptian calendar.

What now follows[25] is no longer of independent interest. Computing for the year Constantine 315 (indictio 7) October = Philip 942 Pharmouthi (VIII) results from the first three of the "five chapters" (i.e. 926+16 years+7 months) in a double elongation $2\bar{\eta}=199;8^\circ$. From it one obtains $2\bar{\eta}=360^\circ$ by progressing additional 22^d and $8;42^h=8\ 1/2\ 1/5^h$ after Alexandria noon. This moment gives, as expected, for the mean longitudes almost exact opposition: $\lambda_\odot=$ ♎ $17;21$, $\lambda_☾=$ ♈ $17;20$. The corresponding true longitudes are ♎ 15;32 and ♈ 15;30 such that the missing $0;2^\circ$ require only a change to $8;46^h=8\ 1/2\ 1/5\ 1/15$ after noon.[26] The corresponding modern data are[27] $8;2^h$ and $\lambda_☾=$ ♈ $18;46$.

Since all this is based on the meridian of Alexandria a conversion to local time and seasonal hours for Byzantium was carried out.[28] First the equation of time for $\lambda_\odot=$ ♎ $15;32$ is found[29] to be $0;1,52^h=0;28^\circ$ in order to obtain true midnight from mean solar time. Then we are told that the difference of longitude between Alexandria and Byzantium amounts to $4;30^\circ$ – a very bad estimate since Byzantium is only 1° to the west of Alexandria.[30] Since $8\ 1/2\ 1/5\ 1/15=8;46^h$ correspond to $131;30^\circ$ and $4;30+0;28=4;58^\circ$ corrects the Alexandrian meridian to the Byzantine we find for Byzantium true solar time $131;30-4;58=126;32^\circ$ after noon. For $\lambda_\odot=$ ♎ $15;32^\circ$ is the length of one seasonal hour[31] of daytime

$$\text{for clima V: } 14;6^\circ, \qquad \text{for clima VI: } 13;58^\circ$$

[22] The relation (3) agrees with Usener's text (p. 298, 28). Marc. gr. 325 fol. 22ʳ, 13 ff. says one should count from Constantine 16, thus 296 years until Philip 942. Since $296\equiv 942$ mod. 19 this shift has no influence on the value of the epact. The corresponding epoch year, however, is Constantine 19, not 16. Indeed, only for Constantine 19 (= Diocletian 39 = Philip 646 = A.D. 322) does one find for Thoth 15 = Sept. 12 a full moon date (cf. Goldstine's tables).

[23] Cf. above I B 6, 1.

[24] For a variant of these epact computations cf. below p. 1052.

[25] Usener, p. 306, 3–309, 4.

[26] In several cases the fractions are incorrectly (and meaningless) given by Usener as 1/2 1/15′ 1/15″(!).

[27] From Goldstine, New and Full Moons.

[28] Usener, p. 309, 5–310, 17; the following lines (18–22) are out of place here, repeating only the computation of the epact.

[29] Halma, H.T. I, p. 155.

[30] The wrong longitudes agree with Ptolemy's Geography: Alexandria 60 1/2° (Geogr. IV 5, 9) and Byzantium 56° (Geogr. III 11, 5).

[31] From Halma, H.T. III, p. 39 and p. 47, respectively.

thus for Byzantium that is located between these climata the arithmetical mean

$$1^{\text{s.h.}} \text{ of daytime: } 14;2°, \text{ of night}: 15;58°.$$

Thus $6^{\text{s.h.}}$ of daytime correspond to $6 \cdot 14;2 = 84;12°$ and $126;32 - 84;12 = 42;20°$ after sunset in Byzantium or

$$42;20/15;58 \approx 2;39^{\text{s.h.}} = 2\ 1/2\ 1/10\ 1/20^{\text{s.h.}}$$

after sunset at Byzantium. This then is the moment of opposition.

The chapter ends with the remark that for the sake of convenience special tables had been computed for Byzantium, simply using the arithmetical means between the values for the climata V and VI.[32]

The details for the subsequent eclipse computations are unpublished and not investigated in any detail. The same holds for the annular solar eclipse of A.D. 617 Nov. 4, i.e. Constantine 315, Athyr (III) 8, indictio 6,[33] or era Philip 941 Pharmouthi (VIII) 18.[34] It is clear, however, that many concepts are peculiar in this latest phase of ancient Greek astronomy, e.g. the association of the "angles of inclination" (prosneusis) of the individual phases of an eclipse with wind directions.[35]

In general one can observe that this treatise adheres to archaic concepts, e.g. by introducing "steps" (βαθμοί)[36] for lunar latitudes as well as for planetary latitudes and for solar declinations.[37] In the latter case, e.g., the ecliptic is divided into four quadrants, beginning at ♋0°, each of which is divided into six 15°-"steps." Thus a solar longitude of ♈19;15 = ♋289;15 locates the sun in the 4th quadrant[38] and $1/5 + 1/12$ of the second step because $289;15 = 270 + 15 + 4;15$ and $4;15 = (1/5 + 1/12) \cdot 15°$.

Probably it is only an apparent archaism when the planets are named in the text of the treatise by their early names (Φαίνων for Saturn, etc.). It may well be a concession to christianity to avoid the names of the pagan gods. In the tables however, the pagan names are used.

The title of one section could promise some real astronomical interest: "to know when sun, moon, or one of the planets approaches (συνεγγίζει) a fixed star".[39] Unfortunately the procedure described is rather trivial: beginning with a given date one computes longitude and latitude of the celestial body in question and finds from a catalogue of stars a fixed star with coordinates as close as possible to the position of the planet. Such a catalogue of stars is assumed to give the

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[32] Usener, p. 310, 11-17. This is the basis for Halma's unfortunate invention of a "clima VIII"; cf. above p. 969, n. 6.

[33] Usener, p. 293, p. 329 (from Vat. gr. 1059 fol. 529ᵛ with reference to "Stephanus, philosopher and Great Teacher"); Marc. gr. 325 fol. 55ʳ, 19.

[34] CCAG 9, 1, p. 50: Cromwell, 12, p. 1205. Cf. for this eclipse Schröter, Kanon, map 4a.

[35] Usener, Kl. Schr. III, p. 297, 8; passim in Marc. gr. 325, e.g. fol. 54ᵛ-55ʳ. For the prosneusis cf. above I B 6, 7.

[36] Cf. above IV B 5.

[37] Cf. above p. 670.

[38] From Usener's Chap. 9 (Marc. gr. 325 fol. 21ʳ, 8-18).

[39] Usener Chap. 26 (Marc. 325 fol. 78ᵛ, 5-80ʳ, 6).

longitudes with respect to Regulus[40] whereas the longitude λ of the planet is counted from the vernal point. The text explains at length how one transforms λ into a distance from Regulus – a trivial problem in view of the fact that the first column of the planetary mean motion tables gives the tropical longitude of Regulus as function of time.[41]

The star catalogue of the Handy Tables is an excerpt from the star catalogue of the Almagest but it does not list stars fainter than the 4th magnitude and outside a zone between $\pm 10°$ of latitude.[42] These limits were obviously chosen in view of the planetary approach to fixed stars since Venus can reach latitudes of $\pm 8;56°$.[43] This explains why Olympiodorus assumes a width of 20° for the zodiac;[44] perhaps the observations of Heliodorus[45] reflect a larger tradition of looking out for occultations.

Byzantine historians of the tenth century tell us that Stephanus cast the horoscope of the emperor Heraclius, predicting (wrongly, as we know) that he would die by drowning.[46] Since we also know that Heraclius took elaborate precautions to protect himself against this danger[47] one may consider this as proof for Stephanus' influence.

The famous horoscope of Islam,[48] however, cannot be a work of our Stephanus because its author is familiar with the caliphs up to the end of the eighth century (from then on the predictions are all wrong). To the same period also belongs a praise of the usefulness of astrology by "Stephanus the Philosopher." In it the eras used by Ptolemy, Theon, Heraclius, and Ammonius are correctly named, besides the Persian and Arabic years used by "more recent" authors.[49] Also Abū Ma'shar (ninth century) knows of an astrologer Stephanus.[50] Another short astrological text, supposedly by "Stephanus of Alexandria" has come to light.[51] It concerns conjunctions of Saturn and Jupiter but was actually written in the 11th century.

There are also alchemical works which circulate under the name of Stephanus of Alexandria.[52] Festugière considers the identification with the astronomer of

[40] The star catalogues of the Handy Tables are of this type (cf. Halma, H.T. III, p. 44–58; Vat. gr. 1291 fol. 90ᵛ–94ᵛ; Marc. gr. 325 fol. 168ʳ–170ʳ, etc.). Cf. also above p. 1026f.

[41] Cf. above V C 4, 3 C or p. 1002.

[42] Cf. note 40.

[43] Cf. above p. 1014 (1). The same extremal latitudes for the planets are mentioned also in our treatise (Usener Chap. 22; Marc. gr. 325 fol. 68ʳ, 21–68ᵛ, 2), excepting Saturn's northern extremum of 3;3° instead of 3;2°.

[44] Cf. above p. 1045 and p. 583. The "Quadrivium" of Pachymeres (end of 13th cent.) also gives 20° as width of the zodiac (ed. Tannery, p. 330, 12–14).

[45] Cf. above p. 1038 ff.

[46] Theophanes Continuatus (CSHB, ed. Bekker, 1838), p. 338, 10–12, written under Constantine VII (913–959); cf. Ostrogorsky, Geschichte d. Byz. Staates (2), p. 119.

[47] Cf. the passage in question in Theophanes Cont.; also Ostrogorsky, l.c. p. 90.

[48] Published Usener, Kl. Schr. III, p. 266–287. Cf. also Neugebauer-Van Hoesen, Greek Hor., p. 158–160.

[49] CCAG 2, p. 181 f. (in part translated to German by Balss, Antike Astronomie, München 1949, p. 199–201).

[50] Greek version CCAG 5, 1, p. 152, 3 and 9; Latin version, trsl. Thorndike, Isis 45 (1954), p. 30.

[51] Published by D. Pingree, JAOS 82 (1962), p. 501 f.

[52] Greek text, translation and commentary by F. Sherwood Taylor, Ambix 1 (1937), p. 116–139, 2 (1938), p. 38–49 (on making gold); also 4 (1951), p. 92, p. 118 (No. 15), p. 120 (No. 36).

Heraclius' time as certain[53] and there seems indeed no reason to doubt that the late Neoplatonists were interested in this new occult science.[54]

C. Fragments

The commentary (or "Explanation") by Stephanus to the Handy Tables was written only three years before the Prophet's flight to Medina. Hence Stephanus' treatise belongs to the very latest phase of Greek astronomy, still outside any influence from Islamic sources. Under such circumstances it is a sad commentary on the level of modern "History of Science" that such a work has not attracted more attention than Usener's excerpts from two chapters, published in 1880. Unfortunately this is not an isolated case of neglect. Not even the Handy Tables are adequately published (Halma's edition of 1822/25 can hardly be so considered), nor has Theon's "Great Commentary" to these tables seen any edition whatsoever. Hence one is forced to rely on more or less accidental studies[1] and discoveries instead of working with a "Corpus" of critical editions of Byzantine astronomical treatises which are still extant in abundance in all major libraries of Europe.

For astrological sources we are a little better informed thanks to the twelve volumes of inventories and excerpts, known as the "CCAG." For our present subject, however, the astrological texts are only of auxiliary significance, e.g. by demonstrating an increase in astronomical sophistication in the techniques of the later astrologers in comparison with the methodology up to the time of Vettius Valens,[2] i.e. until Ptolemy's work became generally known. Nevertheless, residues of earlier levels remain recognizable to the very end of medieval astronomy. It is for this reason frequently impossible to assign a definite period to methods or parameters which are found scattered through compilations of the most diverse astronomical material in manuscripts from all periods of Byzantine history. The subsequent examples illustrate this situation.

For unknown motives the names of Neoplatonists – Proclus, Ammonius, Stephanus – were all associated, often against clear evidence, with astronomical writings which could not have been their own.[3] Fragments of one such treatise, dubiously ascribed to Heliodorus, has come down to us in the astrological literature.[4] This short text is of interest insofar as it contains elements that are strongly reminiscent of Babylonian astronomy. Not only do we find here the "goal-year periods" from the early stage of Babylonian procedures[5]

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[53] Festugière, RHT I[(2)], p. 239f. For the text cf. Usener, Kl. Schr. III, p. 248. Also Ruska, Arabische Alchemisten (Heidelberg 1924), p. 33, etc.

[54] Cf. Westerink, Anon. p. XXV; also Ingeborg Hammer-Jensen, Die älteste Alchymie, p. 146ff. (Danske Vid. Selsk., Hist.-fil. Medd. 4, 2 [1921]). Cf. also above p. 1045.

[1] For some recent studies in Byzantine Greek treatises and their relation to Islamic astronomy cf. Mogenet [1962], Neugebauer [1960, 1], Pingree [1962], [1964], [1971], [1973].

[2] Cf., e.g., Neugebauer-Van Hoesen, Greek Hor., p. 176ff.

[3] E.g. Proclus, Sphaera (cf. p. 1036); Ammonius, Astrolabe (p. 878); Stephanus (above p. 1050).

[4] Texts: Monac. 287 fol. 117ᵛ–119ᵛ, published CCAG 7, p. 119, 27–122, 15; Vat. gr. 208, fol. 131ʳ, published Neugebauer [1958, 2], p. 243f. Unpublished versions: Scor. R.I. 14 fol. 146 (CCAG 11, 1, p. 7 and A. Revilla, Catál. I, p. 31 No. 11); Laurent. 28. 46 fol. 200ʳ, 20–201ʳ, 19; Paris. Coislin. 338 fol. 194ᵛ (CCAG 8, 2, p. 27). Cf. also Tihon [1973] No. XIV; for Heliodorus above p. 1038ff.

[5] Cf. above p. 151 (2) and (4) and p. 391 (12).

Saturn:	59 years	Venus:	8 years
Jupiter:	83	Mercury:	46
Mars:	79		

but also subdivisions of the synodic motions (arc lengths, times, velocities) in typical Babylonian fashion.[6] Unfortunately the text is so badly transmitted that one cannot go much beyond this general statement.[7]

On the other hand this text also contains remnants of an epicyclic theory from which values of the anomaly are cited that are associated with the intervals of invisibility:

	Ω	Γ		Σ	Ξ
Saturn:	343°	17°	Venus:	347;36°	12;24°
Jupiter:	344	16	Mercury:	322	38
Mars:	318	42			

We discussed in an earlier section[8] the question of how far these parameters resemble Ptolemy's theory of planetary phases. It seems evident that they belong to an earlier stage in the development of epicyclic theory.

Another set of rules which has no parallel in the known material comes from a manuscript in the Escorial,[9] representing a rather chaotic collection of astrological and astronomical topics, some well-known, and some new to us. To the latter group belongs a set of rules for the determination of syzygies connected with lunar eclipses,[10] rules most likely no longer understood by the author of the present compilation.

The "first method" is trivial: one should add 177 days to a previous calendar date (obviously one that is connected with an eclipse). Since $177 = 6 \cdot 29;30$ this rule only implies that eclipses are six months apart.

The "second method" is explained by an example for a year with $N = 13$ as its place in a 19-year cycle,[11] March 13, a date which is $t = 72$ days distant from January 1. We are told to compute

$$5N + t = 65 + 72 = 137, \quad \text{add to it} \quad 137/60 = 2\ 1/5\ 1/12;$$

to this result should be added $6N = 78$ and the total reduced modulo 30:

$$d = 137 + 2\ 1/5\ 1/12 + 78 = 217\ 1/5\ 1/12 \equiv 7\ 1/5\ 1/12 \text{ mod. } 30. \qquad (1)$$

Then d is "the day of the moon in the given month"; if $d = 15$ a lunar eclipse would be certain. We are told furthermore that "the lunar year begins in June, the 19-year cycle ends with Diocletian 51, while year 1 of the cycle begins with January."

To explain these statements and computational rules we note first that Diocletian 51 = A.D. 335/36 and that 335 June 1 = Thoth 1 in the Egyptian calendar.

[6] Cf. above II A 5.
[7] What can be recovered of numerical data is discussed on p. 239f. in my publication [1958, 2].
[8] Above p. 259.
[9] Scor. II.Ψ.17 (written in the 15th cent.); we shall come back to this text in connection with rules for the layout of ephemerides (below p. 1055).
[10] Fol. 44ᵛ, 16–47ʳ, 8 (unpublished).
[11] The text says incorrectly "15-year" cycle, obviously a mixup with the *indictio*.

Thus, what the text calls "lunar year" is an error for Egyptian year, the ordinary calendaric basis for astronomical computations.

The significance of the rules leading to (1) becomes clear when one writes them as a general formula:

$$d \equiv (5N+t) + 1/60\,(5N+t) + 6N = 133/12\,N + 61/60\,t = 11;5N + 1;1\,t \text{ mod. } 30. \quad (2)$$

To multiply a number t of days by 1;1 means to introduce "tithis".[12] Hence the modul 30 is also to be understood in tithis and 30^{τ} represent by definition the length of one mean synodic month. Similarly $11;5^{\tau}$ is the "epact," i.e. the excess of one year over 12 synodic months.[13] The corresponding length of the year is then 12;22,10 mean synodic months, a value very near to the standard Babylonian value of 12;22,8.[14]

Formula (2) therefore tells us how many tithis (or in practice, days) the given day t of year N of the 19-year cycle reaches into a synodic month, provided that our cycle starts with a new moon. This latter condition is indeed satisfied since the first day of January of Diocletian 51 (i.e. 336 Jan. 1) is a new moon date. Thus $d=15$ indicates closeness to a full moon date. There remains, of course, the investigation of the positions of the nodes, a problem not mentioned in our text. Nevertheless one can say that the starting point was well chosen because the preceding opposition, 335 Dec. 16, coincided with a total lunar eclipse.[15] It seems plausible to assume that these rules were composed in the fourth century when predictions on this basis had a fair chance of being correct.

The existence of this method of computing syzygies is not only of interest by its use of tithis but also by its reference to January 1 in contrast to the better known "Alexandrian" style which operates with Alexandrian Thoth 1.[16] The "January style" shows that astronomical tables were also compiled for use in the western half of the empire, a fact not surprising for the fourth century.[17] By that time the easter computus added new interest in the epacts. Consequently the Handy Tables contain a table of epacts, e.g. Vat. gr. 1291 for the years Diocletian 30 to 257 (i.e. from 313 to 541).[18]

A few fragments of notes, probably made in lectures explaining the Handy Tables and related topics, have reached us in a group of manuscripts which usually contain Ptolemy's Introduction to the Handy Tables, followed by the computations for the solar eclipse of 364 June 16, observed by Theon in Alexandria.[19] These notes, fifteen in number, were recently published by A. Tihon[20] who showed that

[12] Cf. above p. 358 and below p. 1070.
[13] Cf. above p. 1047.
[14] Cf. above p. 396 (5 b).
[15] Cf. Ginzel, Kanon, p. 153.
[16] Cf. above p. 967.
[17] Cf. the existence of a Latin introduction *(preceptum)* to the Handy Tables from the year 534 (cf. above p. 970). Cf. also Cassiodorus (Institutiones II, 3, ed. Mynors, p. 156, 3-6) on the ecclesiastic usefulness of the *canones*.
[18] Cf. above p. 978 fol. 47r. The epacts are computed with $r \equiv N-1$ (mod. 19) as is correct for years N of the era Diocletian (cf. above p. 966 (1)).
[19] Cf. above p. 965. Tihon [1973]. p. 49 f. suggests that these computations were the work of a student, not of Theon himself.
[20] Tihon [1973] under the title "Les scolies des Tables Faciles de Ptolémée" although one usually connects a much narrower mearning with the term "scholia".

they were the product of the Alexandrian school, most likely from the turn of the 5th to the 6th century.[21]

The first six notes are of interest in so far as they reveal the existence of some variants in the parameters of Ptolemy's planetary models, not attested elsewhere. As we have seen before,[22] Ptolemy in the lunar theory of the Almagest reckons as 60 units the total of eccentricity and deferent radius (i.e. $OM+MC=e+R=60$) whereas in the Canobic Inscription and in the Planetary Hypotheses R alone is given the length 60. The reason for the norm in the Almagest is, of course, the connection of the new lunar model with its predecessor, the "simple" model, in which the radius of the deferent is always $OC=60$. In the planetary theory, however, Ptolemy had no reason to introduce a norm in which not all deferents have a fixed radius $R=60$. This also holds for Mercury where the center N of the deferent rotates on a small circle of radius e about a fixed point M (such that $OM=2e$).[23] Now the Note V introduces the norm $R+e=60$ which causes the eccentricity to become 2;51 and the radius of the epicycle 21;26 (from Ptolemy's 3 and 22;30, respectively, multiplied by the factor 1,0/1,3).

I see no advantage in this modification of the parameters which is also extended (in Note VI) to the remaining four planets, although without giving the numerical values. Since it is possible that this new norm appears some day in another treatise I give in the following table all values for e and r. Numbers in [ ] are not explicitly given in our texts.

	$R=60$		$R+e=60$	
	e	r	e	r
☾	12;28 or ;30	6;20	10;19 or 10;20	5;15
☿	3	22;30	[2;51,25] ≈ 2;51	[21;25,42] ≈ 21;26
♀	1;15	43;10	[1;13,28]	[42;17, 9]
♂	6	39;30	[5;27,16]	[35;54,32]
♃	2;45	11;30	[2;37,46]	[10;59,46]
♄	3;25	6;30	[3;13,57]	[6; 8,59]

The terminology in these notes is peculiar. For example, the deferent in general position (i.e. not in the apogee of the eccenter) is called "invisible" (ἀφανής) and indeed it is not shown in the drawings which accompany the text;[24] hence the epicycle seems to hang unsupported in space. On the other hand (in the figure to No. VI) the eccenter is called "undivided", in contrast to the "divided" eccenter[25] which has the equant as its center. This terminology is obviously chosen to indicate that angles are reckoned with respect to the equant and not with respect to the midpoint of the deferent.

[21] Tihon [1973]. e.g. p. 106.
[22] Cf. above p. 903 and Fig. 84 there.
[23] Cf. above p. 162 and I, Fig. 147.
[24] Marc. gr. 314 fol. 220ʳ.
[25] διαιρετός/ἀδιαίρετος; only the "divided" circle is shown in the diagram to No. VI.

3. Ephemerides

In using the term "ephemerides" for a large class of cuneiform texts of the Seleucid-Arsacid period we did not strictly follow the modern terminology, although at least some of these texts tabulate the day by day positions of the moon or of a planet. The essential feature that characterizes all Babylonian "ephemerides" is their step by step progress over a certain chronological interval, e.g. by giving a list of the consecutive occurrences of the same planetary phase.

The Greek and demotic "planetary tables" of the Roman imperial period[1] represent a different type of astronomical tables: they are primarily concerned with the dates of entry of the planets into the consecutive zodiacal signs. Thus the more or less constant time intervals of the Babylonian ephemerides are replaced by the emphasis on equidistant longitudinal motion, a point of view which is very convenient for astrological purposes in contrast to the Babylonian lunar ephemerides and the sequences of planetary phases which are of very little astrological interest.

In the latest period of Greek astronomy, however, we return to texts which can be called "ephemerides" even in the narrower sense of the word since they tabulate lunar and planetary positions in day by day, or at least in equidistant, intervals. The purpose of these tables is undoubtedly astrological but in effect we have progress here towards a systematic tabulation of astronomical phenomena, regardless of the use one may make of the results.

The first occurrence of the term "ephemerides", known to me, is found in connection with observations by Heliodorus in A.D. 509 and 510 of near approaches of Mars and Venus to Jupiter[1a]. Also Philoponus, in his treatise on the astrolabe, mentions an "ephemeris" for the sun[1b]. In later periods the term is no longer rare.

Ephemerides must have become a quite common type of astronomical tables because we have from the Byzantine period a short instruction of how an ephemeris should be laid out. Delambre first drew attention to this text[2] which he found in Par. gr. 2394 at the end of Theon's Small Commentary to the Handy Tables. Theon's authorship is doubtful, however, since in Par. gr. suppl. 38 foreign material separates Theon's commentary from the rules for the framework of an ephemeris. Curtis and Robbins published the Greek text of this better version and gave an English translation.[3] Exactly the same text is also found in a codex in the Escorial[4], followed by astrological material and other computations. At the end one finds additional rules about ephemerides[5]: eclipses should be listed at the beginning, but the syzygies for all months should be given at the end – exactly as is the case in the Michigan papyrus. Finally, there is an interesting remark: for planetary ephemerides steps of 10 days suffice for Saturn and Jupiter,

[1] Cf. above V A 1, 2 A.
[1a] Cf. above p. 1039.
[1b] Hase [1839], p. 137; cf. also above V B 3, 7 F.
[2] Delambre, HAA II, p. 635 f.
[3] Curtis-Robbins [1935], p. 82 f.; cf. below p. 1058.
[4] Cod. Scor. II.Ψ.17; cf. CCAG 11, 2, p. 35 F. 55ᵛ (55ᵛ, 2 to 56ʳ, 14). This codex does not contain any material from Theon.
[5] Fol. 58ʳ, 14 to 58ᵛ, 6 (unpublished).

5 days for Mars, 3 days for Venus, and 2 days for Mercury; then it will be easy to find by (linear) interpolation latitudes, stations, and phases.

Another important feature that is associated with this latest type of Greek astronomical tables is their arrangement in pages of a "codex" in contrast to a roll.[6] In our material we have fragments from at least six codices, all either Handy Tables or ephemerides.[7] The fact that astronomical tables were written on papyrus codices as early as around A.D. 200[8] should be of considerable interest in the discussion of the history of the transition from the roll to the "book."

The following is a summary of the ephemerides that are preserved from the second to the fifth century.[9]

1. P. Tebt. 449. Badly preserved, 2nd cent. Recto: endings of lines of first column of unknown contents. Column II: lunar positions from Mekheir (VI) day 1 to 21. Verso: letter.

Of the lunar table only the consecutive day numbers are preserved and the zodiacal signs with occasionally a numeral. The preserved traces suffice to show that the longitudes deviate a little from mean longitudes; hence we are dealing with true positions as is to be expected for an ephemeris.

2. P. Harris 60 and P. Ryl. 526. Fragment from an ephemeris, 3rd cent., perhaps from the same text. P. Harris 60 gives the dates in the Roman calendar, beginning with January. Cf. Neugebauer [1962, 1], p. 385 and 388.

3. P. Heid. Inv. 34. Ephemeris for the years 345/6 to 348/9; from a codex. Published Neugebauer [1956, 2]. Dates are given in the Alexandrian calendar. Lunar positions are listed day by day, planetary positions in 5-day intervals.

In my original publication I did not realize that I was not dealing with obverse and reverse of a papyrus roll but with the remnants of four pages from a codex, the fold being located in the middle of the extant sheet. This misled me to declare the lunar positions incorrect. Only after I had understood that 12 pages on three sheets were missing from the middle section of the signature did it become clear[10] that the first two pages (1 and 2) concerned (solar and) lunar positions for the years 345/6 whereas the last two pages (15 and 16) gave the planetary positions for 348/9.

As usual the text is not too carefully written but one can count on 45 to 47 lines for each column. For a planetary table covering one year in 5-day steps one needs $365/5 = 73$ lines plus 14 headings for the month names and the planet's name, i.e. 87 lines or two columns. If each page had 5 columns one needed 2 pages for the 5 planets. This agrees with the remnants of the pages 15 and 16 (cols. IV ff. of the edition) which begin with Saturn and end with Mercury. For the moon we should expect $365 + 14 = 379$ lines, hence 8 columns. Two columns of solar tables in 5-day intervals would again complete 10 columns or 2 pages, i.e. our pages 1 and 2. Since each year requires four pages we see that the pages 5 to 8 and 9 to 12 for the years 346/7 and 347/8 are missing together with the end of 345/6 and the beginning of 348/9.

[6] Both types use papyrus for their writing material. For the history of the "codex" see, e.g., C.H. Roberts in Proc. of the British Acad. 40 (1954), p. 169–204, though written before evidence from astronomical tables was available (with the exception of P. Mich. Inv. 1454, Curtis-Robbins [1935], p. 82).

[7] Neugebauer [1962, 1] Nos. 12, 17, 23, 29, 34, 45; cf. also No. 27.

[8] The Handy Tables of P. Lond. 1278; cf. above p. 977.

[9] For the papyri and ostraca of the preceding period cf. above V C 2, 2.

[10] Cf. Neugebauer [1962, 1], p. 385, No. 12.

From the preserved fragment of page 1 and from the lunar tables on page 2 one sees that some additional material had been squeezed in ahead of the regular lunar ephemeris. What this additional material consisted of is difficult to say. Only one short section of 12 lines[11] is sufficiently well preserved to see that each line began with $\omega\rho$ ("hour"), some followed by η (thus "day" – but no case for "night"?). The second line has the entry $\tau\eta$ and perhaps $\div$, i.e. zero, thus 308;0 which is reminiscent of an entry "310 minutes" in an ephemeris for 467,[12] indicating maximum lunar latitude (5;10°). The other entries in this section could indicate the moments of entry into a sign since similar entries are found in the ephemeris of 467.[13] It is impossible to say what the remaining two or three columns contained. The end of the lunar tables (from month VIII on) must have spilled over on p. 3 and thus pushed subsequent tables forward.

Burckhardt [1958], in comparing the planetary positions in our ephemeris with longitudes resulting from the use of the Handy Tables (of course in the Theonic version) found excellent general agreement excepting a practically constant difference 1;40° = text – computation. Van der Waerden[14] explained this deviation as resulting from the application of a correction for the vernal point according to a "trepidation" theory, known to us through Theon, which indeed leads to this difference.[15] Hence one can say that the longitudes in this ephemeris are sidereal, accepting a theory of trepidation for the distance between sidereal and tropical zero point.

4. P. Vindob. 29370 and 29370b. Fragment of a lunar ephemeris for A.D. 348, published by Gerstinger-Neugebauer [1962]. An additional small fragment comes from a similar text, probably for the year 341 or 360. The main text belongs to a codex in which 12 pages or 3 sheets are combined for one year. The extant fragment comes from the inner part of the pages 3 and 4 (March and April) and 9 and 10 (September and October).

Each page consists of 8 columns of about 30 lines for the consecutive days in one month. Column I gives the days in the Roman calendar (i.e. with backward count), col. II the julian calendar, col. III the Alexandrian calendar, and IV the lunar dates, beginning with new moon. The columns V and VI give the lunar longitudes, signs, degrees and minutes, VII the longitude of the moon's ascending node; finally VIII gives the character of the day, "good," "bad," "neutral" and a few other entries, e.g. "eclipsing." On the left margin a hook marks every 7th day, corresponding to the weekday of January 1,[16] i.e. a procedure that leads in the Middle Ages to the "*littera dominicalis*."[17]

In the original publication I suggested two possible dates: A.D. 348 and 424. It is not difficult, however, to exclude the second date when one makes use of a remark by Fotheringham[18] saying that the note "eclipsing" means only nearness of the moon to the node. We have three such cases in our fragment:

[11] Headed σελῆνοι [.
[12] Below p. 1058; cf. Curtis-Robbins [1935] col. VI, Sept. 27.
[13] Curtis-Robbins [1935] col. VI.
[14] Burckhardt [1958], p. 86.
[15] This can be checked in our little table above p. 632, n. 10 by linear interpolation.
[16] Friday in this case.
[17] Cf. Ginzel. Handb. III, p. 125 f.
[18] In a short note on P. Mich. Inv. 1454; Classical Review 49 (1935), p. 242.

April 28/29, October 7/8 and 20/22. For the nodes we have only the degrees and minutes, no signs being preserved. The lunar longitudes are also lost but we can easily estimate these for the given dates. Now we make use of the fact that $76=4\cdot 19$ (julian) years increase the (mean) longitude of the moon only by about 3° but decrease the position of the node by almost exactly 30°. Thus to nearly the same lunar position we must obtain two nodal positions one sign apart. Only one of these two cases can bring the moon near to an ascending (↑) or descending (↓) node. Indeed we find

	Text	348	Computed	424	
	Node	Moon	Node	Moon	Node
Apr. 29	0;26	♏ 4	♏ 0;30 ↓	♏ 17	♎ 0;30 ↓
Oct. 8	21;54	♎ 3	♎ 21;54 ↓	♎ 25	♍ 21;54 ↓
Oct. 22	21;10	♈ 14	♈ 21;12 ↑	♈ 19	♓ 21;12 ↑

The zodiacal signs in 348 are correct, in 424 wrong; thus we can now restore the signs for the ascending nodes: ♉0;26 in April 29, ♈21;54 in October 8 and ♈21;10 in October 22.

For three other cases where the moon is near to a node, as we can now establish for 348, we find in column VI (signs of $\lambda_{\text{☾}}$) letters $\pi\rho$ or $\alpha\varphi$ which are obviously abbreviations for "additive" or "subtractive", respectively. The plausible hypothesis that it indicates "ascending" or "descending" latitude near the node is confirmed by computation. Unfortunately the entry $\pi\rho$ is also found (once) in the ephemeris for A.D. 467 but then near a descending node. Since this is the only preserved case in this text we cannot say whether this is a simple scribal error or not.

5. *P. Mich. Inv. 1454.* Fragment of an ephemeris for moon, sun, and planets; from a codex. The two preserved pages cover the days from Sept. 19 to Oct. 31 of A.D. 467. Published by Curtis-Robbins [1935].

This latest text in our papyrological material is an ephemeris in the strict sense of the word. The first two columns give month after month the consecutive days in the Roman and in the Alexandrian calendar, followed by lunar days beginning at new moon. The next three columns (IV to VI) concern the moon; the first two give the longitudes (sign, degrees and minutes), the third one indicates the hour at which the moon crosses the boundary of a sign. Then follow six columns (VII to XII) with longitudes (degrees and minutes) for the sun and the planets (from Saturn to Mercury). The last column (XII) describes the character of the day ("good," "bad," etc.). Outside the first column every seventh day shows a number such that Saturday, August 26 = Epag. 3 would be number 1. The reason for this choice is not clear to me.[19]

Burckhardt [1958] p. 87–92 recomputed all longitudes with the Handy Tables and found excellent agreement without any correction for trepidation.[20]

19 Jan. 1 of 467 is a Sunday. For a motivation of using a Saturday cf. Curtis-Robbins [1935], p. 85.
20 In contrast to the ephemeris for 348/9; cf. above p. 1057.